Carry Lookahead Adder (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Young W. Lim
11/13/15

G and P

Generate $G_i = a_i \cdot b_i$

Propagate $P_i = a_i \oplus b_i$

$c_{out} = G_i + P_i c_i$
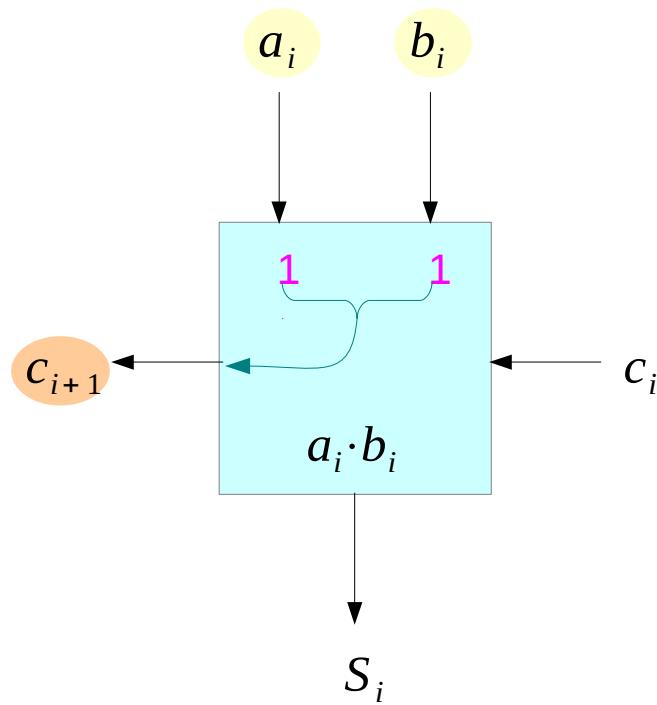
Generate c_{i+1}
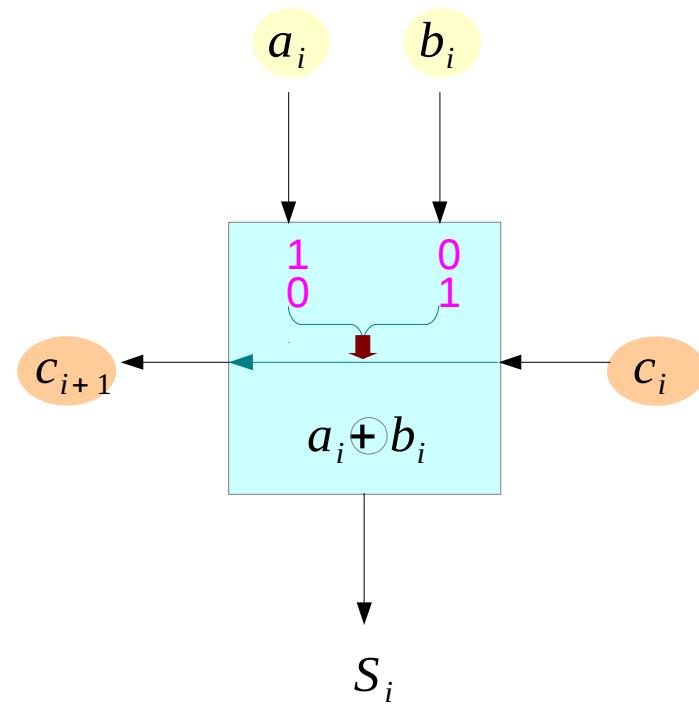
Propagate c_i

Carry Equations

$$c_{i+1} = G_i + P_i c_i$$

$$c_1 = G_0 + P_0 c_0 \qquad\qquad c_1 = G_0 + P_0 c_0$$

$$c_2 = G_1 + P_1 c_1 \qquad\qquad c_2 = G_1 + P_1 \big[G_0 + P_0 c_0 \big]$$

$$c_3 = G_2 + P_2 c_2 \qquad\qquad c_3 = G_2 + P_2 \big[G_1 + P_1 \big[G_0 + P_0 c_0 \big] \big]$$

$$c_4 = G_3 + P_3 c_3 \qquad\qquad c_4 = G_3 + P_3 \big[G_2 + P_2 \big[G_1 + P_1 \big[G_0 + P_0 c_0 \big] \big] \big]$$

$$G_0 + P_0 c_0 = c_1$$

$$G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$$

$$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$$

$$G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$$

Carry Equations into Gates (1)

$$c_{i+1} = G_i + P_i c_i$$

$$c_1 = G_0 + P_0 c_0 \qquad\qquad c_1 = G_0 + P_0 c_0$$

$$c_2 = G_1 + P_1 c_1 \qquad\qquad c_2 = G_1 + P_1 \left[G_0 + P_0 c_0 \right]$$

$$c_3 = G_2 + P_2 c_2 \qquad\qquad c_3 = G_2 + P_2 \left[G_1 + P_1 \left[G_0 + P_0 c_0 \right] \right]$$

$$c_4 = G_3 + P_3 c_3 \qquad\qquad c_4 = G_3 + P_3 \left[G_2 + P_2 \left[G_1 + P_1 \left[G_0 + P_0 c_0 \right] \right] \right]$$
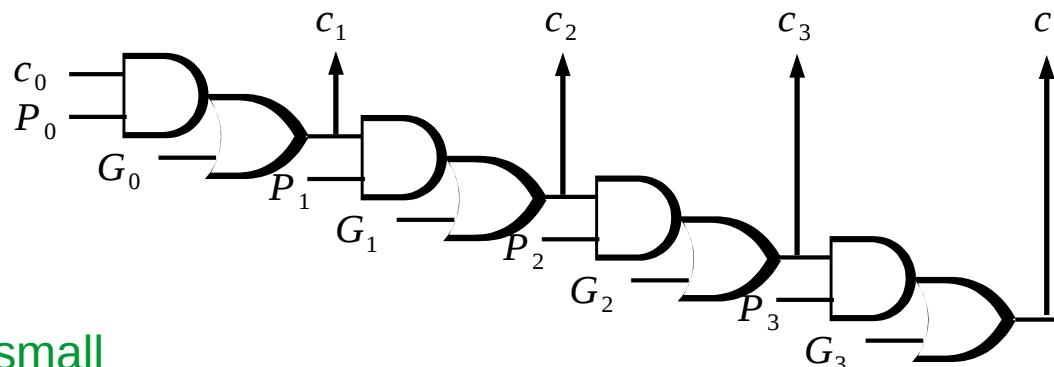
Fan-in number: small
Stage number: large

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$$G_0 + P_0 c_0 = c_1$$

$$G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$$

$$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$$

$$G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$$
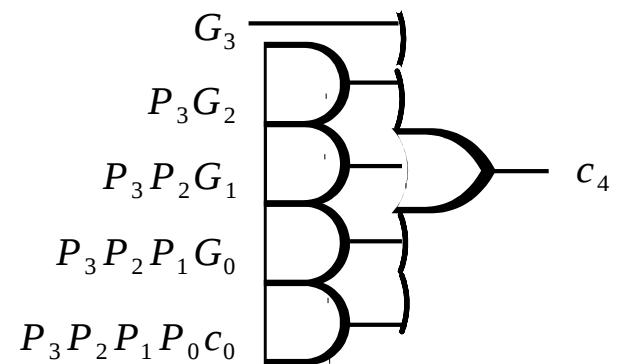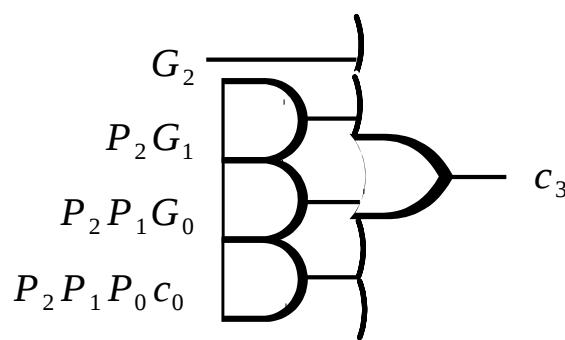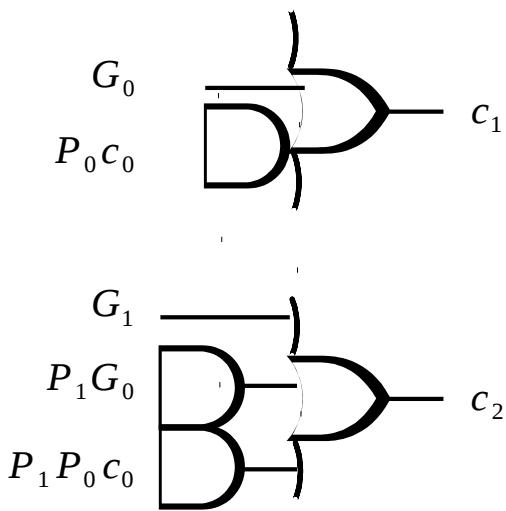
Fan-in number: large
Stage number: small

Full Carry Lookahead

AND2, OR2 $$G_0 + P_0 c_0 = c_1$$

AND3, OR3 $$G_1 + P_1 G_0 + P_1 P_0 c_0 = c_2$$

AND4, OR4 $$G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0 = c_3$$

AND5, OR5 $$G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0 = c_4$$

AND32, OR32 $$= c_{32}$$

Large number of fan-in : Impractical

{ High Radix Addition (2^g)

Multi-level Lookahead

4-bit Carry Lookahead Logic – interface

All G_i's P_i's are computed
<u>simultaneously</u> from a_i and b_i

each c_i's takes 2 gate delays

$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$

c_3 c_2 c_1 c_0

G_3 P_3 G_2 P_2 G_1 P_1 G_0 P_0

c_4 c_0

4-bit Carry Lookahead Logic
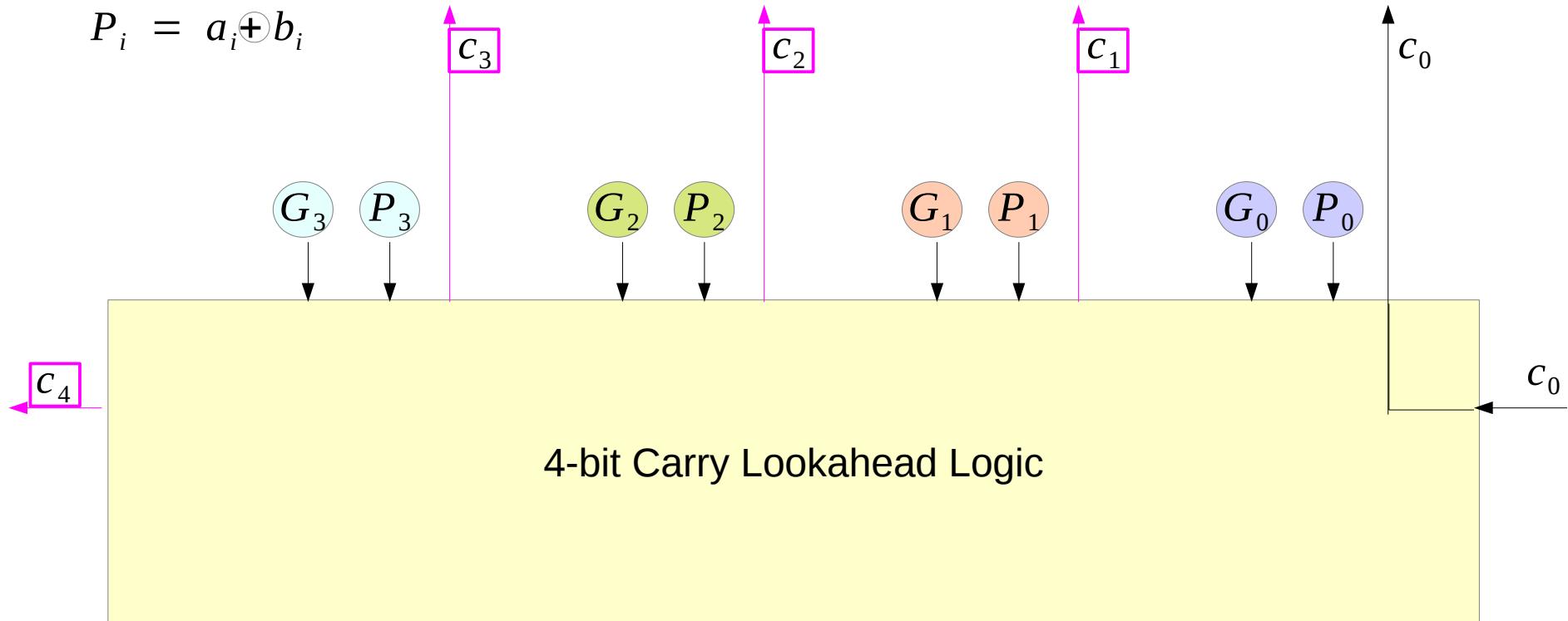
4-bit Carry Lookahead Logic – boolean equations

All G_i's P_i's are computed
simultaneously from a_i and b_i

each c_i's takes 2 gate delays

$$G_i \; = \; a_i \cdot b_i$$
$$P_i \; = \; a_i \oplus b_i$$

c_3

c_2

c_1

c_0

G_3 P_3 G_2 P_2 G_1 P_1 G_0 P_0

Carry Lookahead Logic

c_4

$c_1 = \qquad\qquad\qquad\qquad\qquad G_0 \qquad\qquad + P_0 c_0$

$c_2 = \qquad\qquad\quad G_1 \qquad + P_1 G_0 \qquad + P_1 P_0 c_0$

$c_3 = \qquad G_2 \quad + P_2 G_1 \quad + P_2 P_1 G_0 \quad + P_2 P_1 P_0 c_0$

$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0$

c_0
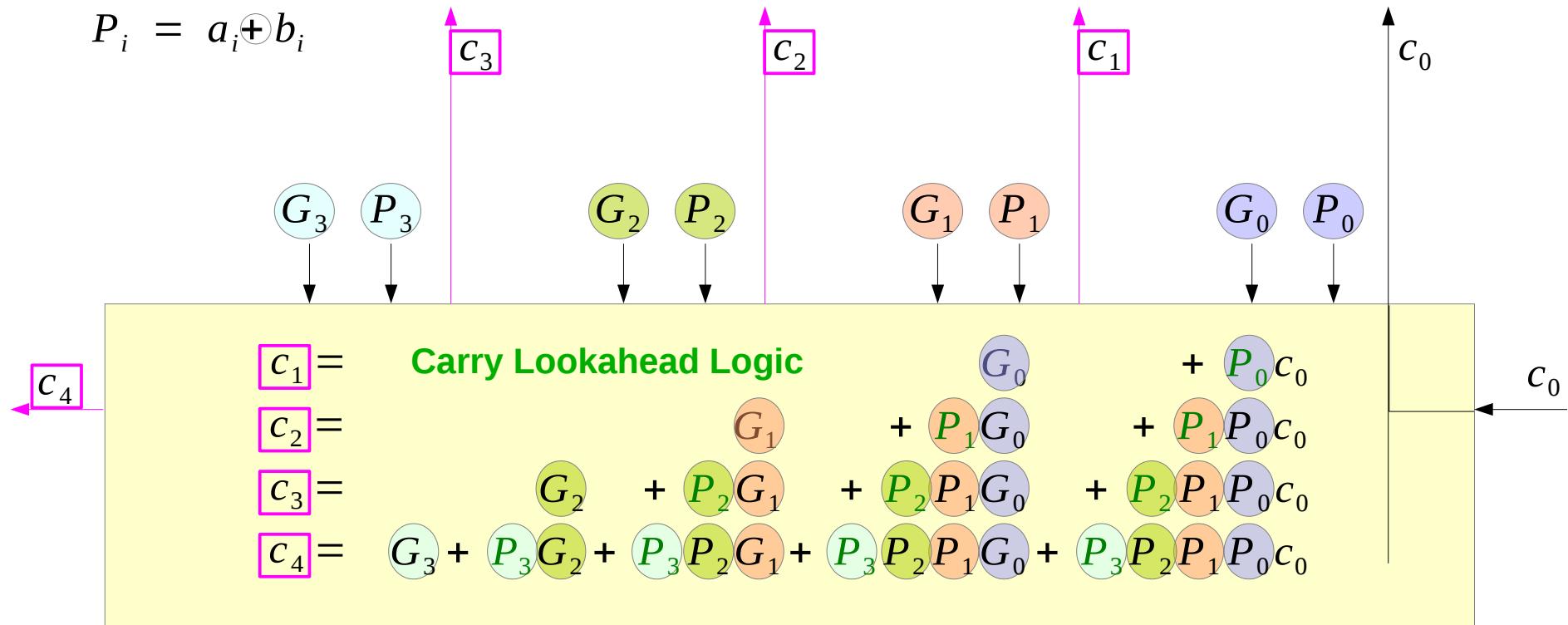
4-bit Carry Lookahead Logic – POS

All G_i's P_i's are computed
<u>simultaneously</u> from a_i and b_i

each c_i's takes 2 gate delays

$$G_i \; = \; a_i \cdot b_i$$
$$P_i \; = \; a_i \oplus b_i$$

Fan-in number: 5
Stage number: small
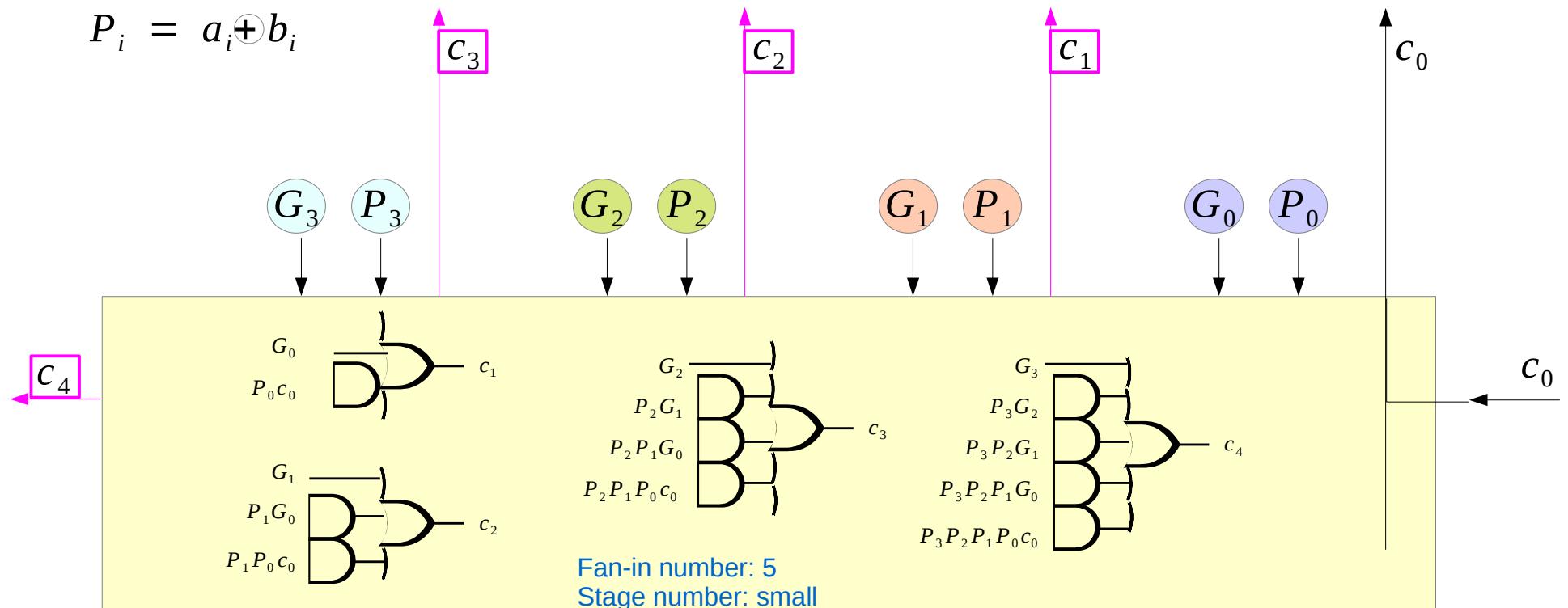
4-bit CLA

Multi-level Carry Lookahead

3											2											1										
1	0	9	8	7	6	5	4	3	2	1	0	9	8	7	6	5	4	3	2	1	0	9	8	7	6	5	4	3	2	1	0
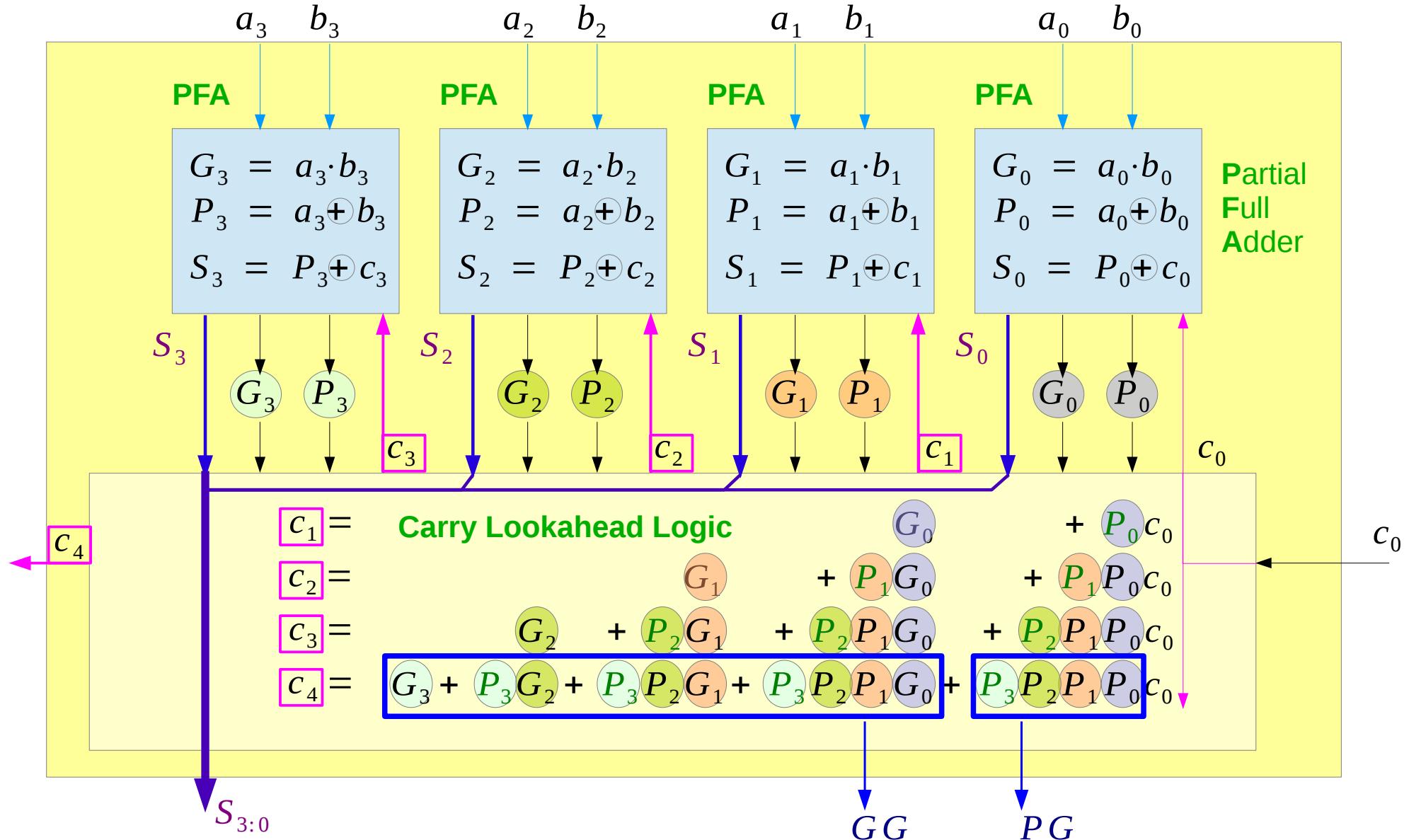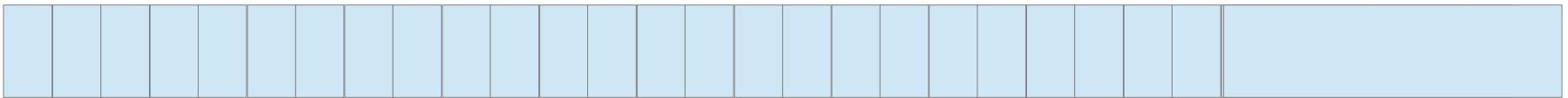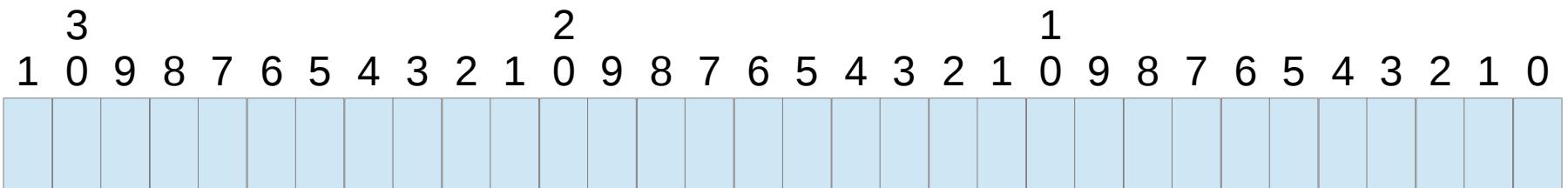
High Radix Addition (2^g)

Multi-level Lookahead

Multi-level Carry Lookahead

1															
5	4	3	2	1	0	9	8	7	6	5	4	3	2	1	0
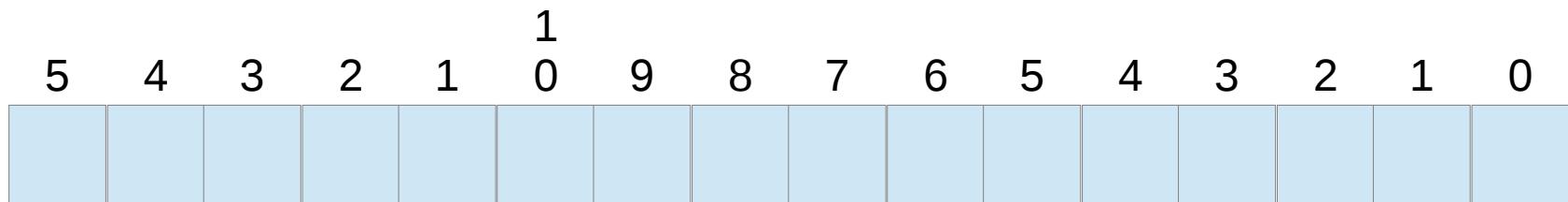
Overlapping

Contiguous

c_6 ← | 6-bit Lookahead | ← c_0

GG_0 PG_0

c_9 ← | 4-bit Lookahed | ← c_5

GG_1 PG_1

c_{14} ← | 5-bit Lookahead | ← c_9

GG_2 PG_2

c_{16} ← | 3-bit Lookahead | ← c_{13}

GG_3 PG_3

Group G's & P's

c_{14} c_9 c_6 no need ⟹ | 4-bit Lookahed | ← c_0 ← c_{16}
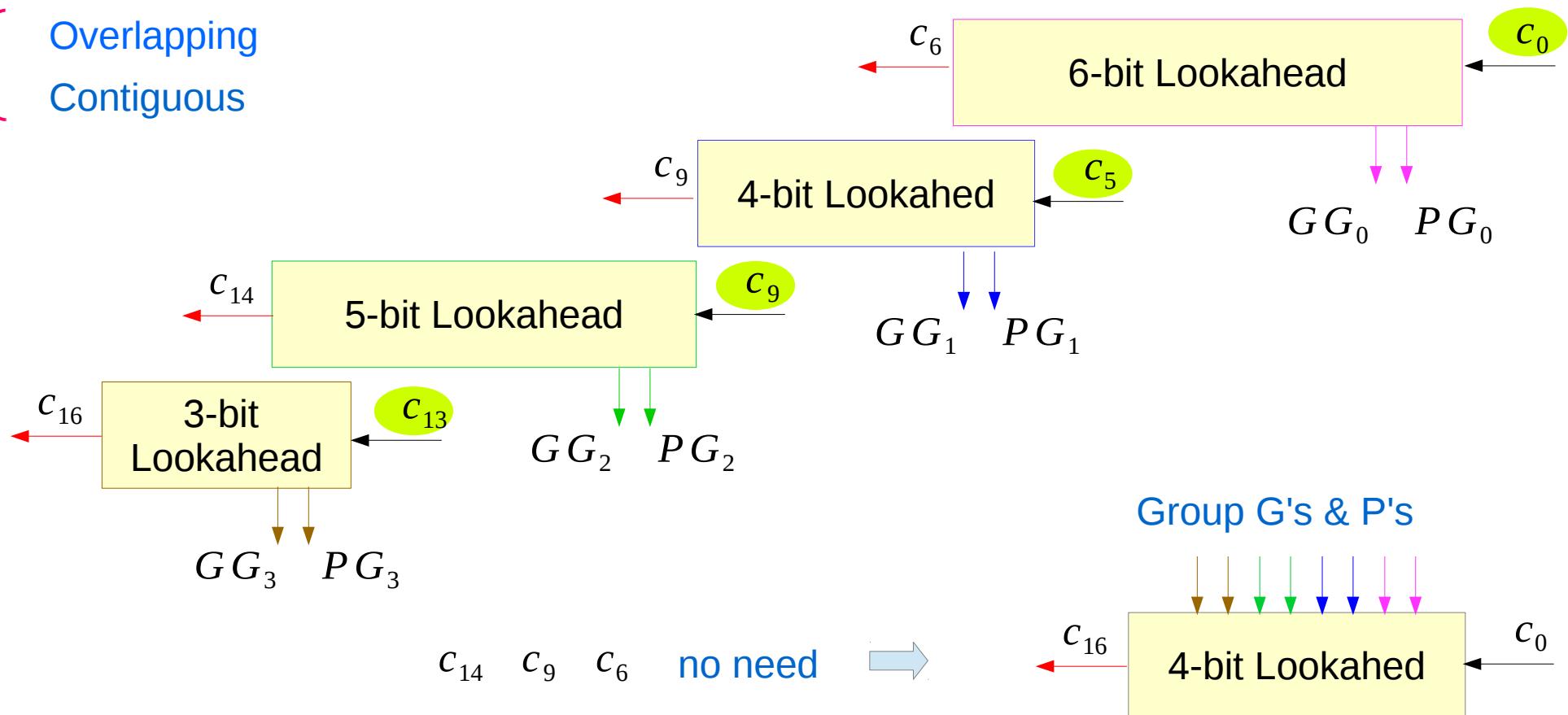
Contiguous Multi-level Carry Lookahead

5	4	3	2	1	1 0	9	8	7	6	5	4	3	2	1	0

c_4 ← 4-bit Lookahead ← c_0

GG_0 PG_0

c_8 ← 4-bit Lookahed ← c_4

GG_1 PG_1

c_{12} ← 4-bit Lookahead ← c_8

GG_2 PG_2

c_{16} ← 5-bit Lookahead ← c_{12}

GG_3 PG_3

c_{12} c_8 c_4 no need ⇒

Group G's & P's

c_{16} ← 4-bit Lookahed ← c_0

Young W. Lim
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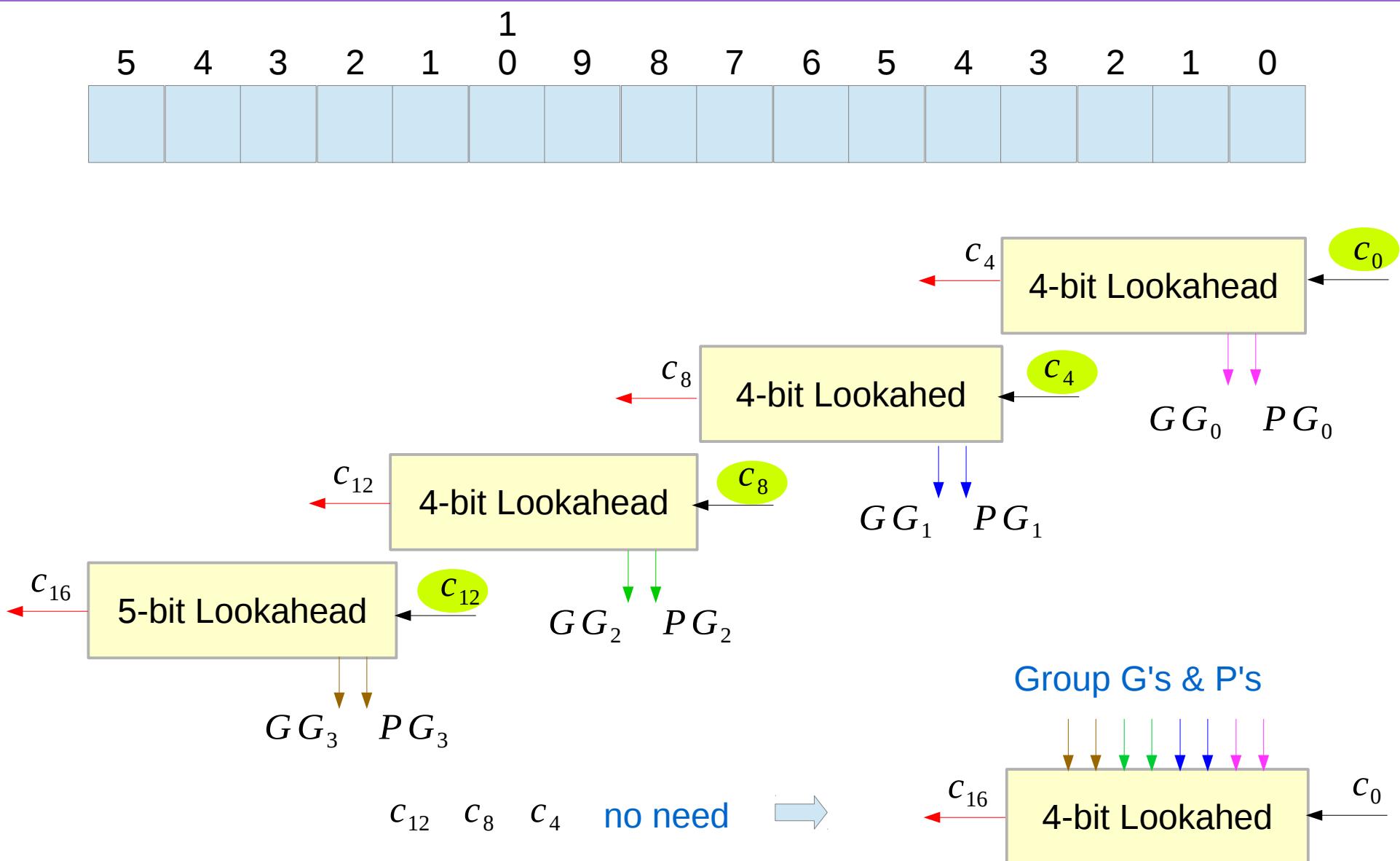
Group Carry Lookahead Logic

$$GG_3 \quad PG_3 \qquad GG_2 \quad PG_2 \qquad GG_1 \quad PG_1 \qquad GG_0 \quad PG_0$$

c_{12} c_8 c_4

c_{16}

$$G_3 \; P_3 \qquad c_3 \qquad G_2 \; P_2 \qquad c_2 \qquad G_1 \; P_1 \qquad c_1 \qquad G_0 \; P_0$$

c_0

$$c_1 = G_0 + P_0 c_0$$

$$c_2 = G_1 + P_1 G_0 + P_1 P_0 c_0$$

$$c_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0$$

$$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0$$
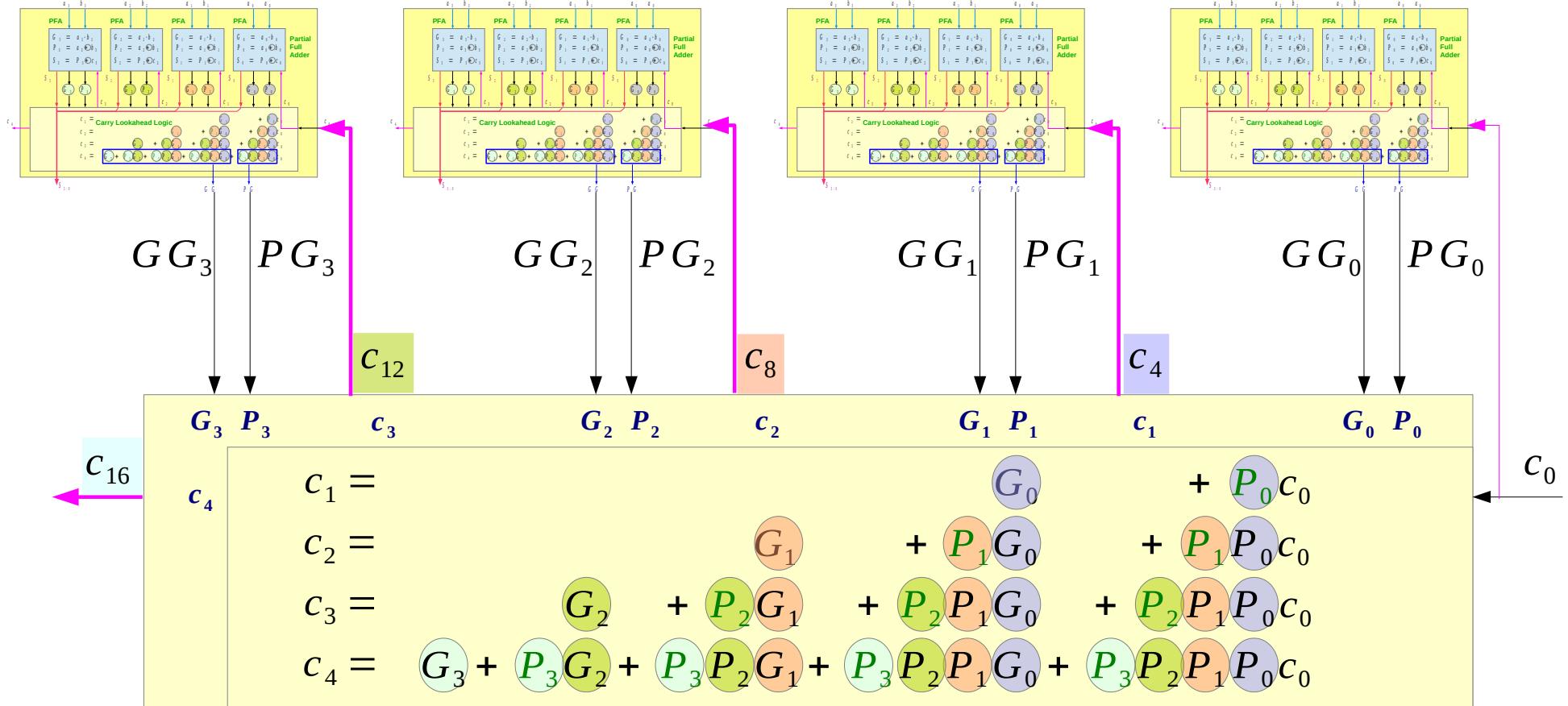
GG and PG (1)

$$c_1 = G_0 + P_0 c_0$$

$$c_2 = G_1 + P_1 \big[G_0 + P_0 c_0 \big]$$

$$c_3 = G_2 + P_2 \big[G_1 + P_1 \big[G_0 + P_0 c_0 \big] \big]$$

$$c_4 = G_3 + P_3 \big[G_2 + P_2 \big[G_1 + P_1 \big[G_0 + P_0 c_0 \big] \big] \big]$$

$$c_4 = \boxed{G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0} + \boxed{P_3 P_2 P_1 P_0} c_0 = \boxed{GG_0} + \boxed{PG_0} c_0$$

$$c_5 = G_4 + P_4 \big[c_4 \big]$$

$$c_6 = G_5 + P_5 \big[G_4 + P_4 \big[c_4 \big] \big]$$

$$c_7 = G_6 + P_6 \big[G_5 + P_5 \big[G_4 + P_4 \big[c_4 \big] \big] \big]$$

$$c_8 = G_7 + P_7 \big[G_6 + P_6 \big[G_5 + P_5 \big[G_4 + P_4 \big[c_4 \big] \big] \big] \big]$$

$$c_8 = \boxed{G_7 + P_7 G_6 + P_7 P_6 G_5 + P_7 P_6 P_5 G_4} + \boxed{P_7 P_6 P_5 P_4} c_4 = \boxed{GG_1} + \boxed{PG_1} c_4$$

GG and PG (2)

$$c_9 = G_8 + P_8 \boxed{c_8}$$

$$c_{10} = G_9 + P_9 \Big[G_8 + P_8 \boxed{c_8} \Big]$$

$$c_{11} = G_{10} + P_{10} \Big[G_9 + P_9 \big[G_8 + P_8 \boxed{c_8} \big] \Big]$$

$$c_{12} = G_{11} + P_{11} \Big[G_{10} + P_{10} \big[G_9 + P_9 [G_8 + P_8 \boxed{c_8}] \big] \Big]$$

$$c_{12} = \boxed{G_{11} + P_{11} G_{10} + P_{11} P_{10} G_9 + P_{11} P_1 P_9 G_8} + \boxed{P_{11} P_{10} P_9 P_8} c_8 = \boxed{GG_2} + \boxed{PG_2} c_8$$

$$c_{13} = G_{12} + P_{12} \boxed{c_{12}}$$

$$c_{14} = G_{13} + P_{13} \Big[G_{12} + P_{12} \boxed{c_{12}} \Big]$$

$$c_{15} = G_{15} + P_{15} \Big[G_{13} + P_{13} \big[G_{12} + P_{12} \boxed{c_{12}} \big] \Big]$$

$$c_{16} = G_{16} + P_{16} \Big[G_{15} + P_{15} \big[G_{13} + P_{13} [G_{12} + P_{12} \boxed{c_{12}}] \big] \Big]$$

$$c_{16} = \boxed{G_{16} + P_{16} G_{15} + P_{16} P_{15} G_{14} + P_{16} P_{15} P_{14} G_{13}} + \boxed{P_{16} P_{15} P_{14} P_{13}} c_0 = \boxed{GG_3} + \boxed{PG_3} c_{12}$$

Critical Path in the Carry Lookahead Logic

All G_i's P_i's are computed simultaneously from a_i and b_i

each c_i's takes 2 gate delays

$$G_i = a_i \cdot b_i$$
$$P_i = a_i \oplus b_i$$

c_3 c_2 c_1 c_0

G_3 P_3 G_2 P_2 G_1 P_1 G_0 P_0

1Δ

c_4

$c_1 =$ **Carry Lookahead Logic** G_0 $+ P_0 c_0$

$c_2 =$ G_1 $+ P_1 G_0$ $+ P_1 P_0 c_0$

$c_3 =$ G_2 $+ P_2 G_1$ $+ P_2 P_1 G_0$ $+ P_2 P_1 P_0 c_0$

$c_4 =$ $G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0$

c_0

2Δ
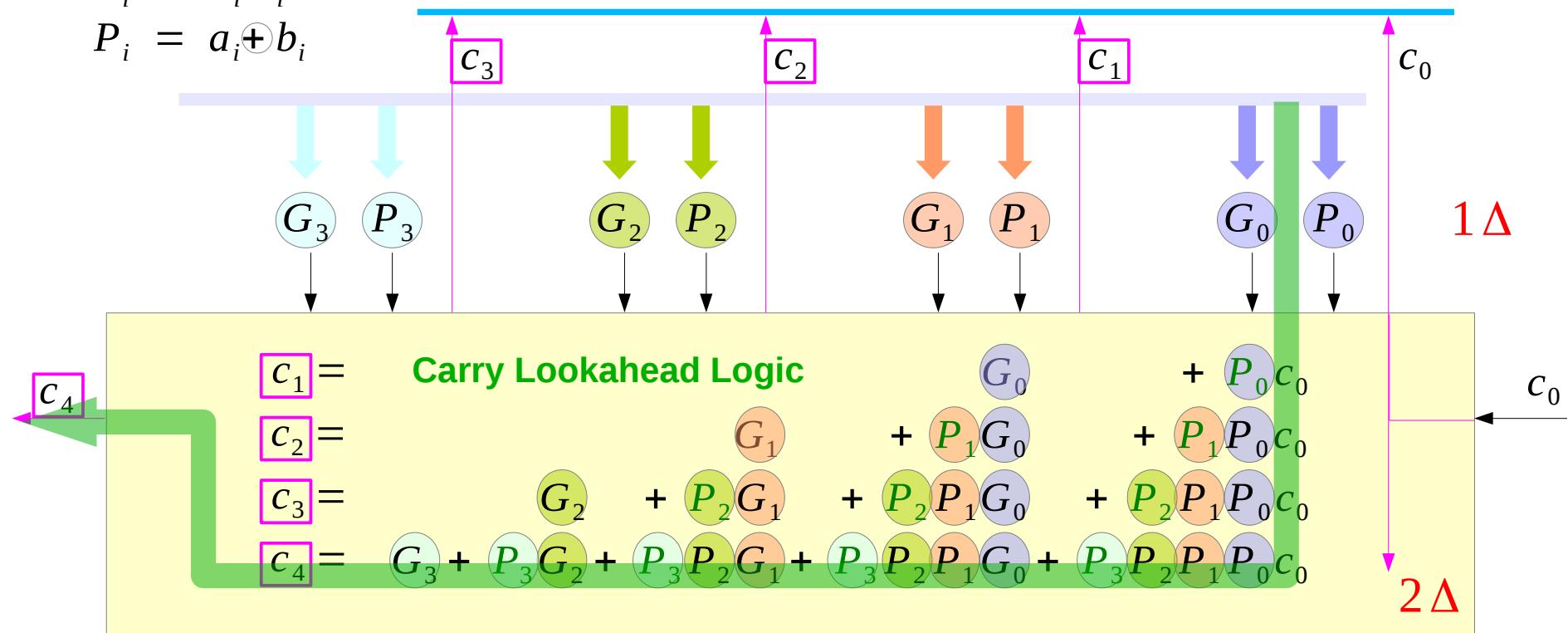
Critical Path in the 4-bit CLA

a_3 b_3 a_2 b_2 a_1 b_1 a_0 b_0

PFA   **PFA**   **PFA**   **PFA**

1Δ $G_3 = a_3 \cdot b_3$ $G_2 = a_2 \cdot b_2$ $G_1 = a_1 \cdot b_1$ $G_0 = a_0 \cdot b_0$ **P**artial

1Δ $P_3 = a_3 \oplus b_3$ $P_2 = a_2 \oplus b_2$ $P_1 = a_1 \oplus b_1$ $P_0 = a_0 \oplus b_0$ **F**ull

1Δ $S_3 = P_3 \oplus c_3$ $S_2 = P_2 \oplus c_2$ $S_1 = P_1 \oplus c_1$ $S_0 = P_0 \oplus c_0$ **A**dder

S_3 S_2 S_1 S_0

G_3 P_3 G_2 P_2 G_1 P_1 G_0 P_0 1Δ

c_3 c_2 c_1 c_0

2Δ c_4

Carry Lookahead Logic

$c_1 =$ G_0 $+$ $P_0 c_0$ c_0

$c_2 =$ G_1 $+$ $P_1 G_0$ $+$ $P_1 P_0 c_0$

$c_3 =$ G_2 $+$ $P_2 G_1$ $+$ $P_2 P_1 G_0$ $+$ $P_2 P_1 P_0 c_0$

$c_4 = G_3$ $+$ $P_3 G_2$ $+$ $P_3 P_2 G_1$ $+$ $P_3 P_2 P_1 G_0$ $+$ $P_3 P_2 P_1 P_0 c_0$

2Δ

$S_{3:0}$

GG PG 2Δ

$$c_1 = G_0 + P_0 c_0$$

$$c_2 = G_1 + P_1 G_0 + P_1 P_0 c_0$$

$$c_3 = G_2 + P_2 G_1 + P_2 P_1 G_0 + P_2 P_1 P_0 c_0$$

$$c_4 = G_3 + P_3 G_2 + P_3 P_2 G_1 + P_3 P_2 P_1 G_0 + P_3 P_2 P_1 P_0 c_0$$