

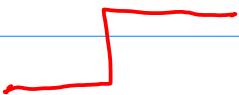
Time Responses (H.1) System Types

20150529

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$$\text{Step fn } u(t) \iff \frac{1}{s}$$

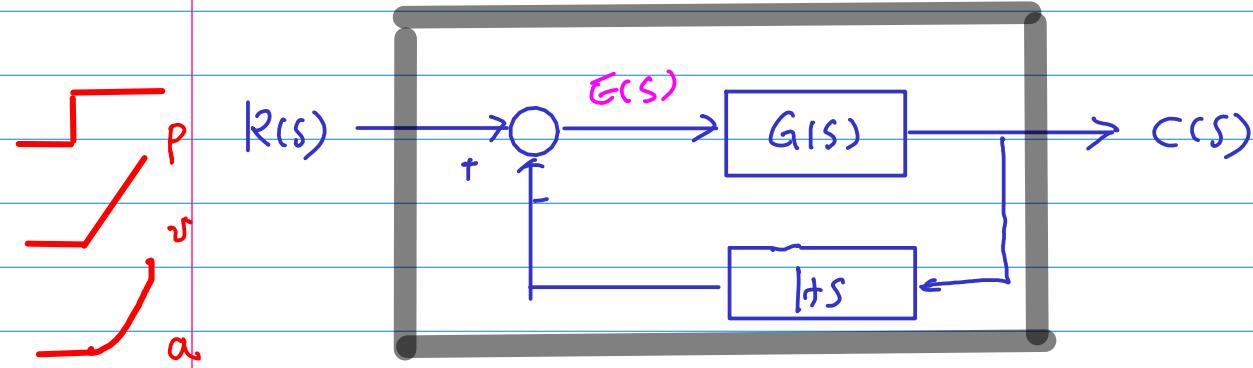


$$\text{ramp fn } t u(t) \iff \frac{1}{s^2}$$



$$\text{Parabola fn } t^2 u(t) \iff \frac{2}{s^3}$$





$$\frac{G(s)}{1 + G(s) H(s)} \quad \frac{1}{1 + G(s) H(s)} \times G(s)$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s) H(s)}$$

$$E(s) = \frac{1}{1 + G(s) H(s)} \cdot R(s)$$

$$u(t) \quad \text{unit step} \quad E_p(s) = \frac{1}{1 + G(s) H(s)} \cdot \frac{1}{s}$$

$$t u(t) \quad \text{unit ramp} \quad E_r(s) = \frac{1}{1 + G(s) H(s)} \cdot \frac{1}{s^2}$$

$$\frac{t^2 u(t)}{2} \quad \text{unit parabola} \quad E_a(s) = \frac{1}{1 + G(s) H(s)} \cdot \frac{1}{s^3}$$

Steady State : $t \rightarrow \infty$

$$y'' + ay' + b = x(t)$$

$$y_h = C_1 e^{-m_1 t} + C_2 e^{-m_2 t} \quad m_1 > 0 \\ m_2 > 0$$

$$t \rightarrow \infty \quad y_h \rightarrow 0$$

$$y_p = x(t) \text{ 와 } \theta \text{ 일치한 } y_p : \text{ Undetermined Coefficient}$$

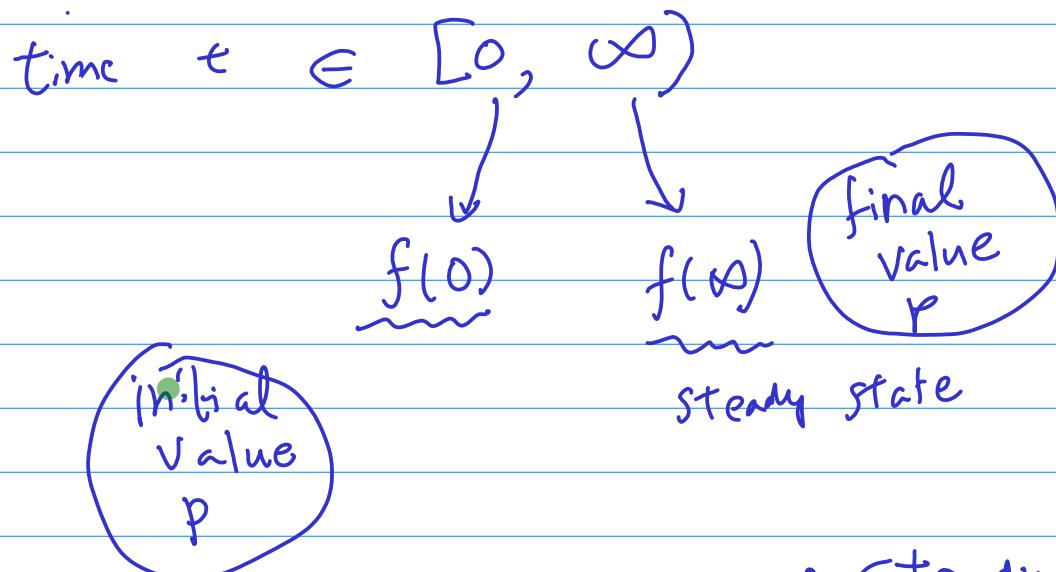
$$f(t) \iff F(s)$$

- **Initial value theorem:**

$$f(0^+) = \lim_{s \rightarrow \infty} sF(s).$$

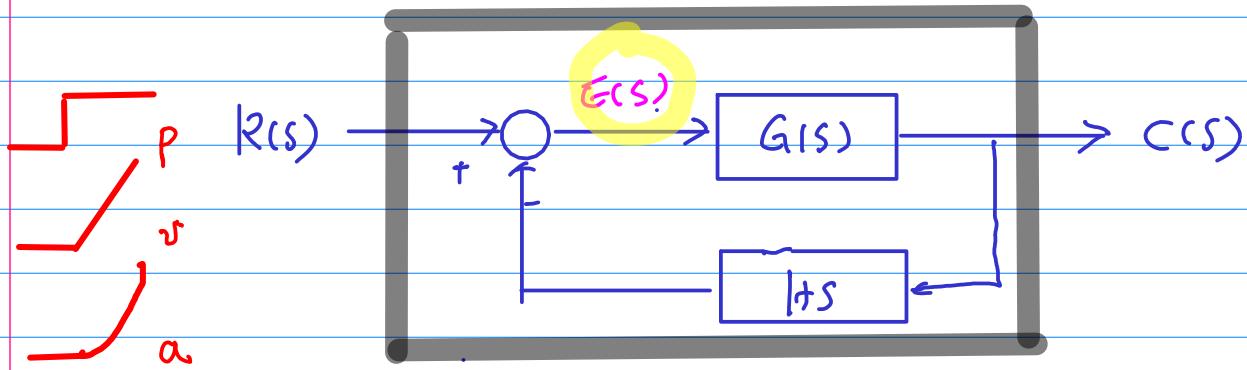
- **Final value theorem:**

$$f(\infty) = \lim_{s \rightarrow 0} sF(s), \text{ if all poles of } sF(s) \text{ are in the left half-plane.}$$



$$f(\infty) = \lim_{s \rightarrow 0} s F(s) \quad : \text{Steady State}$$

$$f(0) = \lim_{s \rightarrow \infty} s F(s)$$



$$E(s) = \frac{1}{1 + G(s) H(s)}$$

$$e(\infty) = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} \frac{s R(s)}{1 + G(s) H(s)}$$

u_{lt}

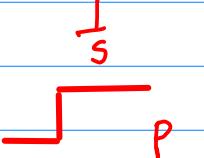
$$e_p(\infty) = \lim_{s \rightarrow 0} \frac{s}{1 + G(s) H(s)} \left(\frac{1}{s} \right)$$

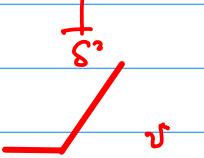
$t u_{lt}(t)$

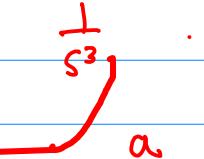
$$e_v(\infty) = \lim_{s \rightarrow 0} \frac{s}{1 + G(s) H(s)} \left(\frac{1}{s^2} \right)$$

$\frac{t^2}{2} u_{lt}$

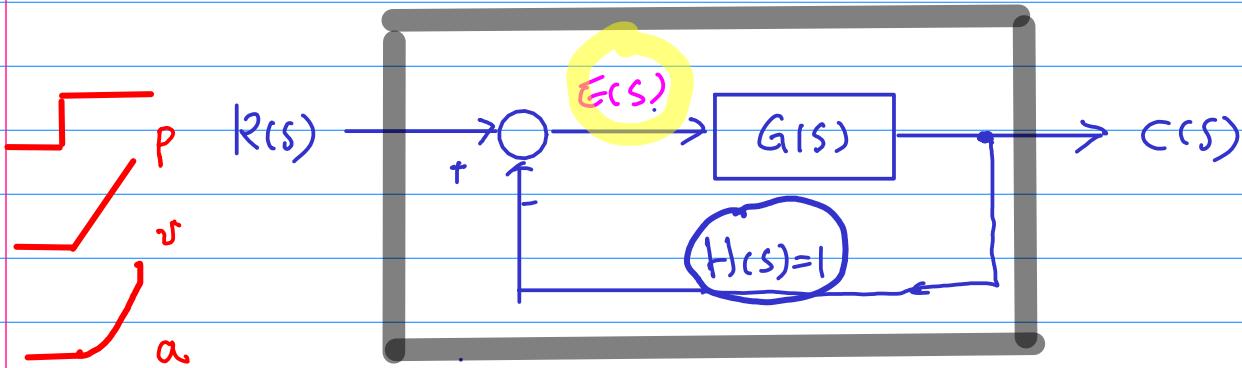
$$e_a(\infty) = \lim_{s \rightarrow 0} \frac{s}{1 + G(s) H(s)} \left(\frac{1}{s^3} \right)$$

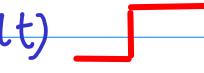
ult)  $e_p(\infty) = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)}$

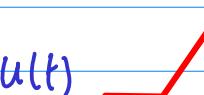
tult)  $e_r(\infty) = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)} \frac{1}{s}$

$\frac{t^2}{2}$ ult)  $e_a(\infty) = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)} \frac{1}{s^2}$

* Unit feedback System

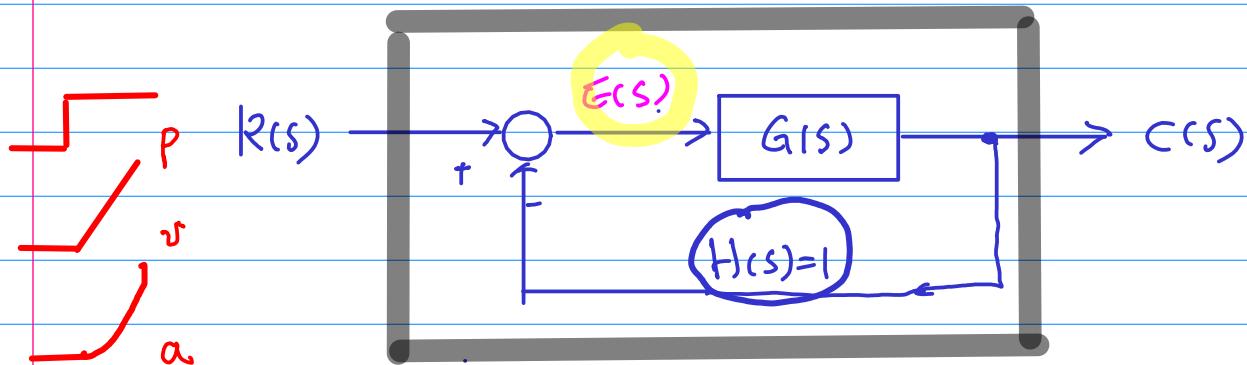


ult)  $e_p(\infty) = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)} = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)}$

tult)  $e_r(\infty) = \lim_{s \rightarrow 0} \frac{1}{s + G(s)} = \lim_{s \rightarrow 0} \frac{1}{sG(s)}$

$\frac{t^2}{2}$ ult)  $e_a(\infty) = \lim_{s \rightarrow 0} \frac{1}{s^2 + s^2G(s)} = \lim_{s \rightarrow 0} \frac{1}{s^2G(s)}$

* Unit feedback System



$$\text{Position } e_p(\infty) = \lim_{s \rightarrow 0} \frac{1}{1 + G(s)} = \frac{1}{1 + K_p}$$

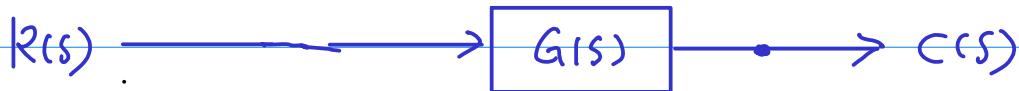
$$\text{Velocity } e_v(\infty) = \lim_{s \rightarrow 0} \frac{1}{sG(s)} = \frac{1}{K_v}$$

$$\text{Acceleration } e_a(\infty) = \lim_{s \rightarrow 0} \frac{1}{s^2G(s)} = \frac{1}{K_a}$$

Position $K_p = \lim_{s \rightarrow 0} G(s)$

Velocity $K_v = \lim_{s \rightarrow 0} sG(s)$

Acceleration $K_a = \lim_{s \rightarrow 0} s^2G(s)$



$$G(s) = \frac{(s+z_1)(s+z_2) \dots}{s^n (s+p_1)(s+p_2) \dots}$$

$n=0$ $G(s) = \frac{(s+z_1)(s+z_2) \dots}{(s+p_1)(s+p_2) \dots}$ Type(0) System

$n=1$ $G(s) = \frac{(s+z_1)(s+z_2) \dots}{s (s+p_1)(s+p_2) \dots}$ Type(1) System

$n=2$ $G(s) = \frac{(s+z_1)(s+z_2) \dots}{s^2 (s+p_1)(s+p_2) \dots}$ Type(2) System

$n=3$ $G(s) = \frac{(s+z_1)(s+z_2) \dots}{s^3 (s+p_1)(s+p_2) \dots}$

Type 0 System

~~$\int s$: Integrator~~

$N=0$

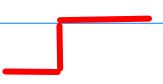
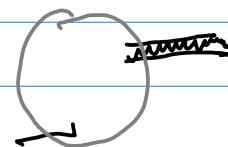
$$G(s) = \frac{(s+z_1)(s+z_2)\dots}{(s+p_1)(s+p_2)\dots} \quad \text{Type 0 System}$$

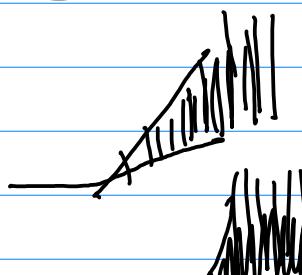
$$\lim_{s \rightarrow 0} G(s) = \frac{z_1 z_2 \dots}{p_1 p_2 \dots}$$

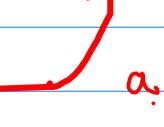
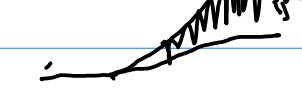
Position $K_p = \lim_{s \rightarrow 0} G(s) = \frac{z_1 z_2 \dots}{p_1 p_2 \dots}$

Velocity $K_v = \lim_{s \rightarrow 0} s \cdot G(s) = \lim_{s \rightarrow 0} s \cdot \frac{z_1 z_2 \dots}{p_1 p_2 \dots} \Rightarrow 0$

Acceleration $K_a = \lim_{s \rightarrow 0} s^2 \cdot G(s) = \lim_{s \rightarrow 0} s^2 \cdot \frac{z_1 z_2 \dots}{p_1 p_2 \dots} \Rightarrow 0$

$u(t)$  $e_p(\infty) = \frac{1}{1+K_p}$ 

$t u(t)$  $e_v(\infty) = \frac{1}{K_v} \rightarrow \infty$ 

$\frac{t^2 u(t)}{2}$  $e_a(\infty) = \frac{1}{K_a} \rightarrow \infty$ 

Type 1 System

$\frac{1}{s}$: Integrator

$n=1$

$$G(s) = \frac{(s+z_1)(s+z_2)\dots}{s(s+p_1)(s+p_2)\dots} \text{ Type 1 System}$$

$$\lim_{s \rightarrow 0} G(s) = \frac{z_1 z_2 \dots}{p_1 p_2 \dots}$$

Position $K_p = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{z_1 z_2 \dots}{s p_1 p_2 \dots} \Rightarrow \infty$

Velocity $K_v = \lim_{s \rightarrow 0} s \cdot G(s) = \lim_{s \rightarrow 0} \frac{z_1 z_2 \dots}{p_1 p_2 \dots}$

acceleration $K_a = \lim_{s \rightarrow 0} s^2 \cdot G(s) = \lim_{s \rightarrow 0} s \cdot \frac{z_1 z_2 \dots}{p_1 p_2 \dots} \Rightarrow 0$

$e_p(\infty) = \frac{1}{1+K_p} \rightarrow 0$

$e_v(\infty) = \frac{1}{K_v}$

$e_a(\infty) = \frac{1}{K_a} \rightarrow \infty$

Type 2 System

\int : Integrator 2^{th}

$\eta=2$

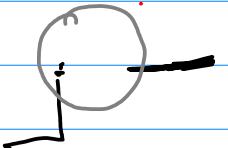
$$G(s) = \frac{(s+z_1)(s+z_2) \dots}{s^2(s+p_1)(s+p_2) \dots} \quad \text{Type 2 System}$$

$$\lim_{s \rightarrow 0} G(s) = \frac{z_1 \cdot z_2 \dots}{p_1 \cdot p_2 \dots}$$

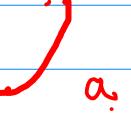
Position $K_p = \lim_{s \rightarrow 0} G(s) = \lim_{s \rightarrow 0} \frac{1}{s^2 p_1 p_2 \dots} \Rightarrow \infty$

Velocity $K_v = \lim_{s \rightarrow 0} s \cdot G(s) = \lim_{s \rightarrow 0} \frac{1}{s} \frac{z_1 z_2 \dots}{p_1 p_2 \dots} \Rightarrow \infty$

acceleration $K_a = \lim_{s \rightarrow 0} s^2 \cdot G(s) = \lim_{s \rightarrow 0} \frac{1}{s^2} \frac{z_1 z_2 \dots}{p_1 p_2 \dots}$

(u(t))  $e_p(\infty) = \frac{1}{1 + K_p} \rightarrow 0$ 

(t u(t))  $e_v(\infty) = \frac{1}{K_v} \rightarrow 0$ 

$\frac{t^2 u(t)}{2}$  $e_a(\infty) = \frac{1}{K_a}$ 

System type	error constant			Steady state error		
	K_p	K_r	K_a	$e_p(\infty)$	$e_r(\infty)$	$e_a(\infty)$
$n=0$	C	0	0	$1/(C+K_p)$	∞	∞
$n=1$	∞	C	0	0	$1/K_r$	∞
$n=2$	∞	∞	C	0	0	$1/K_a$
$n=3$	∞	∞	∞	0	0	0

