

CT Sinc Function (1B)

- Continuous Time Sinc Function

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Sinc Function (1)

Normalized Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$\text{sinc}(-t) = \frac{\sin(-\pi t)}{-\pi t} = \text{sinc}(t)$$

an even function

$$\lim_{t \rightarrow 0} \frac{\sin(\pi t)}{\pi t} = 1$$

Maximum: $\text{sinc}(0) = 1$ when $t = 0$

$$t = n \quad n: \text{integer} \ (n \neq 0)$$

$$\text{Zeros: } \text{sinc}(t) = \frac{\sin(n\pi)}{n\pi} = 0$$

Unnormalized Sinc function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

$$\text{sinc}(-x) = \frac{\sin(-x)}{-x} = \text{sinc}(x)$$

an even function

$$\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$$

Maximum: $\text{sinc}(0) = 1$ when $x = 0$

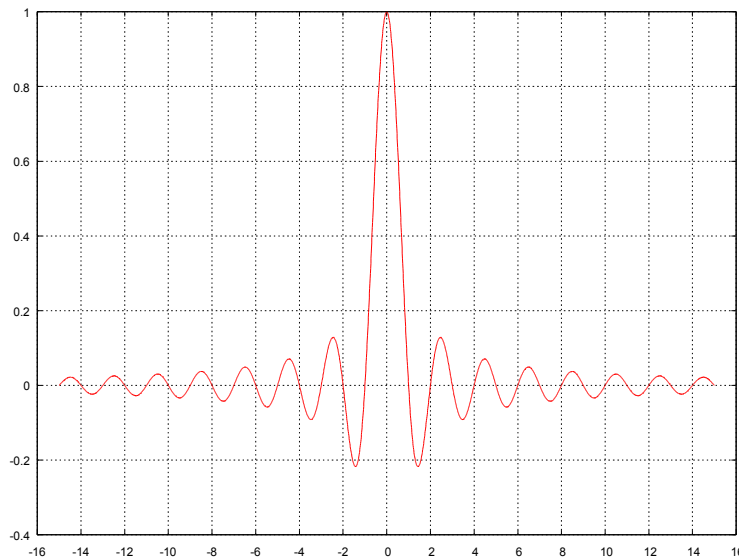
$$x = \pi n \quad n: \text{integer} \ (n \neq 0)$$

$$\text{Zeros: } \text{sinc}(x) = \frac{\sin(x)}{x} = 0$$

Sinc Function (2)

Normalized Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



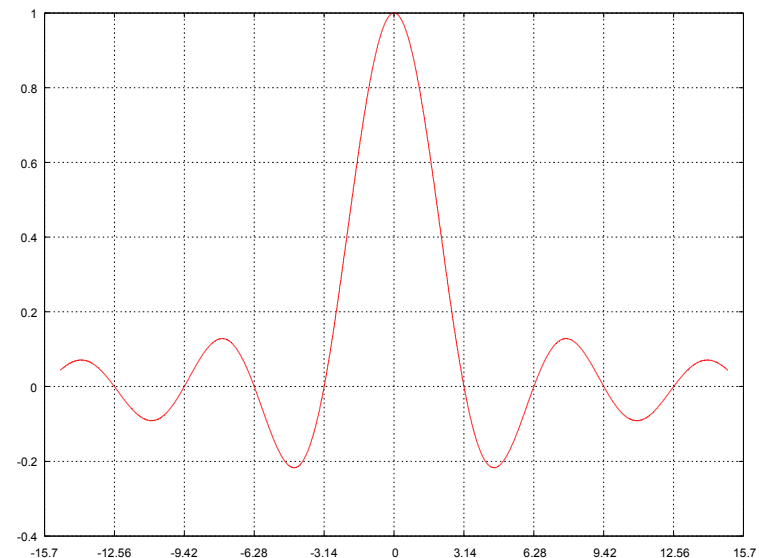
Zeros at $t = n$ n : integer ($n \neq 0$)

Normalized Sinc function

Octave `sinc(x) = sin(pi*x)/(pi*x)`

Unnormalized Sinc function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$



Zeros at $x = \pi n$ n : integer ($n \neq 0$)

Unnormalized Sinc function

Independent Variables: f and ω

Normalized Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



$$t \leftarrow f$$

$$\begin{aligned} \text{sinc}(f) &= \frac{\sin(\pi f)}{\pi f} = \frac{\sin(2\pi f/2)}{2\pi f/2} \\ &= \frac{\sin(\omega/2)}{\omega/2} \end{aligned}$$

Zeros at $t = n$ n : integer ($n \neq 0$)

Unnormalized Sinc function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$



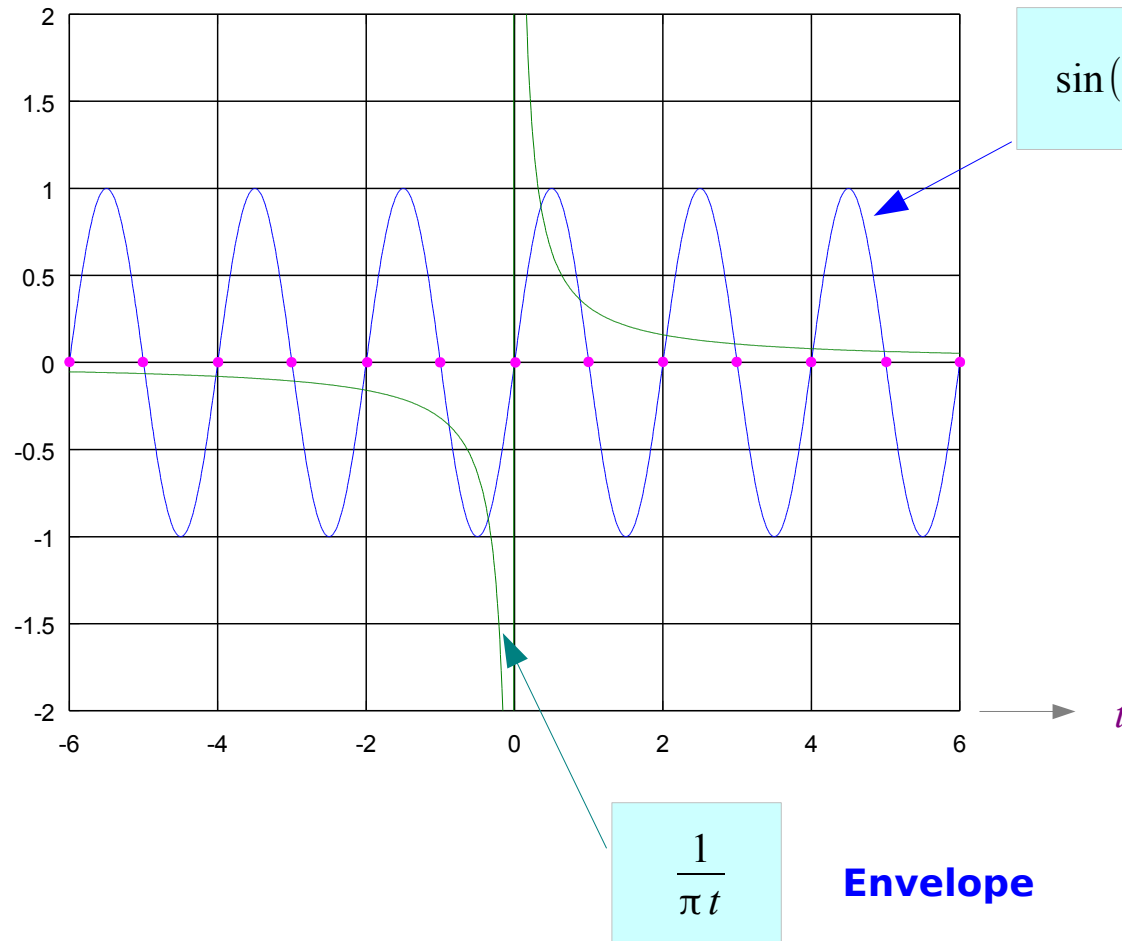
$$x \leftarrow \omega/2$$

$$\text{sinc}(\omega/2) = \frac{\sin(\omega/2)}{\omega/2}$$

Zeros at $x = \pi n$ n : integer ($n \neq 0$)

$$\Rightarrow \omega = 2\pi n$$

$$(1/\pi t) * \sin(\pi t)$$



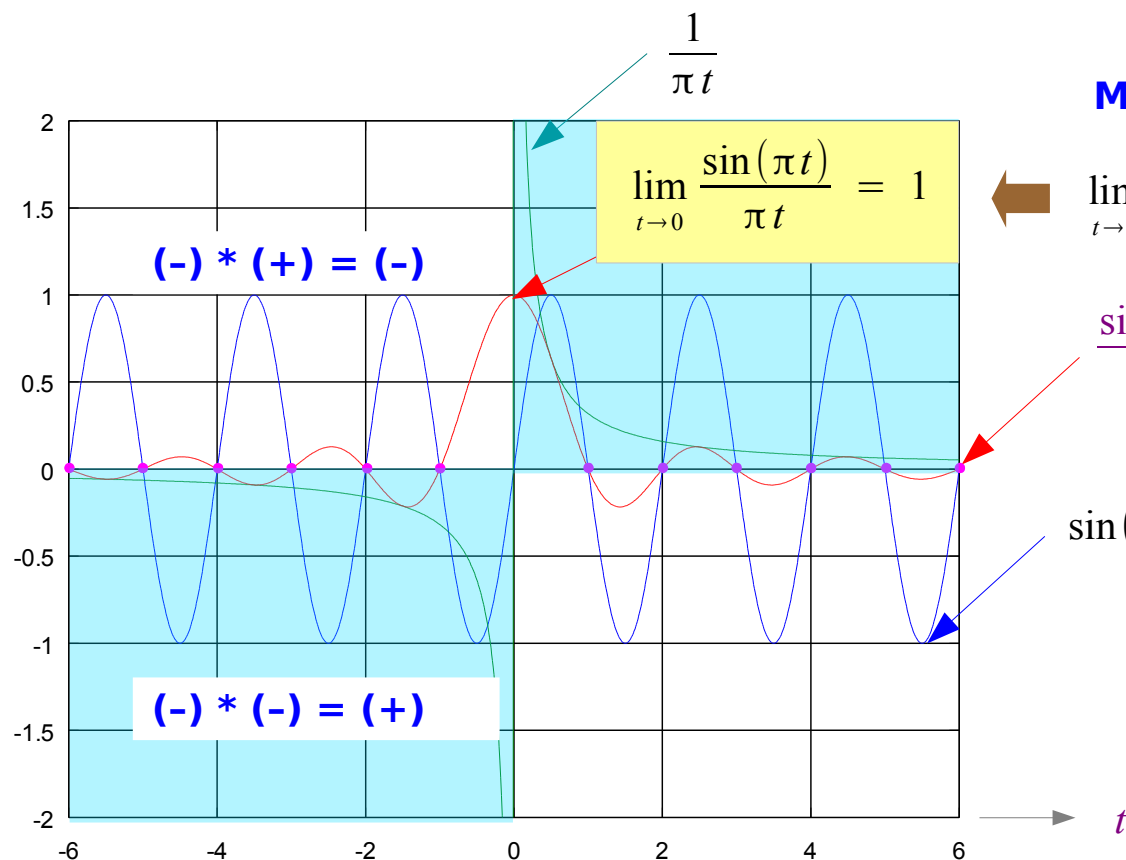
$\sin(\pi t)$

Zero crossings

$t = -2$	$t = -1$	$t = 0$	$t = +1$	$t = +2$
↓	↓	↓	↓	↓
-2π	$-\pi$	0	$+\pi$	$+2\pi$
↓	↓	↓	↓	↓
$\sin(-2\pi)$ $= 0$	$\sin(-\pi)$ $= 0$	$\sin(0)$ $= 0$	$\sin(\pi)$ $= 0$	$\sin(2\pi)$ $= 0$

Envelope

An Even Function



Max Value = 1

$$\lim_{t \rightarrow 0} \frac{(\sin(\pi t))'}{(\pi t)'} = \lim_{t \rightarrow 0} \frac{\pi \cos(\pi t)}{\pi} = 1$$

$$\frac{\sin(\pi t)}{\pi t}$$

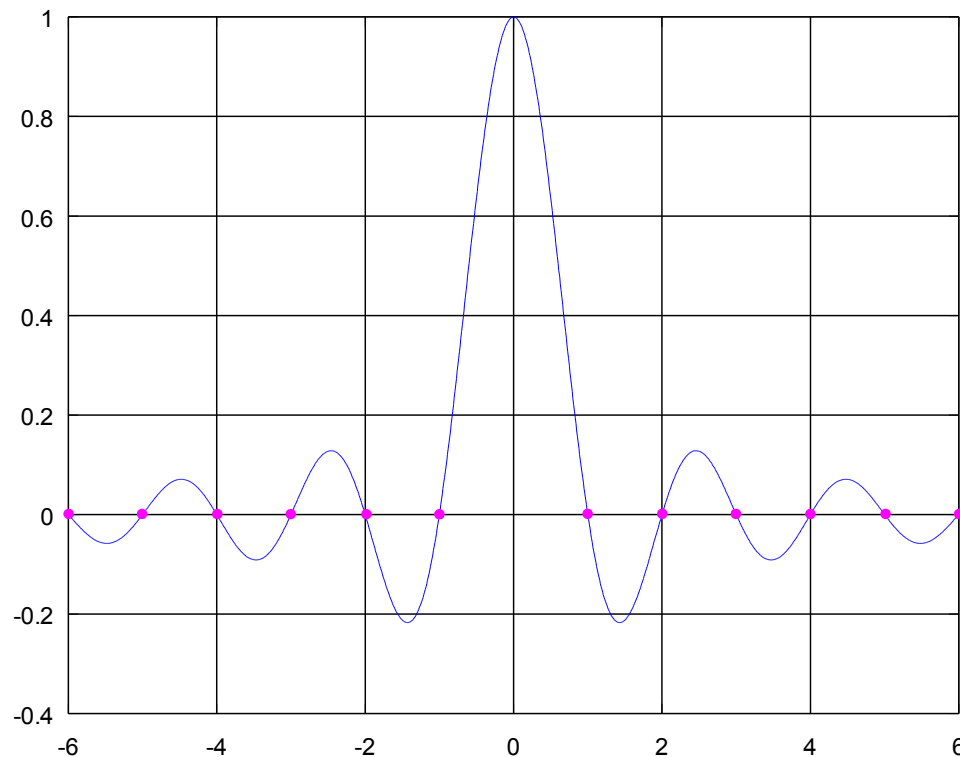
$$\sin(\pi t)$$

An Even Function

$$\text{sinc}(-t) = \text{sinc}(t)$$

$$\frac{\sin(-\pi t)}{-\pi t} = \frac{-\sin(\pi t)}{-\pi t} = \frac{\sin(\pi t)}{\pi t}$$

Zeros



$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$t = -2$	$t = -1$	$t = 0$	$t = +1$	$t = +2$
↓	↓	↓	↓	↓
-2π	$-\pi$	0	$+\pi$	$+2\pi$

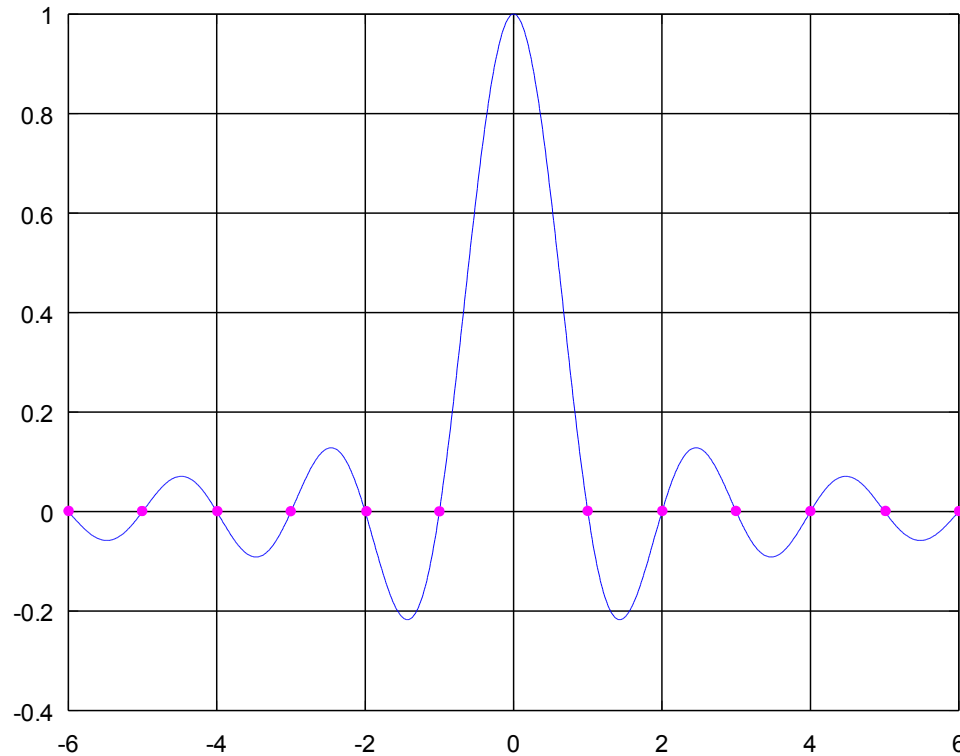
$$t = 0 \quad \Rightarrow \quad \text{sinc}(0) = 1$$

$$\text{sinc}(0) = 1 \quad (t = 0)$$

$$t = n \quad \Rightarrow \quad \sin(\pi t) = 0 \quad n: \text{integer} \quad (n \neq 0)$$

$$\text{sinc}(t) = 0 \quad (t = \pm 1, \pm 2, \pm 3, \dots)$$

Independent Variables: f and ω



$f = -2$	$f = -1$	$f = 0$	$f = +1$	$f = +2$
$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$	$\updownarrow$
$\omega = -2\pi$	$\omega = -\pi$	$\omega = 0$	$\omega = +2\pi$	$\omega = +4\pi$

Normalized Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



$$t \leftarrow f$$

$$\text{sinc}(f) = \frac{\sin(\pi f)}{\pi f}$$

$$= \frac{\sin(2\pi f/2)}{2\pi f/2}$$

$$\text{sinc}(\omega/2) = \frac{\sin(\omega/2)}{\omega/2}$$



$$x \leftarrow \omega/2$$

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$

Unnormalized Sinc function

Example (1)

$$\frac{\sin(\omega T/2)}{\omega/2} = T \frac{\sin(\pi \omega T/2 \pi)}{\pi \omega T/2 \pi}$$

Normalized Sinc function

$$\text{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



$$t \leftarrow \omega T/2 \pi = f T$$

$$\text{sinc}(\omega T/2 \pi) = \frac{\sin(\pi \omega T/2 \pi)}{\pi \omega T/2 \pi}$$

$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T \text{sinc}(\omega T/2 \pi)$$

Zeros at $\omega T/2 \pi = n \Rightarrow \omega = \frac{2\pi}{T} n$

n : integer ($n \neq 0$)

$$\frac{\sin(\omega T/2)}{\omega/2} = T \frac{\sin(\omega T/2)}{\omega T/2}$$

Unnormalized Sinc function

$$\text{sinc}(x) = \frac{\sin(x)}{x}$$



$$x \leftarrow \omega T/2$$

$$\text{sinc}(\omega T/2) = \frac{\sin(\omega T/2)}{\omega T/2}$$

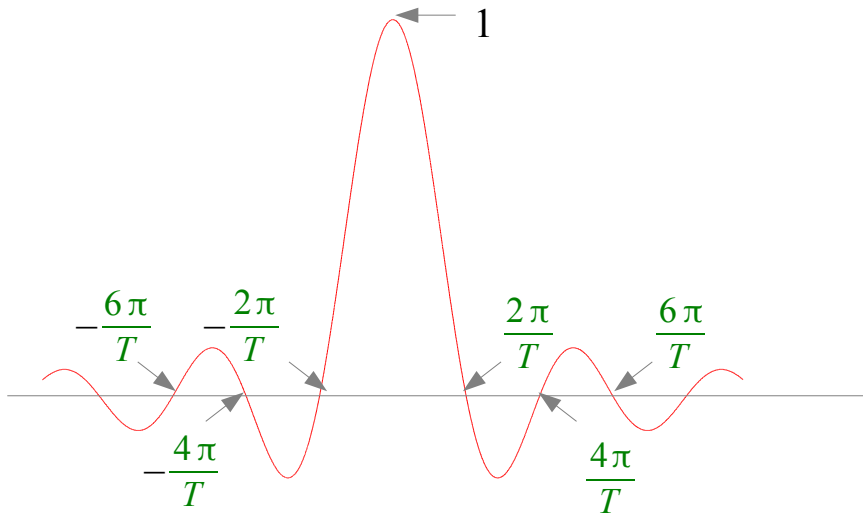
$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T \text{sinc}(\omega T/2)$$

Zeros at $\omega T/2 = \pi n \Rightarrow \omega = \frac{2\pi}{T} n$

n : integer ($n \neq 0$)

Example (2)

$$\frac{\sin(\omega T/2)}{\omega/2} = T \frac{\sin(\pi \omega T/2 \pi)}{\pi \omega T/2 \pi}$$

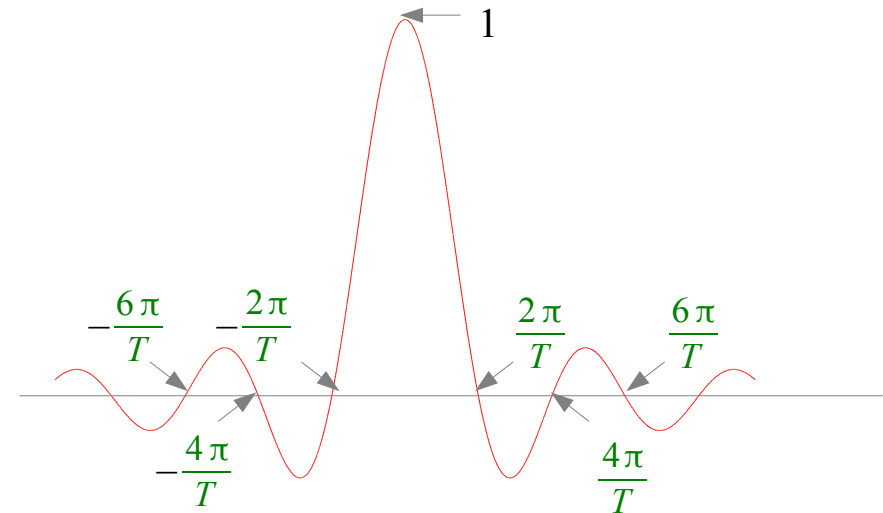


$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T \text{ sinc}(\omega T/2 \pi)$$

Zeros at $\omega T/2 \pi = n \Rightarrow \omega = \frac{2\pi}{T} n$

n : integer ($n \neq 0$)

$$\frac{\sin(\omega T/2)}{\omega/2} = T \frac{\sin(\omega T/2)}{\omega T/2}$$



$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T \text{ sinc}(\omega T/2)$$

Zeros at $\omega T/2 = \pi n \Rightarrow \omega = \frac{2\pi}{T} n$

n : integer ($n \neq 0$)

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] G. Beale, http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html
- [4] C. Langton, <http://www.complextoreal.com/chapters/fft1.pdf>

