Group & Phase Velocities (2B)

- 3-D Group & Phase Velocities

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Phase Velocity (1)

At any point in space $\boldsymbol{x}$

$$s(\boldsymbol{x}, t) = A e^{j(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

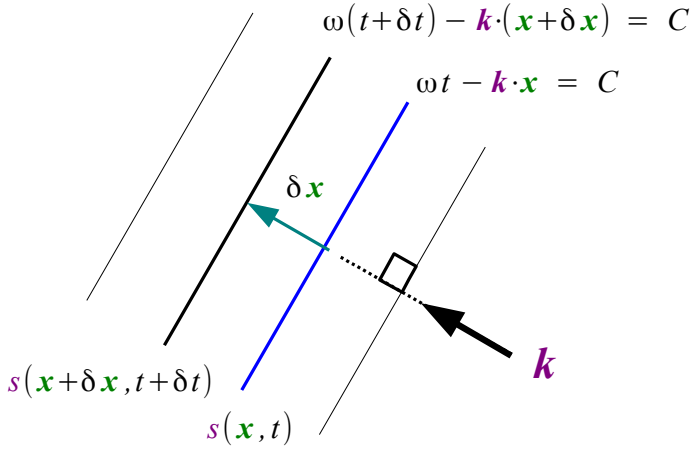
The wave oscillates with a temporal frequency ω

During one period of oscillation $T = \dfrac{2\pi}{\omega}$

In the direction of $\boldsymbol{k}$ (spatial frequency)

The wave propagates forward

By one wavelength $\lambda = \dfrac{2\pi}{|\boldsymbol{k}|}$



Phase Velocity (2)

$$s(\boldsymbol{x}, t) = A e^{j(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

The speed of propagation

The speed at which planes of constant phase $\boldsymbol{k}\cdot\boldsymbol{x} = c$

Phase Velocity

$$|\boldsymbol{v}_p| = \frac{\lambda}{T} = \frac{\omega}{|\boldsymbol{k}|}$$

distance

time

If the directions are the same $\boldsymbol{v}_p$ and $\boldsymbol{k}$

$$\frac{\boldsymbol{k}}{|\boldsymbol{k}|} \cdot |\boldsymbol{v}_p| = \frac{\omega}{|\boldsymbol{k}|} \cdot \frac{\boldsymbol{k}}{|\boldsymbol{k}|}$$

$$\boldsymbol{v}_p = \frac{\omega\,\boldsymbol{k}}{|\boldsymbol{k}|^2}$$

Phase Velocity (2)

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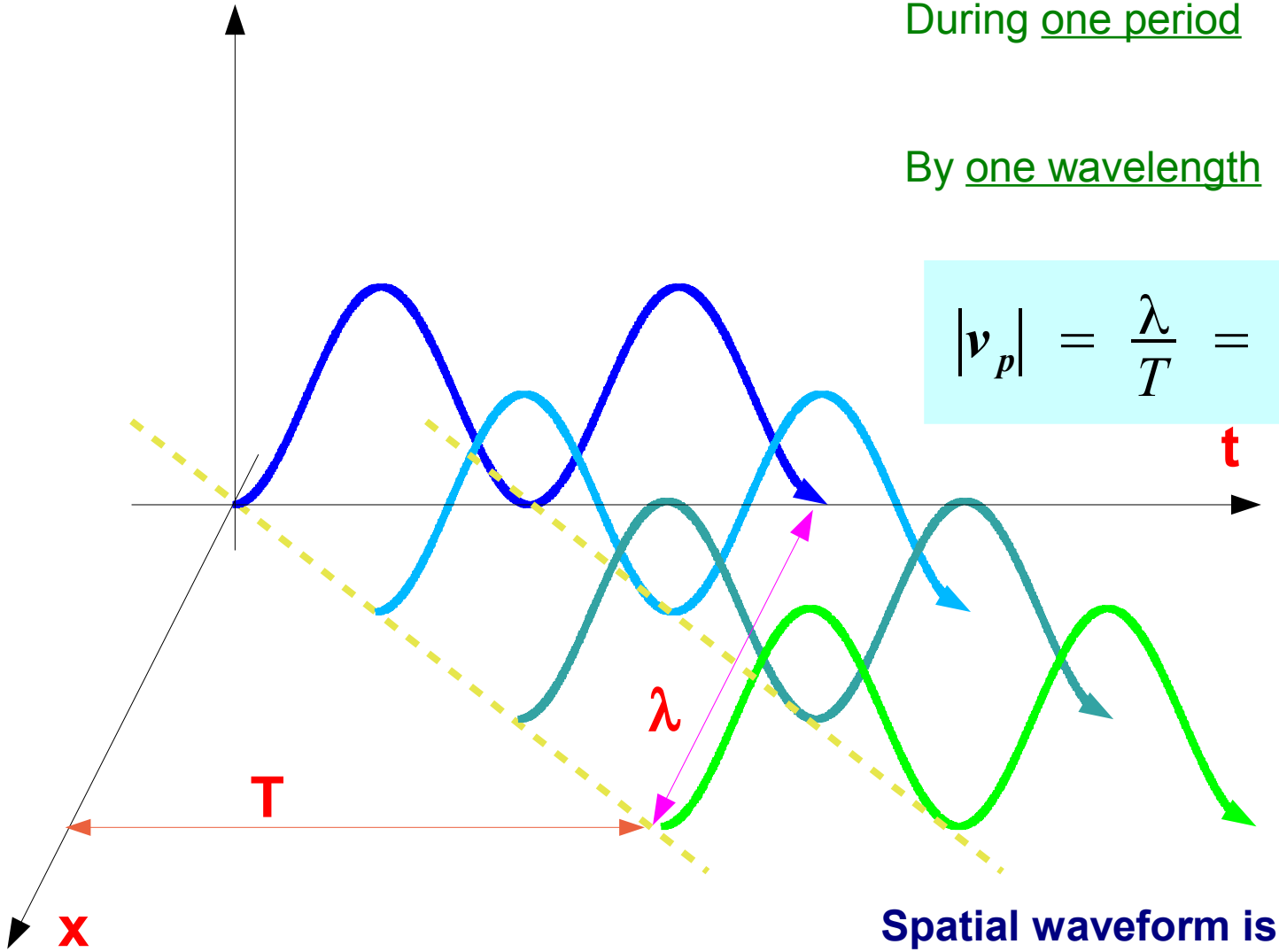
Planes of Constant Phase



During one period $T = \frac{2\pi}{\omega}$

By one wavelength $\lambda = \frac{2\pi}{|k|}$

$$|v_p| = \frac{\lambda}{T} = \frac{\omega}{|k|}$$

distance

time

Spatial waveform is also sinusoid

Acoustic Phonon Dispersion

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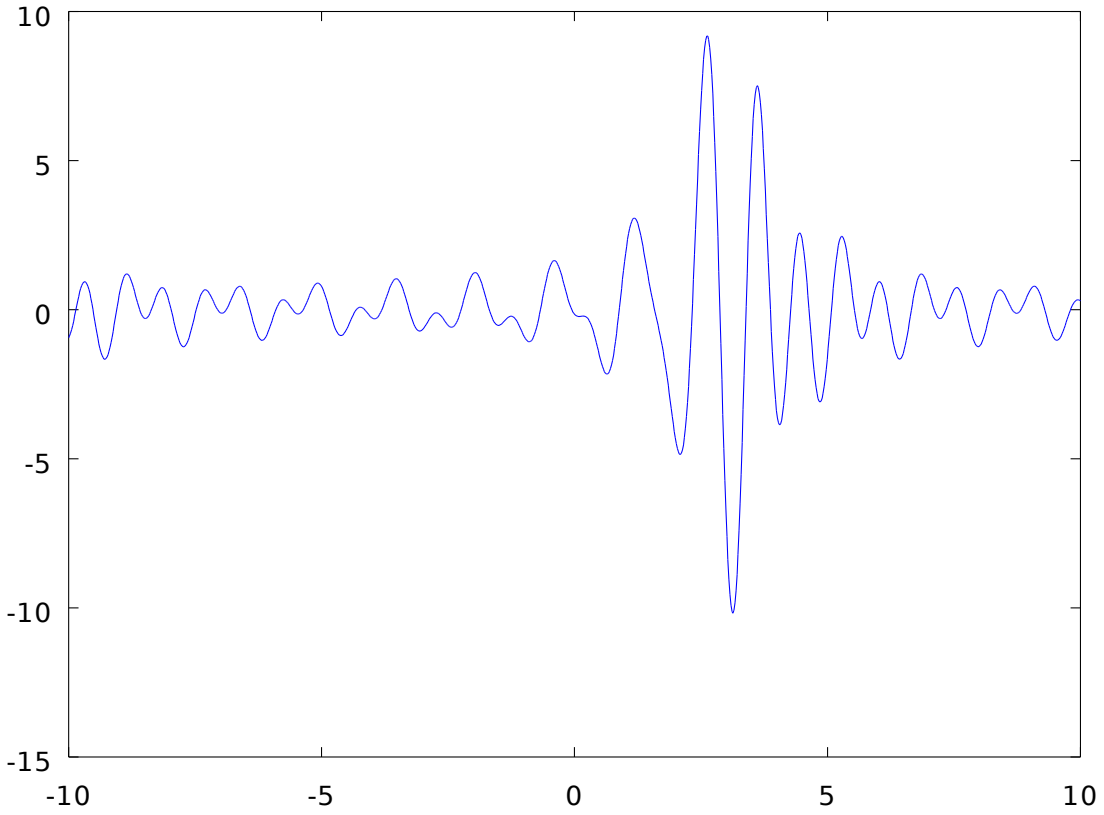


```
x = linspace(-10, +10, 1000);

y = zeros(1, 1000);
for k= 1.0:0.1:2.0
   y = y + cos(4*k*(x-k));
end
plot(x, y);
```

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] http://www.mathpages.com/, Phase, Group, and Signal Velocity
[4] R. Barlow, www.hep.man.ac.uk/u/roger/PHYS10302/lecture15.pdf
[5] P. Hofmann, www.philiphofmann.net/book_material/notes/groupphasevelocity.pdf