# Sequence (1A)

- Arithmetic Mean
- Geometric Mean
- Harmonic Mean
- Arithmetic Progression
- Geometric Progression
- Harmonic Progression

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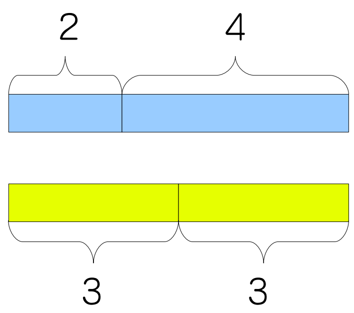
# Arithmetic Mean

2 elements $\{a, b\}$ $$A = \frac{(a+b)}{2}$$

3 elements $\{a, b, c\}$ $$A = \frac{(a+b+c)}{3}$$

n elements $\{a_1, a_2, \ldots, a_n\}$ $$A = \frac{(a_1 + a_2 + \cdots + a_n)}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

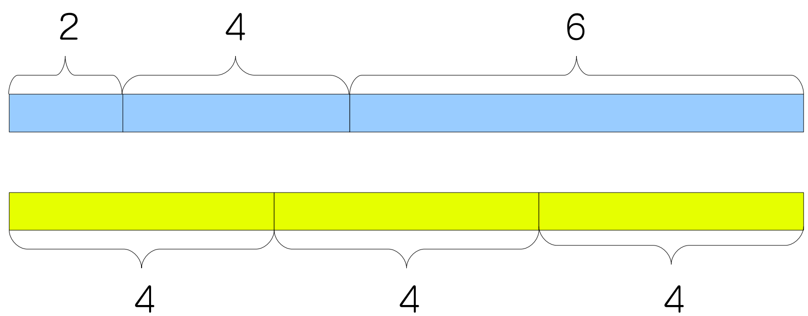
# Arithmetic Mean – Example




- same length

$$(2+4)=(3+3)=2\cdot 3=2\cdot A$$

Arithmetic Mean: A = 3




- same length

$$(2+4+6)=(4+4+4)=3\cdot 4=3\cdot A$$

Arithmetic Mean: A = 4

# Geometric Mean

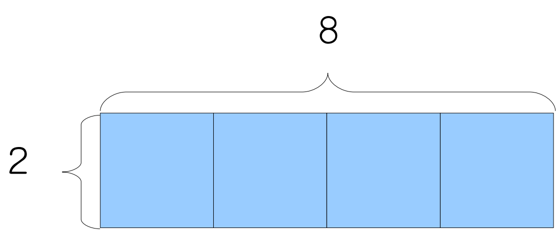
2 elements $\{a, b\}$ $G = \sqrt{a \cdot b}$ $(a > 0,\ b > 0)$

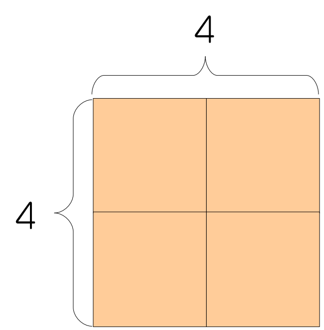
3 elements $\{a, b, c\}$ $G = \sqrt[3]{a \cdot b \cdot c}$ $(a > 0,\ b > 0,\ c > 0)$

n elements $\{a_1, a_2, \ldots, a_n\}$ $G = \sqrt[n]{a_1 \cdot a_2 \cdots a_n} = \left(\prod_{i=1}^{n} a_i\right)^{\frac{1}{n}}$

$(a_i > 0)$

# Geometric Mean – Example

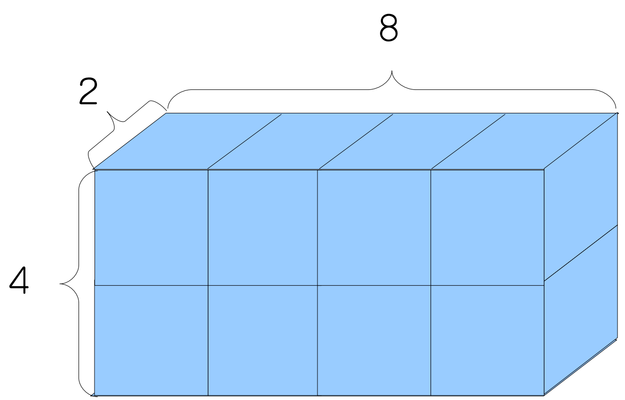


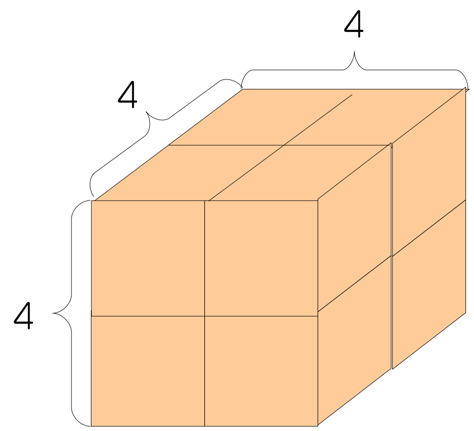




$$(2\cdot 8) = (4\cdot 4) = 4^2 = G^2$$

Geometric Mean: $G = 4$

- same area




$$(2\cdot 4\cdot 8) = (4\cdot 4\cdot 4) = 4^3 = G^3$$

Geometric Mean: $G = 4$

- same volume

# Harmonic Mean

2 elements $(a, b)$

$$H = \frac{2}{\left(\frac{1}{a} + \frac{1}{b}\right)} = \frac{2ab}{a+b}$$

3 elements $(a, b, c)$

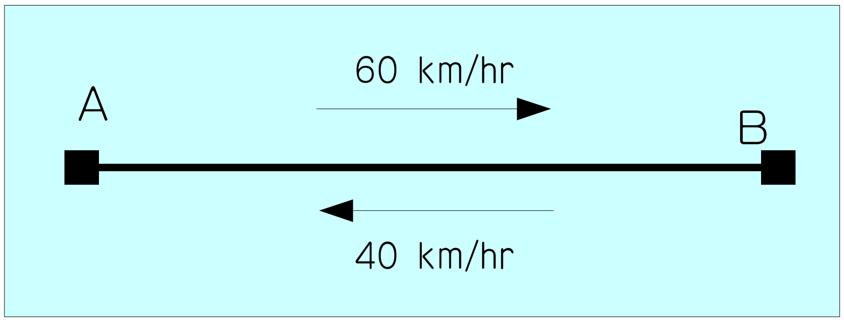
$$H = \frac{3}{\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)} = \frac{3abc}{ab+bc+ca}$$

n elements $(a_1, a_2, \ldots, a_n)$

$$H = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n}\right)} = \frac{n}{\sum_{i=1}^{n} \frac{1}{a_i}}$$

# Harmonic Mean

- same distance = D



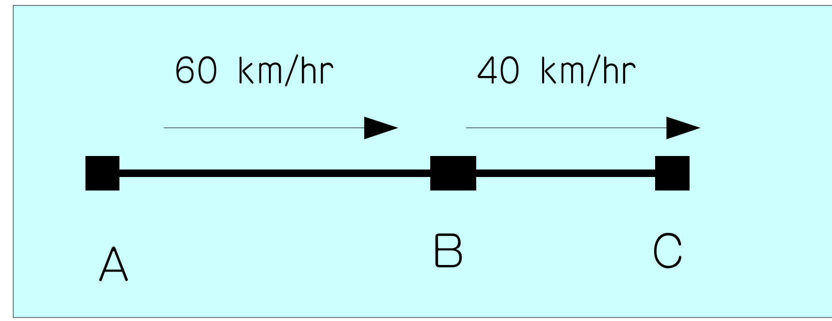

Average Speed

$$= \frac{(\text{total distance})}{(\text{total time})}$$

$$= \frac{2 \cdot D}{\frac{D}{60} + \frac{D}{40}} = \frac{2 \cdot 60 \cdot 40}{60 + 40} = 48 \, km/hr$$

Harmonic Mean: H = 48

- same duration = T



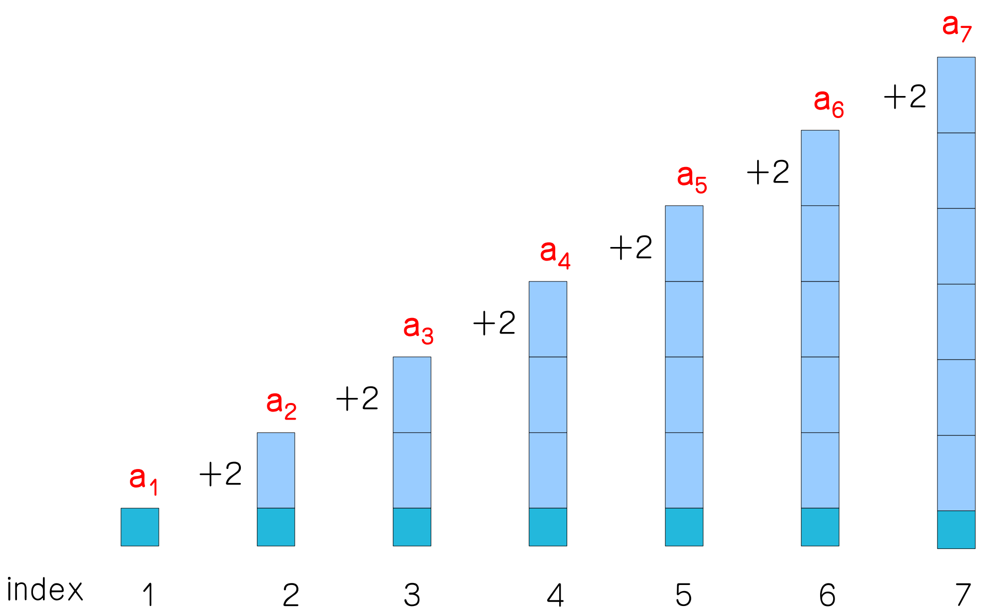

Average Speed

$$= \frac{(\text{total distance})}{(\text{total time})}$$

$$= \frac{60 \cdot T + 40 \cdot T}{2T} = 50 \, km/hr$$

Arithmetic Mean: A = 50

- common difference: d = 2

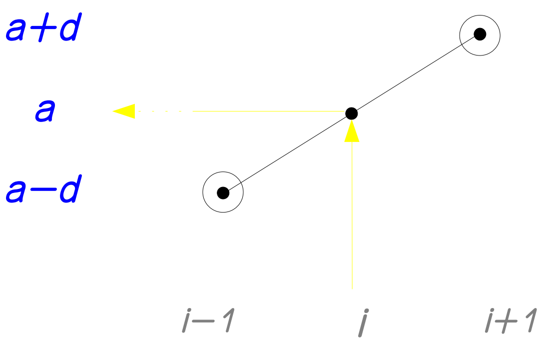



$$a_1 = 1 \qquad = 1$$
$$a_2 = 1 + 2 \qquad = 1 + 1 \cdot 2$$
$$a_3 = 1 + 2 + 2 \qquad = 1 + 2 \cdot 2$$
$$a_4 = 1 + 2 + 2 + 2 \qquad = 1 + 3 \cdot 2$$
$$a_5 = 1 + 2 + 2 + 2 + 2 \qquad = 1 + 4 \cdot 2$$
$$a_6 = 1 + 2 + 2 + 2 + 2 + 2 \qquad = 1 + 5 \cdot 2$$

$$a_n = 1 + (n-1) \cdot 2$$

$$a_n = a + (n-1) \cdot d$$

# Arithmetic Progression – Arithmetic Mean




$a_{i+1} = a+d$

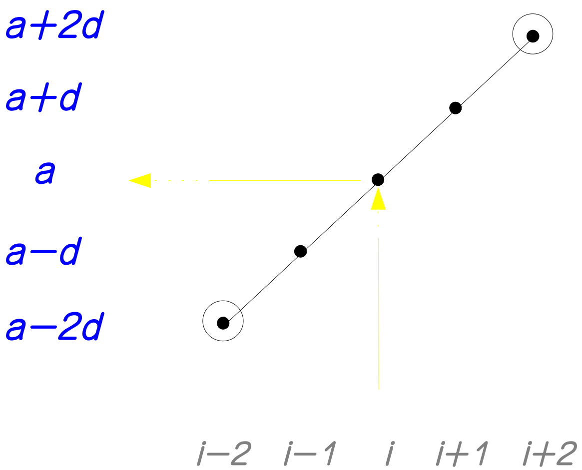
$a_i = a$

$a_{i-1} = a-d$

$$A = \frac{\{(a-d)+a+(a+d)\}}{3}$$

$$A = a$$

$$A = \frac{\{(a-d)+(a+d)\}}{2}$$




$a_{i+2} = a+2d$

$a_{i+1} = a+d$

$a_i = a$

$a_{i-1} = a-d$

$a_{i-2} = a-2d$

$$A = \frac{\{(a-2d)+(a-d)+a+(a+d)+(a+2d)\}}{5}$$

$$A = a$$

$$A = \frac{\{(a-2d)+(a+2d)\}}{2}$$

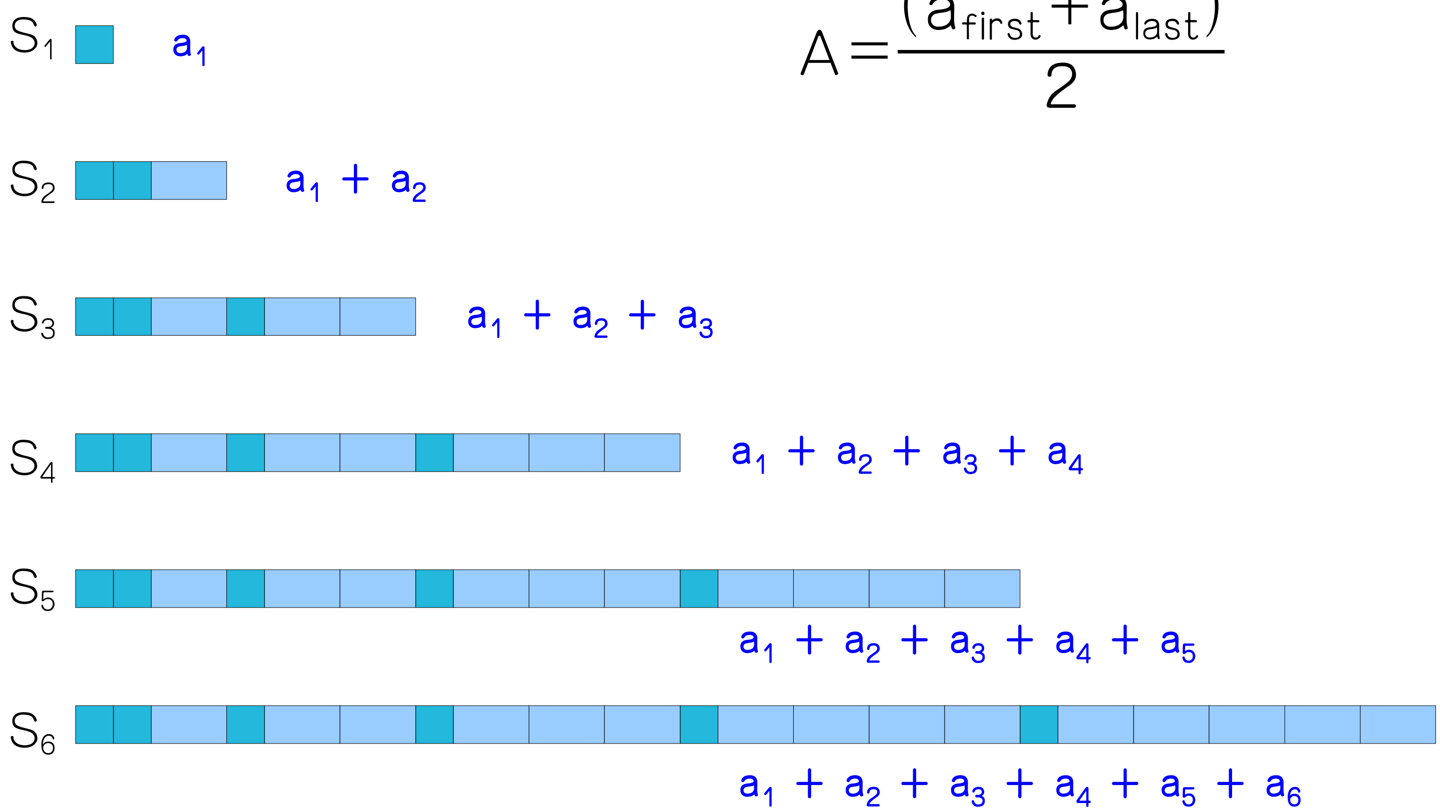
# Arithmetic Progression – Partial Sum

$$S_n = \sum_{i=1}^{n} a_i$$

$$A = \frac{1}{n} \cdot \sum_{i=1}^{n} a_i = \frac{S_n}{n} = \frac{(a_1 + a_n)}{2}$$

$$S_n = \frac{n \cdot (a_1 + a_n)}{2}$$

$$A = \frac{(a_{first} + a_{last})}{2}$$




$$1 = 1$$

$$4 = 2 \cdot \frac{(1+3)}{2}$$

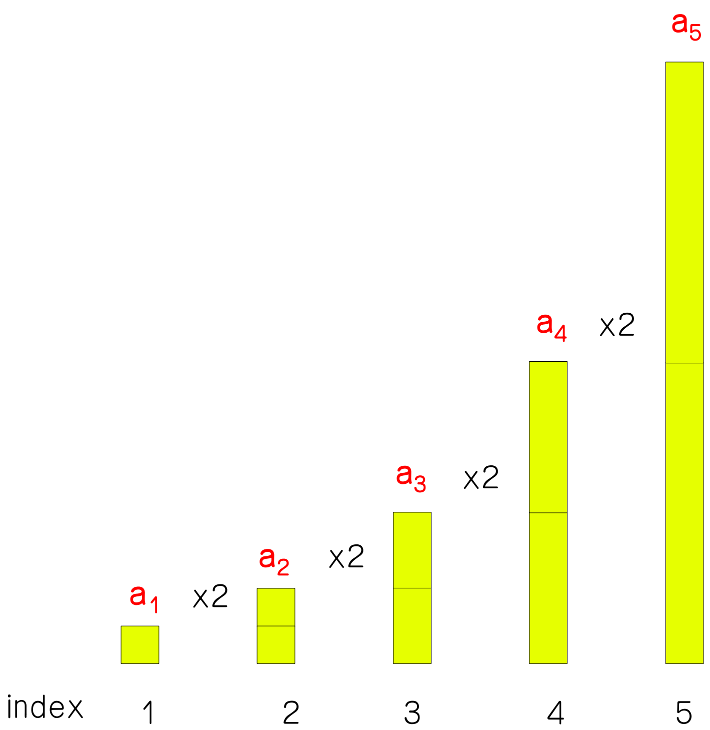
$$9 = 3 \cdot \frac{(1+5)}{2}$$

$$16 = 4 \cdot \frac{(1+7)}{2}$$

$$25 = 5 \cdot \frac{(1+9)}{2}$$

$$36 = 6 \cdot \frac{(1+11)}{2}$$

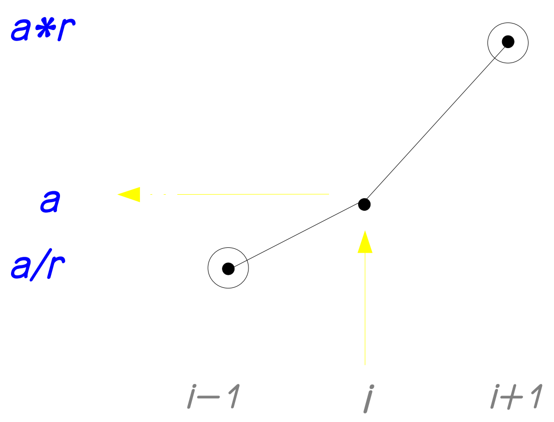
- common ratio: $r = 2$




$$a_1 = 1 \qquad = 1$$
$$a_2 = 1 \cdot 2 \qquad = 1 \cdot 2^1$$
$$a_3 = 1 \cdot 2 \cdot 2 \qquad = 1 \cdot 2^2$$
$$a_4 = 1 \cdot 2 \cdot 2 \cdot 2 \qquad = 1 \cdot 2^3$$
$$a_5 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \qquad = 1 \cdot 2^4$$
$$a_6 = 1 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \qquad = 1 \cdot 2^5$$

$$a_n = 1 \cdot 2^{n-1}$$

$$a_n = a \cdot r^{n-1}$$



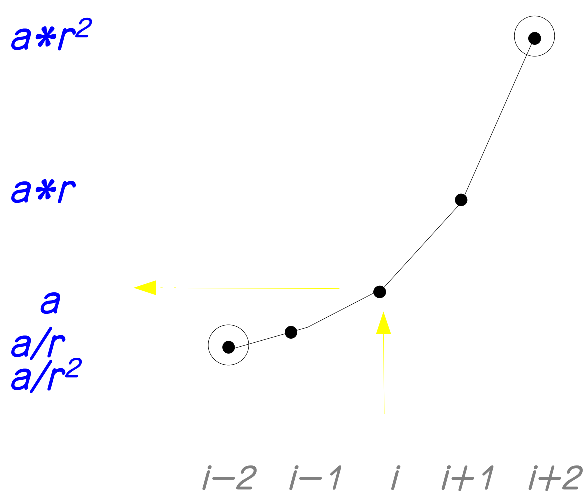

$a_{i+1} = a * r$

$a_i = a$

$a_{i-1} = a / r$

$$G = \sqrt[3]{(ar^{-1}) \cdot (a) \cdot (ar)}$$
$$G = a$$

$$G = \sqrt[2]{(ar^{-1}) \cdot (ar)}$$




$a_{i+2} = a * r^2$

$a_{i+1} = a * r$

$a_i = a$

$a_{i-1} = a / r$

$a_{i-2} = a / r^2$

$$G = \sqrt[5]{(ar^{-2}) \cdot (ar^{-1}) \cdot a \cdot (ar) \cdot (ar^2)}$$
$$G = a$$

$$G = \sqrt[2]{(ar^{-2}) \cdot (ar^2)}$$

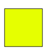
# Geometric Progression – Partial Product

$$P_n = \prod_{i=1}^{n} a_i \quad (a_i > 0)$$

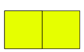
$$G = \left( \prod_{i=1}^{n} a_i \right)^{\frac{1}{n}} = P_n^{\frac{1}{n}} = \sqrt[2]{a_1 \cdot a_n}$$

$$P_n = (a_1 \cdot a_n)^{\frac{n}{2}}$$

$$G = \sqrt[2]{a_{first} \cdot a_{last}}$$



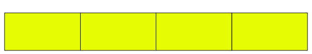
$P_1 = 1$ $a_1$ $1 = 1$



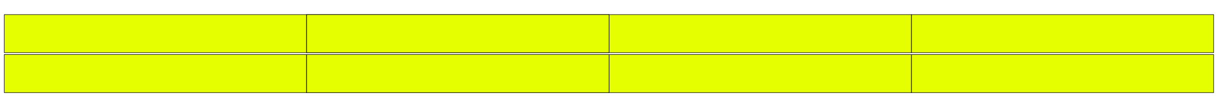
$P_2 = 1^2 \cdot 2^1$ $a_1 \times a_2$

$a_1 = 1,\ a_2 = 2,$
$a_3 = 4,\ a_4 = 8$

$2 = (\sqrt{1 \cdot 2})^2$



$P_3 = 1^3 \cdot 2^{(1+2)}$ $a_1 \times a_2 \times a_3$ $8 = (\sqrt{1 \cdot 4})^3$



$P_4 = 1^3 \cdot 2^{(1+2+3)}$ $a_1 \times a_2 \times a_3 \times a_4$ $64 = (\sqrt{1 \cdot 8})^4$

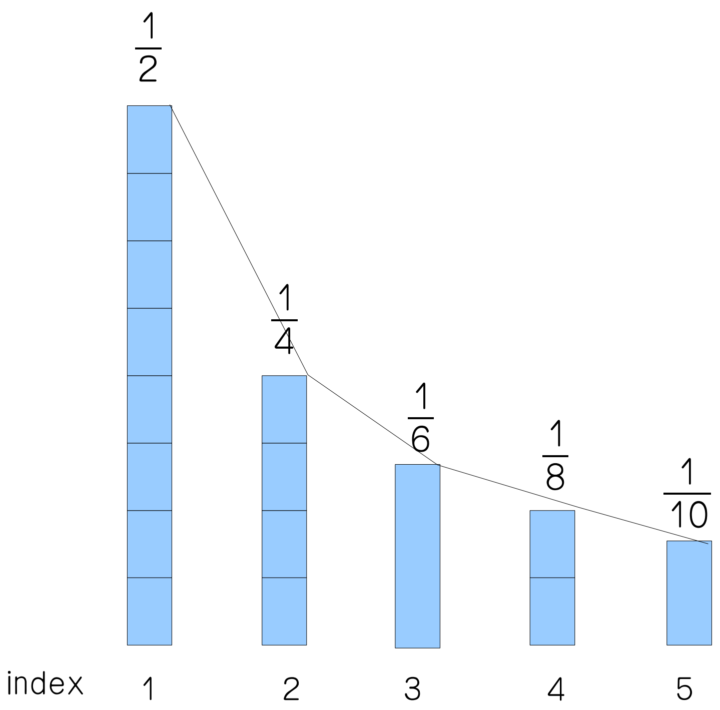
# Harmonic Progression

$$\text{A.P. } (a_n) = \left(a,\ a+d,\ a+2\cdot d,\ a+3\cdot d,\ \ldots\right)$$

$$y = \frac{1}{x}$$

$$\text{H.P. } (b_n) = \left(\frac{1}{a},\ \frac{1}{(a+d)},\ \frac{1}{(a+2\cdot d)},\ \frac{1}{(a+3\cdot d)},\ \ldots\right)$$

# Harmonic Progression – General Term




$$a_1 = \frac{1}{2} \qquad = \frac{1}{2}$$

$$a_2 = \frac{1}{(2+2)} = \frac{1}{4} \qquad = \frac{1}{(2+1\cdot 2)}$$

$$a_3 = \frac{1}{(2+2+2)} = \frac{1}{6} \qquad = \frac{1}{(2+2\cdot 2)}$$

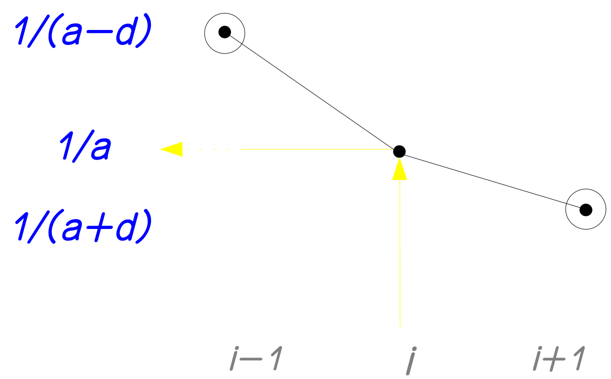
$$a_4 = \frac{1}{(2+2+2+2)} = \frac{1}{8} \qquad = \frac{1}{(2+3\cdot 2)}$$

$$a_5 = \frac{1}{(2+2+2+2+2)} = \frac{1}{10} \qquad = \frac{1}{(2+4\cdot 2)}$$

$$a_n = \frac{1}{(2+(n-1)\cdot 2)}$$

$$a_n = \frac{1}{(a+(n-1)\cdot d)}$$

# Harmonic Progression – Harmonic Mean




$a_{i-1} = 1/(a-d)$

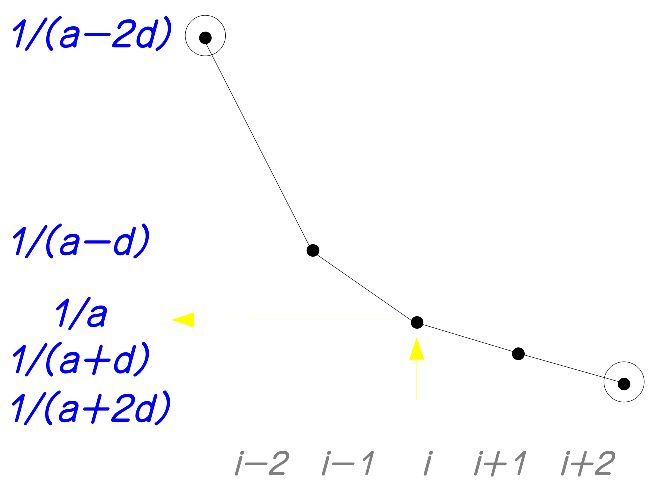
$a_i = a$

$a_{i+1} = 1/(a+d)$

$$A = \frac{3}{\{(a-d)+a+(a+d)\}}$$

$$A = \frac{1}{a}$$

$$A = \frac{2}{\{(a-d)+(a+d)\}}$$




$a_{i-2} = 1/(a-2d\ )$

$a_{i-1} = 1/(a-d)$

$a_i = a$

$a_{i+1} = 1/(a+d)$

$a_{i+2} = 1/(a+2d)$

$$A = \frac{5}{\{(a-2d)+(a-d)+a+(a+d)+(a+2d)\}}$$

$$A = \frac{1}{a}$$

$$A = \frac{2}{\{(a-2d)+(a+2d)\}}$$

# From G.P. To A.P.

G.P. $(a_n) = (a, ar^1, ar^2, ar^3, \ldots)$ $(a>0, \ r>0)$

$y = \log(x)$

A.P. $(b_n) = (\log(a), \ \log(a)+1\cdot\log(r), \ \log(a)+2\cdot\log(r) \ \ldots)$

$= \log(a)+(n-1)\cdot\log(r)$

# From A.P. To G.P.

A.P. $(a_n) = (a,\ a+d,\ a+2\cdot d,\ a+3\cdot d,\ \ldots)$

$y=b^x$

G.P. $(b_n) = \left(b^a,\ b^{(a+d)},\ b^{(a+2\cdot d)},\ b^{(a+3\cdot d)},\ \ldots\right) = (b^a)\cdot(b^d)^{(n-1)}$

## References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] Blitzer, R. "Algebra & Trigonometry." 3rd ed, Prentice Hall
[4] Smith, R. T., Minton, R. B. "Calculus: Concepts & Connections," Mc Graw Hill
[5] 홍성대, "기본/실력 수학의 정석," 성지출판