# Carry Lookahead Adder (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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# Carry Lookahead Adder

Carry Lookahead Adder

$p_i = a_i \oplus b_i$

$g_i = a_i \wedge b_i$

$c_1 = g_0 + p_0 \wedge c_0$

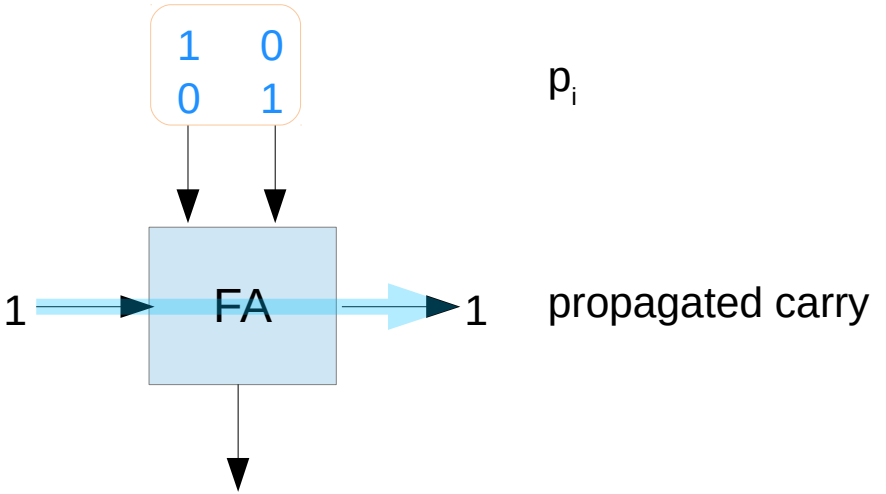
$c_2 = g_1 + p_1 \wedge c_1$

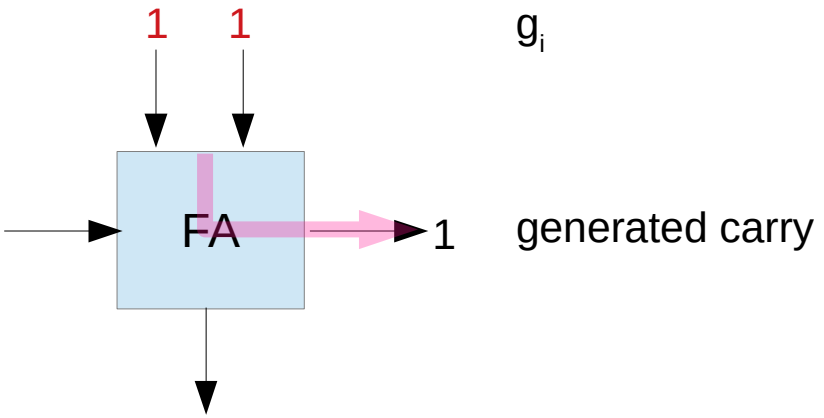
$c_3 = g_2 + p_2 \wedge c_2$

$c_4 = g_3 + p_3 \wedge c_3$

propagated carry

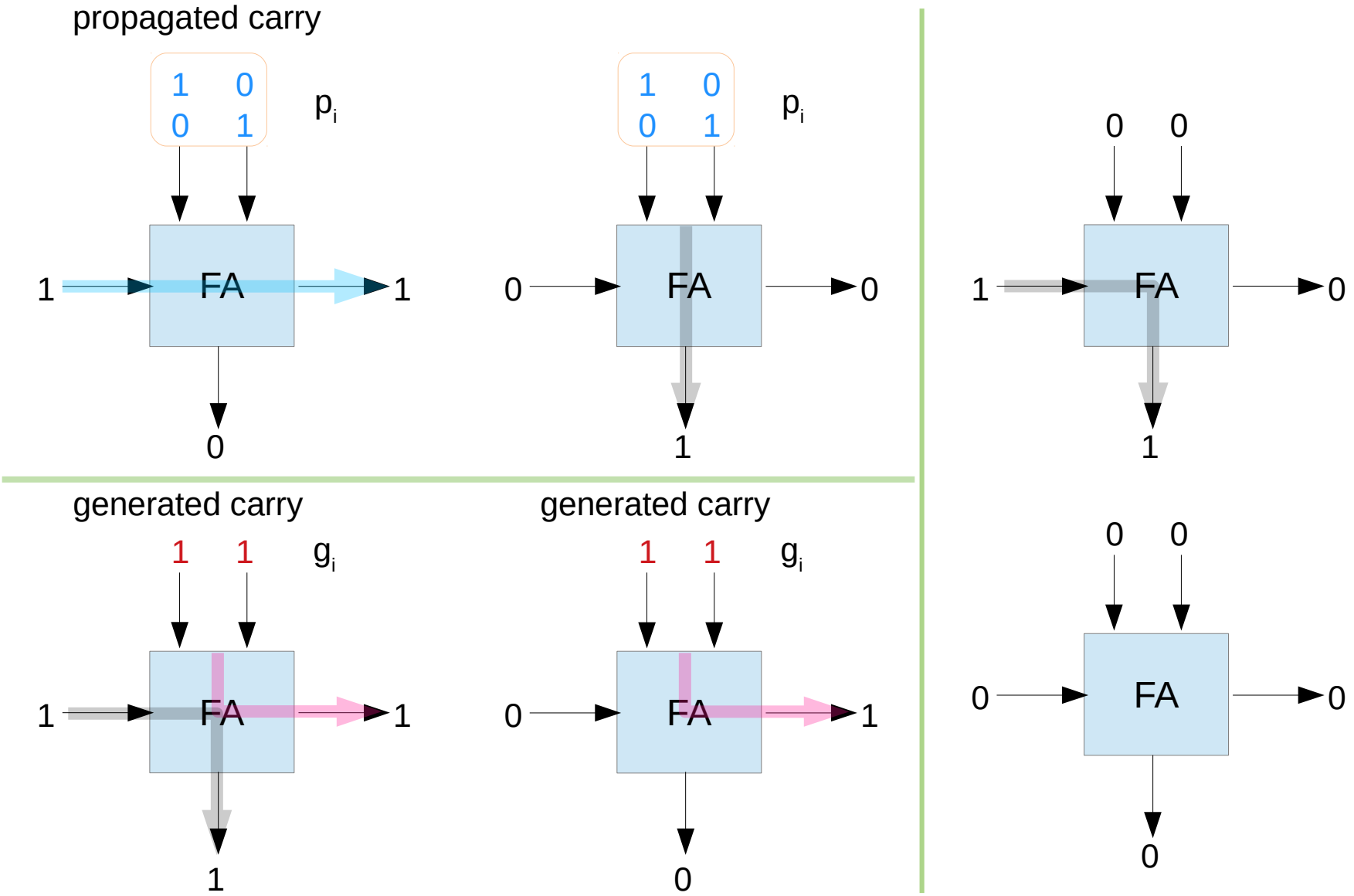
generated carray




https://en.wikipedia.org/wiki/Carry-skip_adder

# Propagated and Generated Carries

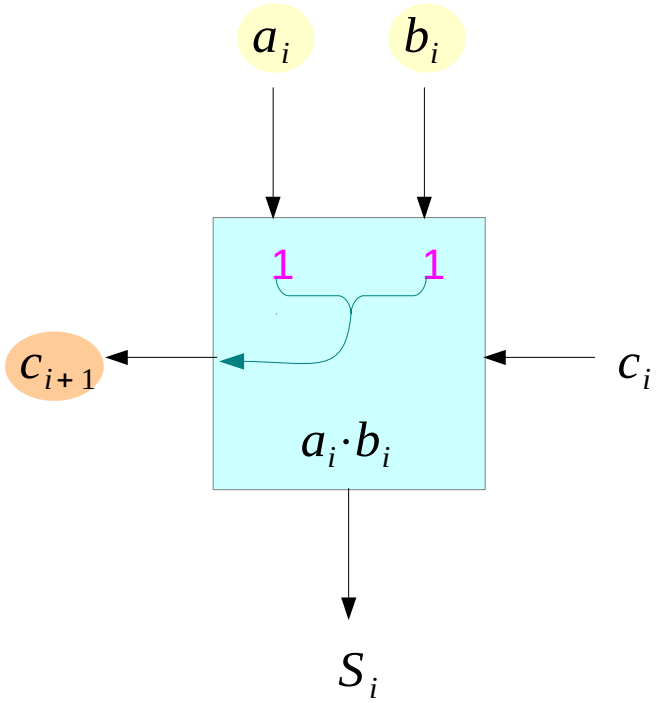


# G and P

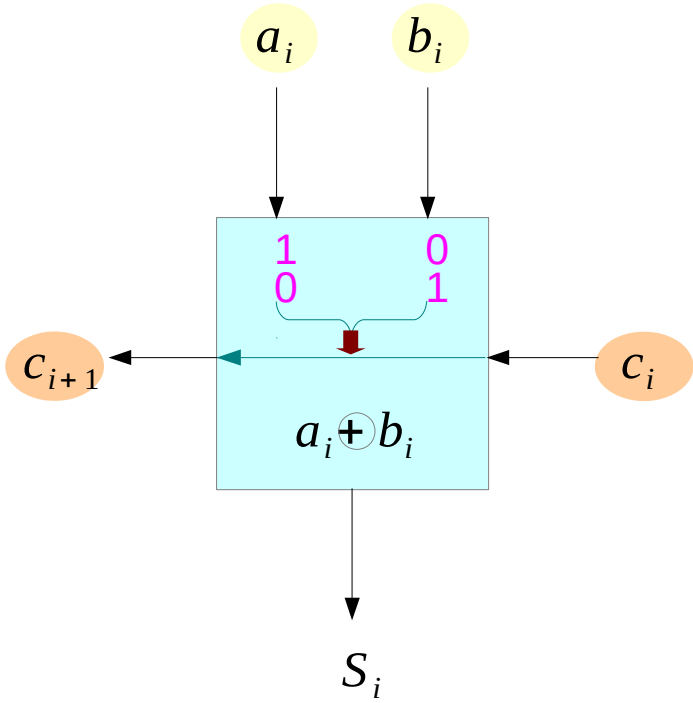
Generate $G_i = a_i \cdot b_i$

Propagate $P_i = a_i \oplus b_i$

$$c_{out} = G_i + P_i c_i$$




Generate $c_{i+1}$




Propagate $c_i$

# Carry Equations

$$c_{i+1} = G_i + P_i c_i$$

$$c_1 = G_0 + P_0 c_0$$
$$c_2 = G_1 + P_1 c_1$$
$$c_3 = G_2 + P_2 c_2$$
$$c_4 = G_3 + P_3 c_3$$

$$c_1 = G_0 + P_0 c_0$$
$$c_2 = G_1 + P_1 \left[ G_0 + P_0 c_0 \right]$$
$$c_3 = G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right]$$
$$c_4 = G_3 + P_3 \left[ G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right] \right]$$

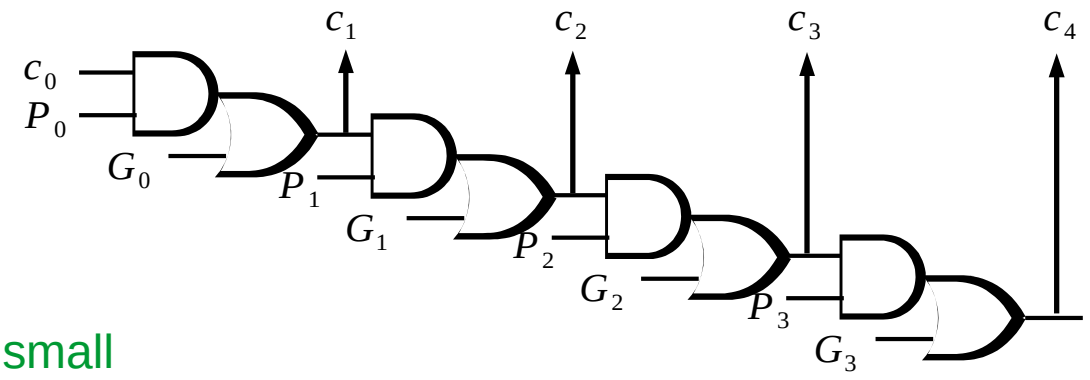
$$G_0 + P_0 c_0 = c_1$$
$$G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$$
$$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$$
$$G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$$

# Carry Equations into Gates (1)

$$c_{i+1} = G_i + P_i c_i$$

$$c_1 = G_0 + P_0 c_0$$
$$c_2 = G_1 + P_1 c_1$$
$$c_3 = G_2 + P_2 c_2$$
$$c_4 = G_3 + P_3 c_3$$

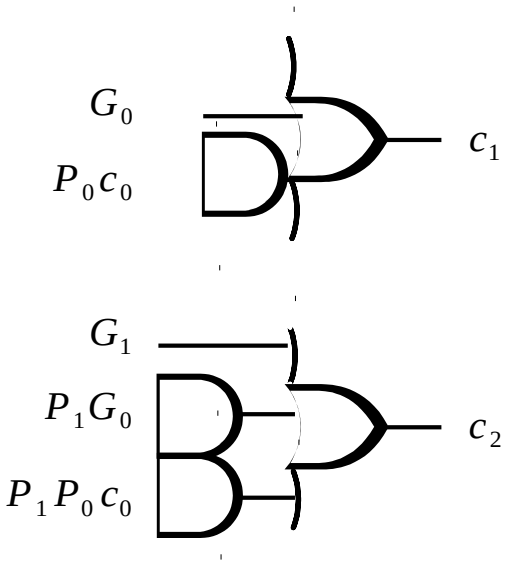
$$c_1 = G_0 + P_0 c_0$$
$$c_2 = G_1 + P_1 \left[ G_0 + P_0 c_0 \right]$$
$$c_3 = G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right]$$
$$c_4 = G_3 + P_3 \left[ G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right] \right]$$

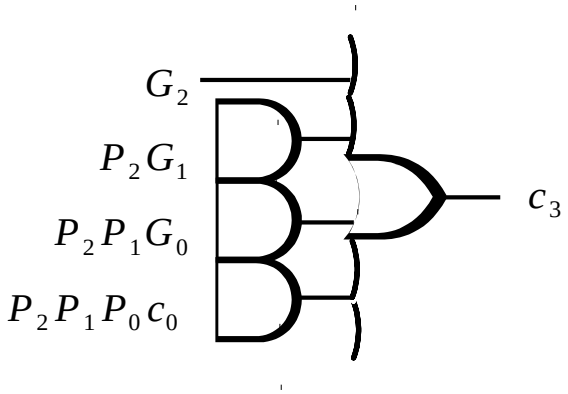



Fan-in number: small
Stage number: large

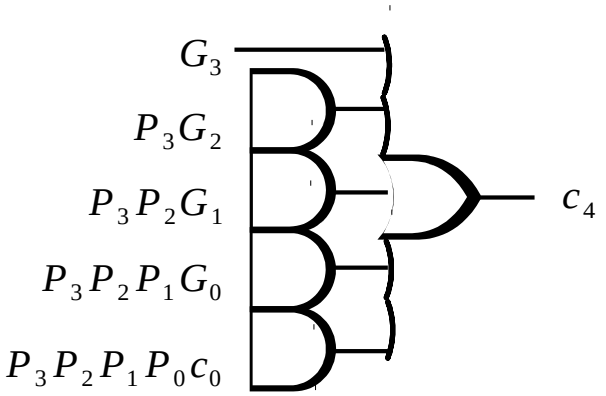
# Carry Equations into Gates (2)

$$G_0 + P_0 c_0 = c_1$$
$$G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$$
$$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$$
$$G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$$




Fan-in number: large
Stage number: small



# Full Carry Lookahead

AND2, OR2 $G_0 + P_0 c_0 = c_1$

AND3, OR3 $G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$

AND4, OR4 $G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$

AND5, OR5 $G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$

AND32, OR32 $= c_{32}$

Large number of fan-in : Impractical

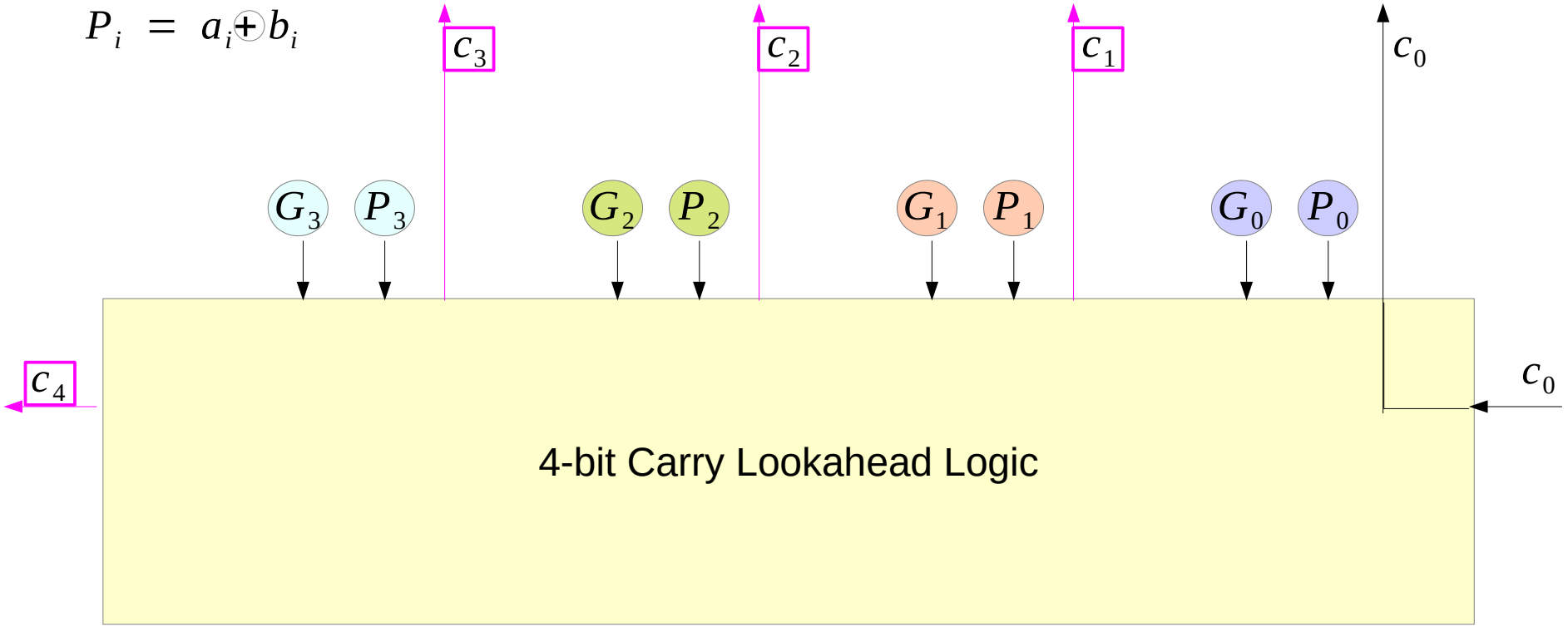
- High Radix Addition (2^g)
- Multi-level Lookahead

# 4-bit Carry Lookahead Logic – interface

All $G_i$'s $P_i$'s are computed simultaneously from $a_i$ and $b_i$

each $c_i$'s takes 2 gate delays

$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$

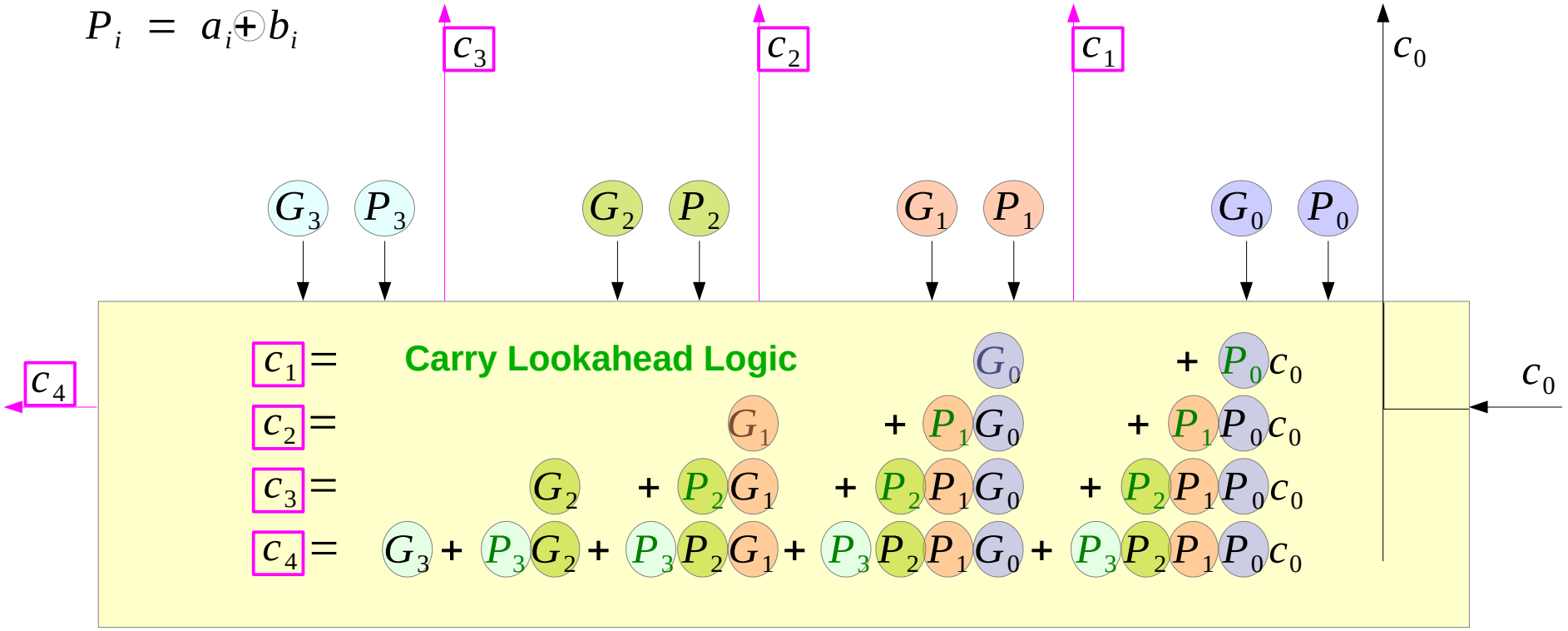


# 4-bit Carry Lookahead Logic – boolean equations

All $G_i$'s $P_i$'s are computed simultaneously from $a_i$ and $b_i$

each $c_i$'s takes 2 gate delays

$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$

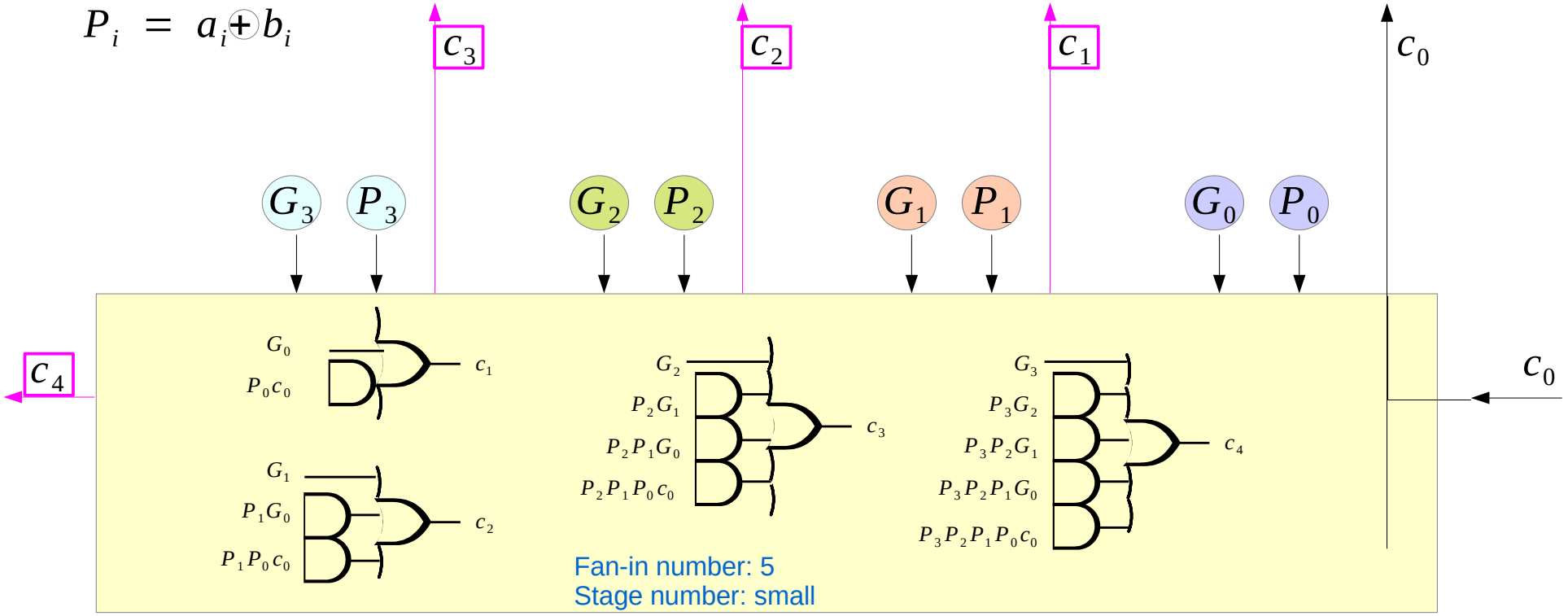


# 4-bit Carry Lookahead Logic – POS

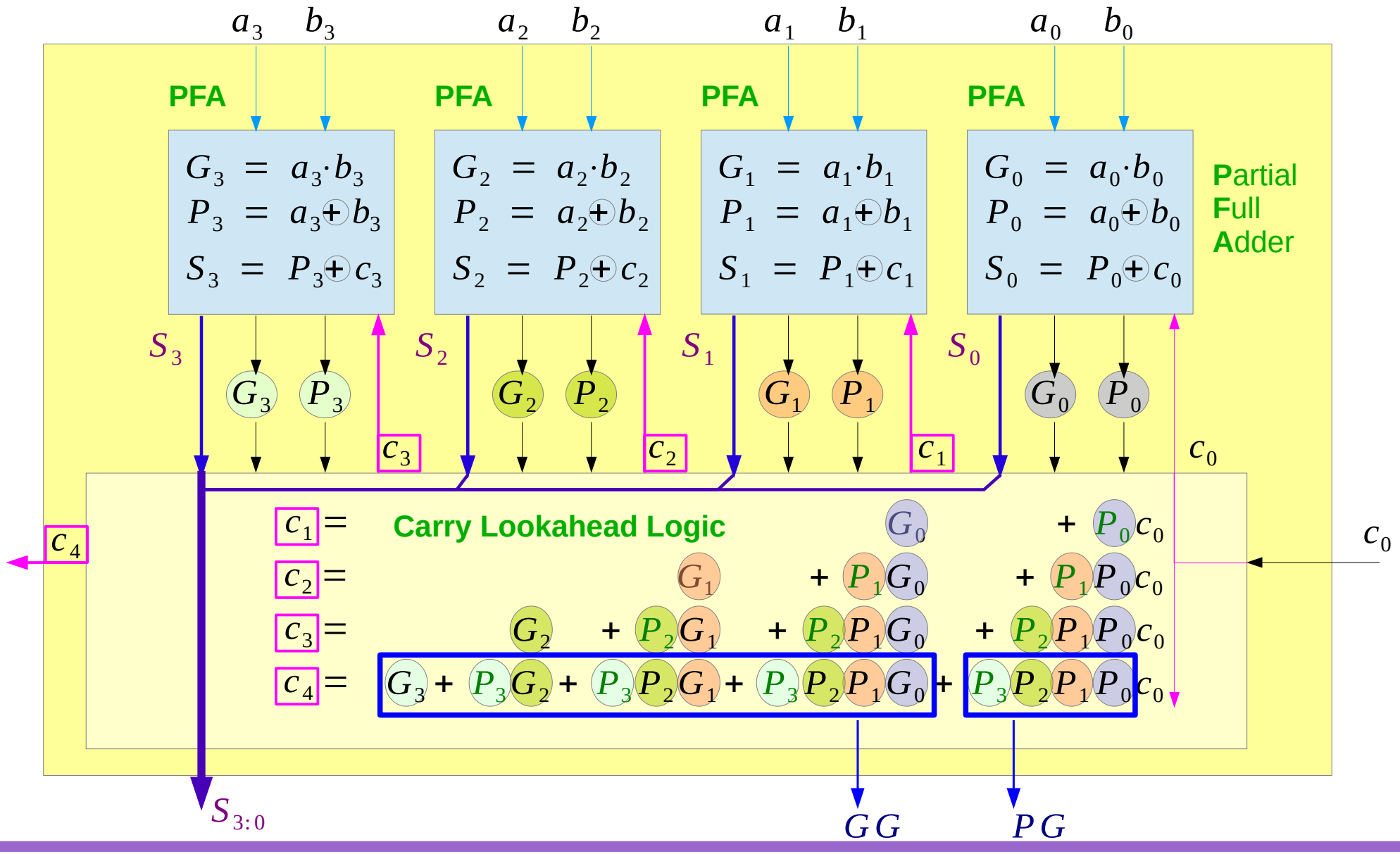
All $G_i$'s $P_i$'s are computed simultaneously from $a_i$ and $b_i$

each $c_i$'s takes 2 gate delays

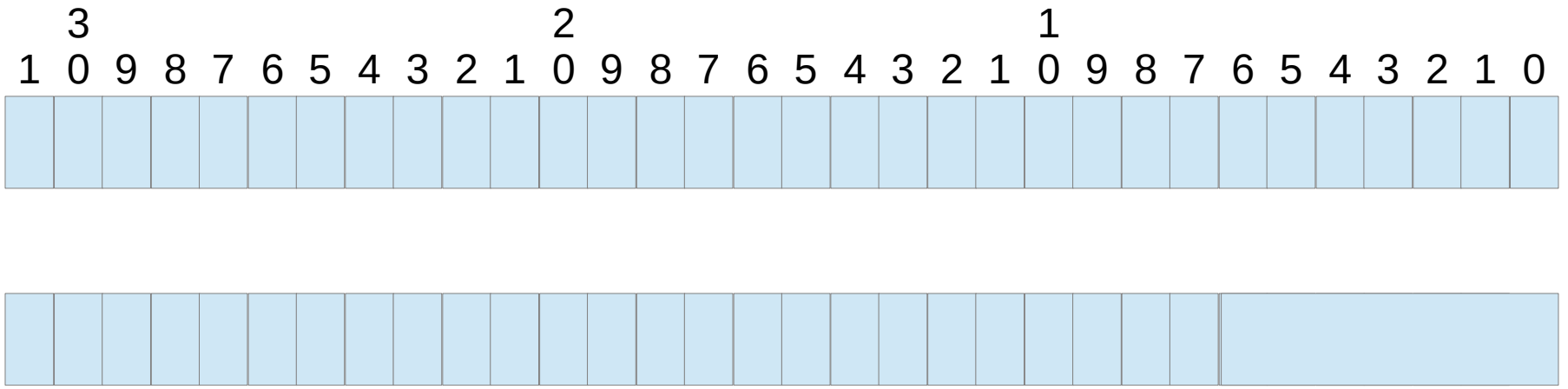
$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$



# 4-bit CLA


$a_3$ $b_3$ $a_2$ $b_2$ $a_1$ $b_1$ $a_0$ $b_0$
PFA
PFA
PFA
PFA
$G_3 = a_3 \cdot b_3$
$P_3 = a_3 \oplus b_3$
$S_3 = P_3 \oplus c_3$
$G_2 = a_2 \cdot b_2$
$P_2 = a_2 \oplus b_2$
$S_2 = P_2 \oplus c_2$
$G_1 = a_1 \cdot b_1$
$P_1 = a_1 \oplus b_1$
$S_1 = P_1 \oplus c_1$
$G_0 = a_0 \cdot b_0$
$P_0 = a_0 \oplus b_0$
$S_0 = P_0 \oplus c_0$
Partial Full Adder
$S_3$ $S_2$ $S_1$ $S_0$
$G_3$ $P_3$ $G_2$ $P_2$ $G_1$ $P_1$ $G_0$ $P_0$
$c_3$ $c_2$ $c_1$ $c_0$
$c_4$
Carry Lookahead Logic
$c_1 = G_0 + P_0 c_0$
$c_2 = G_1 + P_1 G_0 + P_1 P_0 c_0$
$c_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0$
$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0$
$c_0$
$S_{3:0}$
$GG$
$PG$

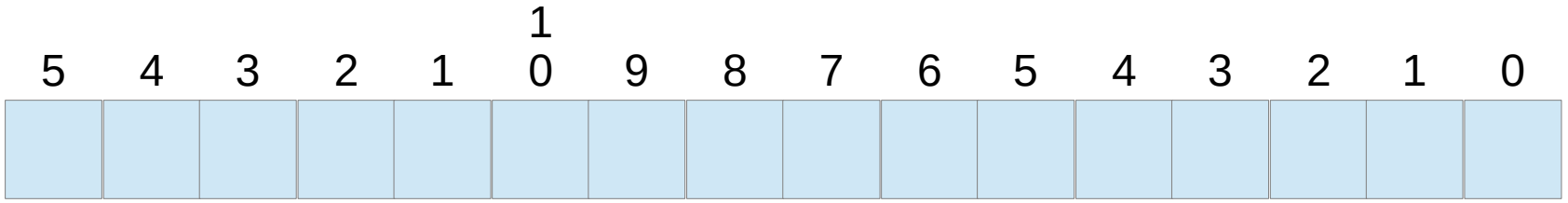
# Multi-level Carry Lookahead




High Radix Addition (2^g)

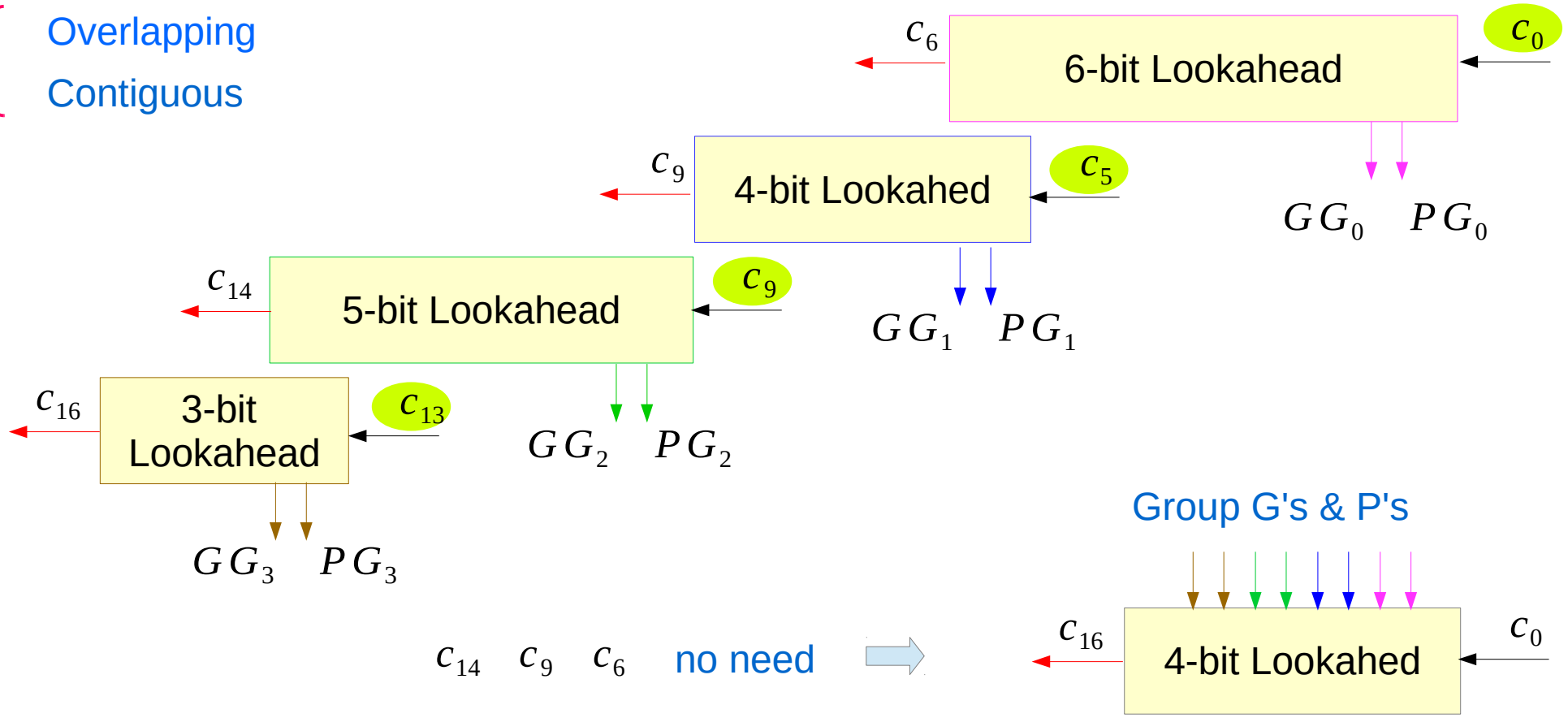
Multi-level Lookahead

# Multi-level Carry Lookahead

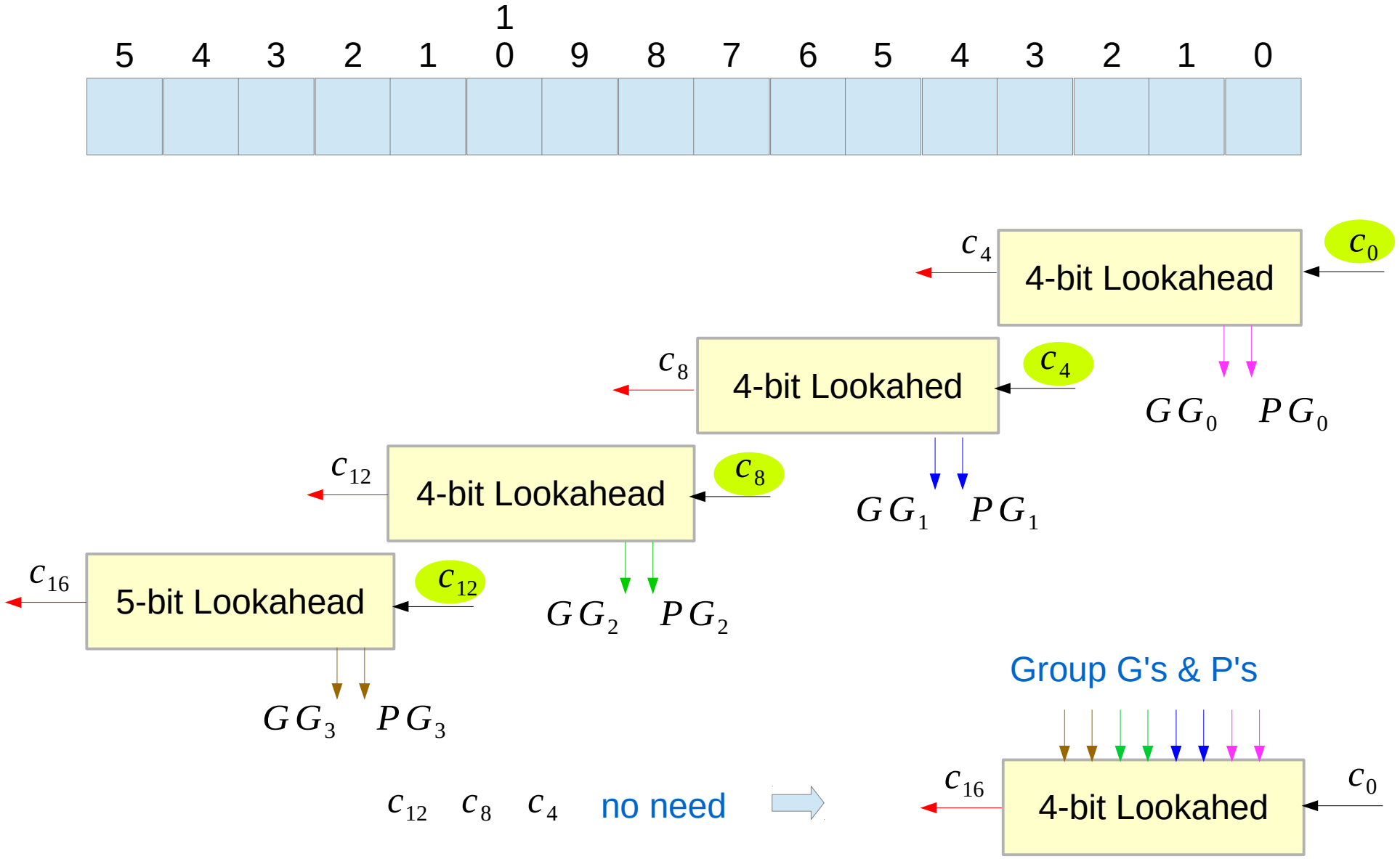



Overlapping

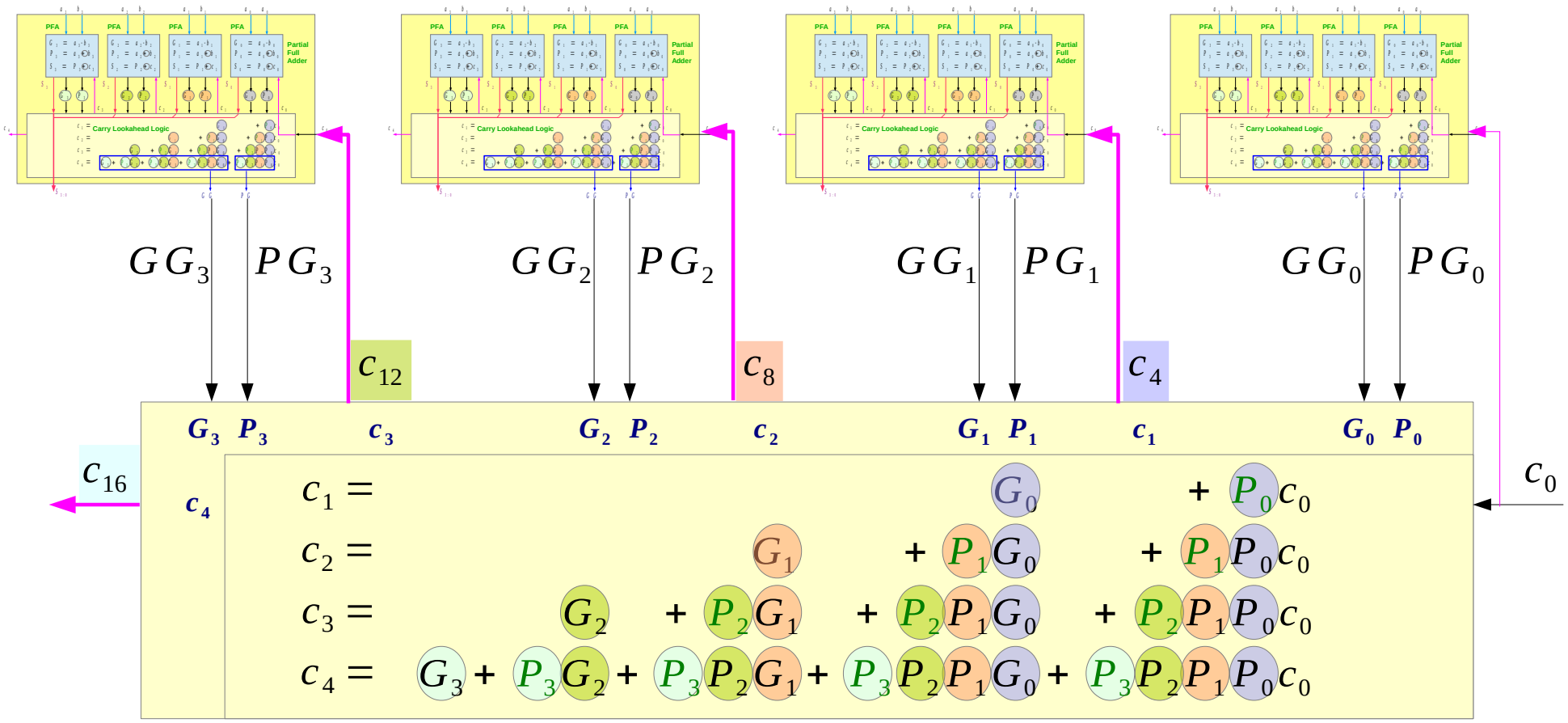
Contiguous



# Contiguous Multi-level Carry Lookahead



# Group Carry Lookahead Logic



# GG and PG (1)

$$c_1 = G_0 + P_0 c_0$$
$$c_2 = G_1 + P_1 \left[ G_0 + P_0 c_0 \right]$$
$$c_3 = G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right]$$
$$c_4 = G_3 + P_3 \left[ G_2 + P_2 \left[ G_1 + P_1 \left[ G_0 + P_0 c_0 \right] \right] \right]$$
$$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = GG_0 + PG_0 c_0$$

$$c_5 = G_4 + P_4 \left[ c_4 \right]$$
$$c_6 = G_5 + P_5 \left[ G_4 + P_4 \left[ c_4 \right] \right]$$
$$c_7 = G_6 + P_6 \left[ G_5 + P_5 \left[ G_4 + P_4 \left[ c_4 \right] \right] \right]$$
$$c_8 = G_7 + P_7 \left[ G_6 + P_6 \left[ G_5 + P_5 \left[ G_4 + P_4 \left[ c_4 \right] \right] \right] \right]$$
$$c_8 = G_7 + P_7 G_6 + P_7 P_6 G_5 + P_7 P_6 P_5 G_4 + P_7 P_6 P_5 P_4 c_4 = GG_1 + PG_1 c_4$$

# GG and PG (2)

$$c_9 = G_8 + P_8 c_8$$

$$c_{10} = G_9 + P_9 \left[ G_8 + P_8 c_8 \right]$$

$$c_{11} = G_{10} + P_{10} \left[ G_9 + P_9 \left[ G_8 + P_8 c_8 \right] \right]$$

$$c_{12} = G_{11} + P_{11} \left[ G_{10} + P_{10} \left[ G_9 + P_9 \left[ G_8 + P_8 c_8 \right] \right] \right]$$

$$c_{12} = G_{11} + P_{11} G_{10} + P_{11} P_{10} G_9 + P_{11} P_1 P_9 G_8 + P_{11} P_{10} P_9 P_8 c_8 = G G_2 + P G_2 c_8$$

$$c_{13} = G_{12} + P_{12} c_{12}$$

$$c_{14} = G_{13} + P_{13} \left[ G_{12} + P_{12} c_{12} \right]$$

$$c_{15} = G_{15} + P_{15} \left[ G_{13} + P_{13} \left[ G_{12} + P_{12} c_{12} \right] \right]$$

$$c_{16} = G_{16} + P_{16} \left[ G_{15} + P_{15} \left[ G_{13} + P_{13} \left[ G_{12} + P_{12} c_{12} \right] \right] \right]$$

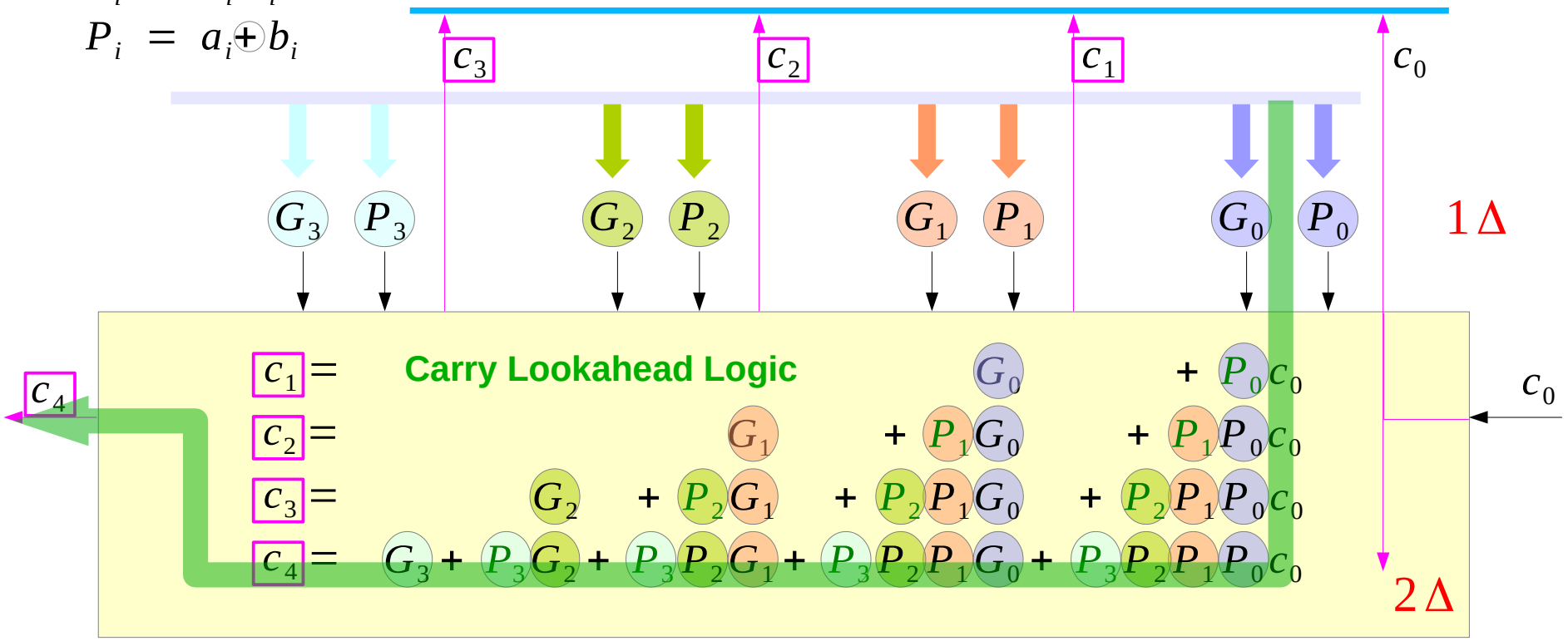
$$c_{16} = G_{16} + P_{16} G_{15} + P_{16} P_{15} G_{14} + P_{16} P_{15} P_{14} G_{13} + P_{16} P_{15} P_{14} P_{13} c_0 = G G_3 + P G_3 c_{12}$$

# Critical Path in the Carry Lookahead Logic

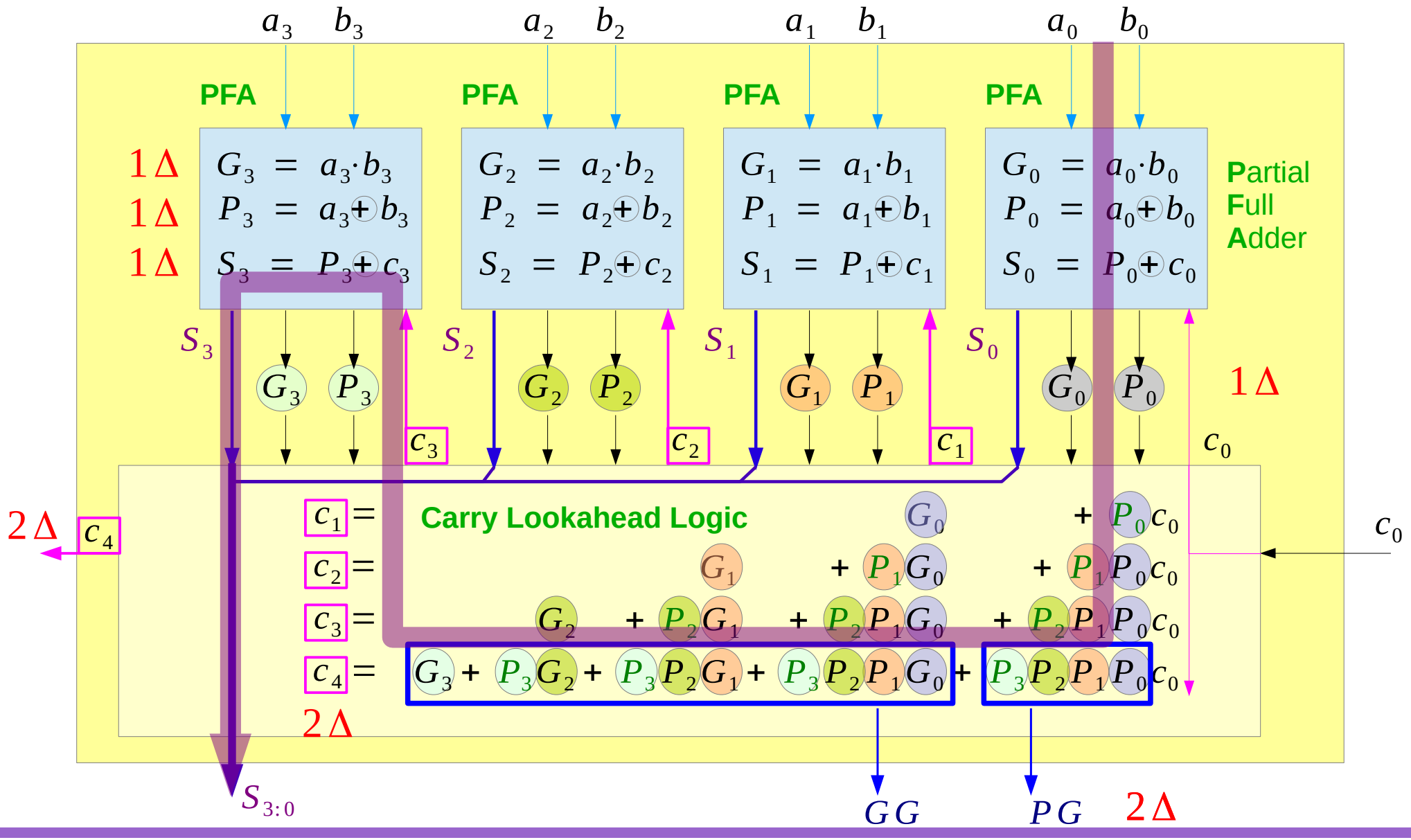
All $G_i$'s $P_i$'s are computed simultaneously from $a_i$ and $b_i$

each $c_i$'s takes 2 gate delays

$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$



# Critical Path in the 4-bit CLA



# Critical Path in the Group Carry Lookahead Logic