CT Sinc Function (1B)

- Continuous Time Sinc Function

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Sinc Function (1)

Normalized Sinc function

$$\mathbf{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

Unnormalized Sinc function

$$\mathbf{sinc}(x) = \frac{\sin(x)}{x}$$

$$\mathbf{sinc}(-t) = \frac{\sin(-\pi t)}{-\pi t} = \mathbf{sinc}(t)$$

an even function

$$\mathbf{sinc}(-x) = \frac{\sin(-x)}{-x} = \mathbf{sinc}(x)$$

an even function

$$\lim_{t \to 0} \frac{\sin(\pi t)}{\pi t} = 1$$

Maximum: $\mathbf{sinc}(0) = 1 \quad when\ t = 0$

$$\lim_{t \to 0} \frac{\sin(x)}{x} = 1$$

Maximum: $\mathbf{sinc}(0) = 1 \quad when\ x = 0$

$t = n$ n: integer $(n \neq 0)$

Zeros: $\mathbf{sinc}(t) = \frac{\sin(n\pi)}{n\pi} = 0$

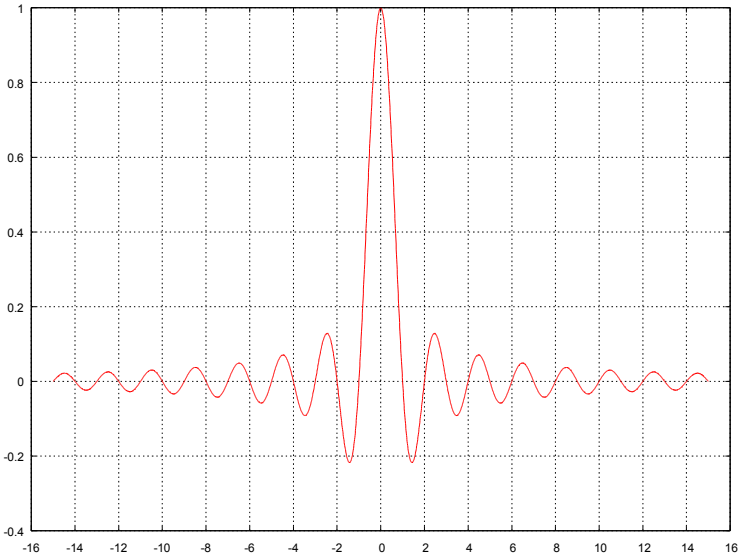
$x = \pi n$ n: integer $(n \neq 0)$

Zeros: $\mathbf{sinc}(x) = \frac{\sin(x)}{x} = 0$

Sinc Function (2)

Normalized Sinc function

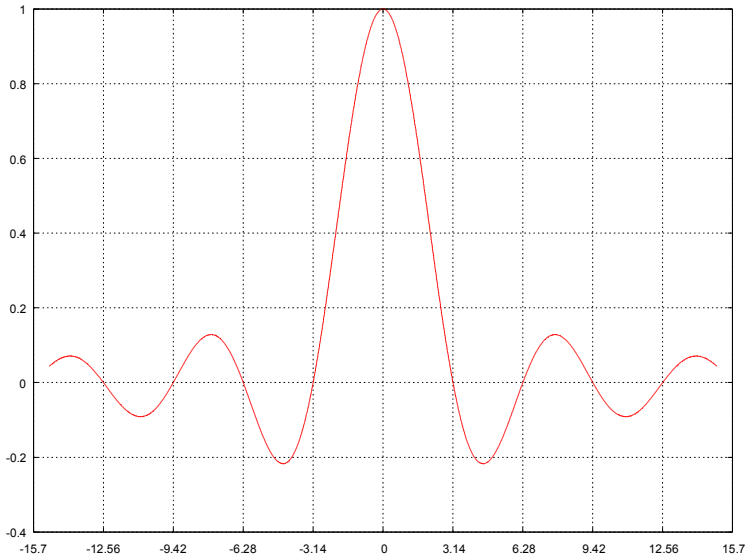
$$sinc(t) = \frac{\sin(\pi t)}{\pi t}$$



Zeros at $t = n$ n: integer $(n \neq 0)$

Unnormalized Sinc function

$$sinc(x) = \frac{\sin(x)}{x}$$



Zeros at $x = \pi n$ n: integer $(n \neq 0)$

Normalized Sinc function

Octave sinc(x) = sin(pi*x)/(pi*x)

Unnormalized Sinc function

Independent Variables: f and ω

Normalized Sinc function

$$sinc(t) = \frac{\sin(\pi t)}{\pi t}$$

$t \leftarrow f$

$$sinc(f) = \frac{\sin(\pi f)}{\pi f} = \frac{\sin(2\pi f/2)}{2\pi f/2}$$

$$= \frac{\sin(\omega/2)}{\omega/2}$$

Zeros at $t = n$ n: integer $(n \neq 0)$

Unnormalized Sinc function

$$sinc(x) = \frac{\sin(x)}{x}$$

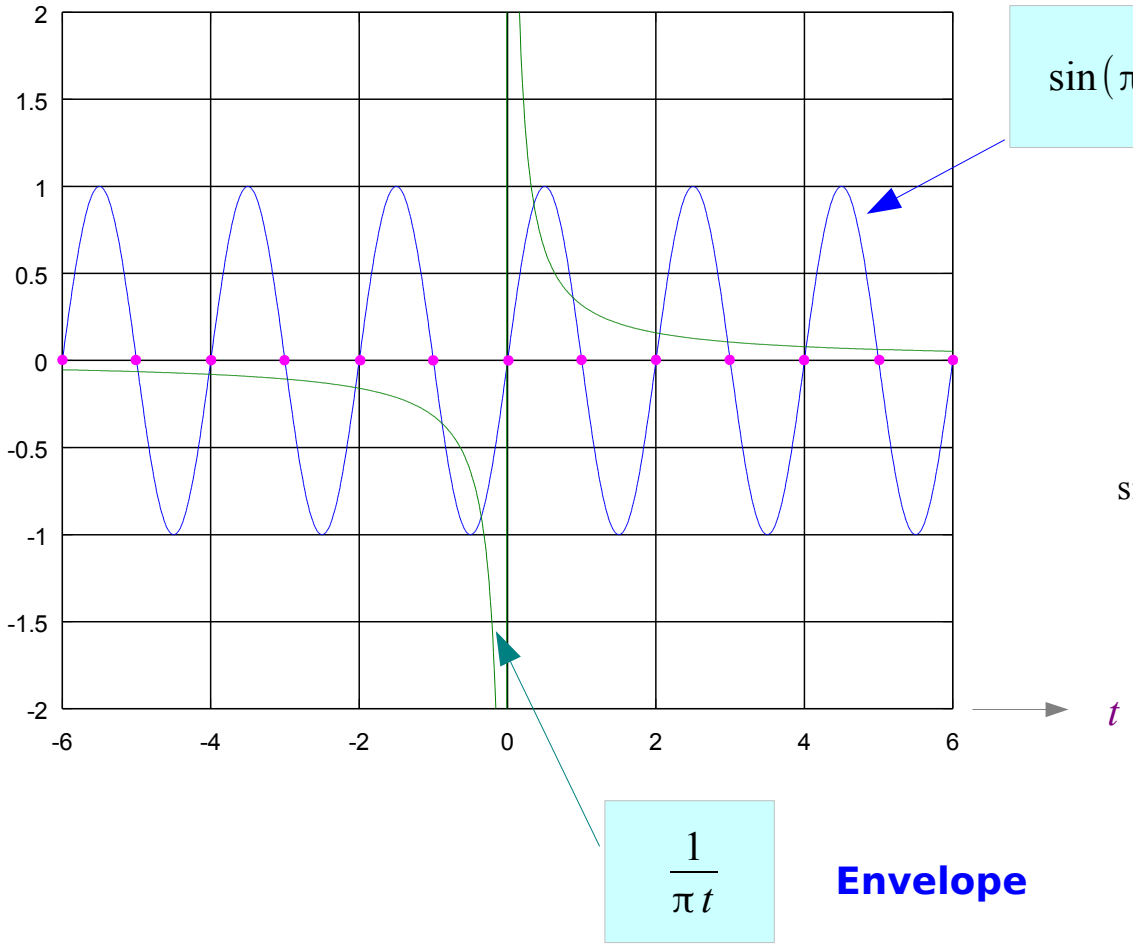
$x \leftarrow \omega/2$

$$sinc(\omega/2) = \frac{\sin(\omega/2)}{\omega/2}$$

Zeros at $x = \pi n$ n: integer $(n \neq 0)$

$\omega = 2\pi n$

(1/πt) * sin(πt)

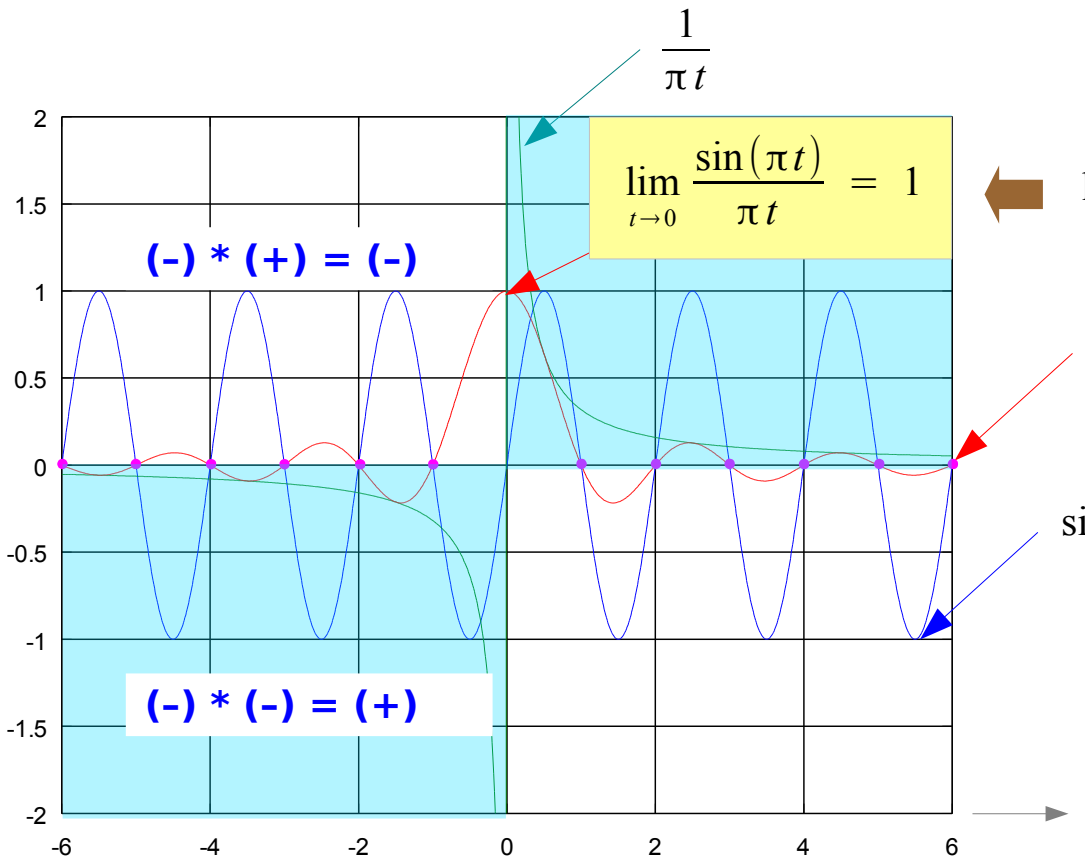


Zero crossings

$t=-2$	$t=-1$	$t=0$	$t=+1$	$t=+2$
-2π	$-\pi$	0	$+\pi$	$+2\pi$
$\sin(-2\pi)=0$	$\sin(-\pi)=0$	$\sin(0)=0$	$\sin(\pi)=0$	$\sin(2\pi)=0$

Envelope

An Even Function



Max Value = 1

$$\lim_{t\to 0}\frac{(\sin(\pi t))'}{(\pi t)'} = \lim_{t\to 0}\frac{\pi\cos(\pi t)}{\pi} = 1$$

$\frac{\sin(\pi t)}{\pi t}$

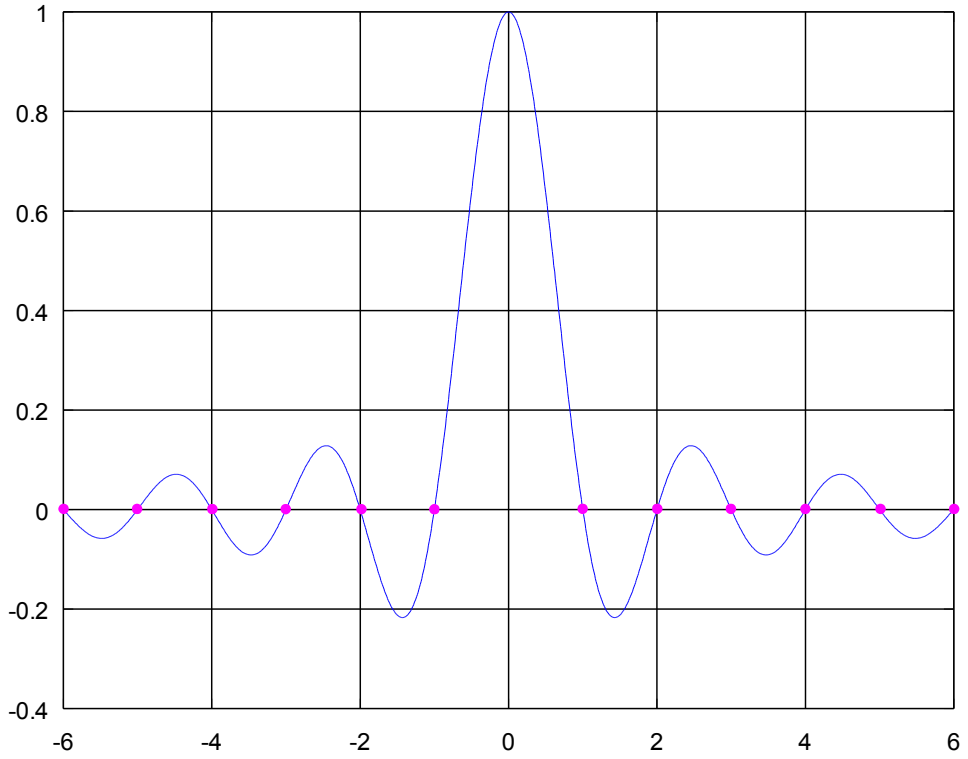
$\sin(\pi t)$

An Even Function

$$sinc(-t) = sinc(t)$$

$$\frac{\sin(-\pi t)}{-\pi t} = \frac{-\sin(\pi t)}{-\pi t} = \frac{\sin(\pi t)}{\pi t}$$

Zeros



$$\mathbf{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$t=-2$ $t=-1$ $t=0$ $t=+1$ $t=+2$

-2π $-\pi$ 0 $+\pi$ $+2\pi$

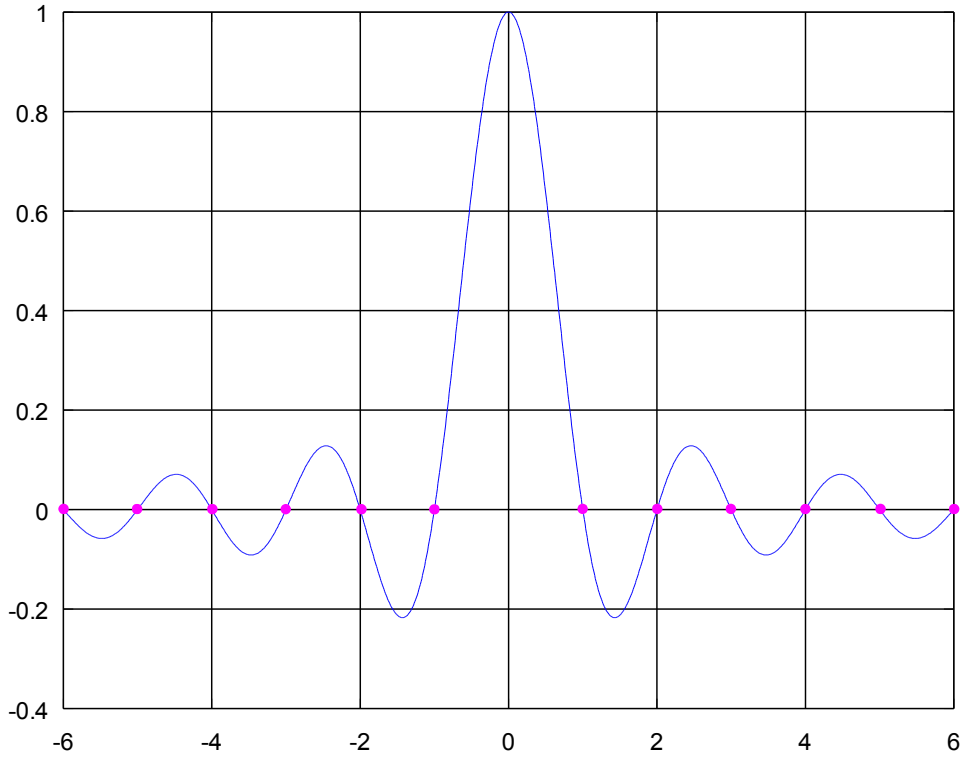
$t = 0 \Rightarrow \mathbf{sinc}(0) = 1$

$\mathbf{sinc}(0) = 1 \quad (t = 0)$

$t = n \Rightarrow \sin(\pi t) = 0$ n: integer $(n \neq 0)$

$\mathbf{sinc}(t) = 0 \quad (t = \pm 1, \pm 2, \pm 3, \cdots)$

Independent Variables: f and ω



$f = -2$ ⇕ $\omega = -2\pi$ | $f = -1$ ⇕ $\omega = -\pi$ | $f = 0$ ⇕ $\omega = 0$ | $f = +1$ ⇕ $\omega = +2\pi$ | $f = +2$ ⇕ $\omega = +4\pi$

Normalized Sinc function

$$sinc(t) = \frac{\sin(\pi t)}{\pi t}$$

$t \leftarrow f$

$$sinc(f) = \frac{\sin(\pi f)}{\pi f} = \frac{\sin(2\pi f/2)}{2\pi f/2}$$

$$sinc(\omega/2) = \frac{\sin(\omega/2)}{\omega/2}$$

$x \leftarrow \omega/2$

$$sinc(x) = \frac{\sin(x)}{x}$$

Unnormalized Sinc function

Example (1)

$$\frac{\sin(\omega T/2)}{\omega/2} = T\frac{\sin(\pi\omega T/2\pi)}{\pi\omega T/2\pi}$$

Normalized Sinc function

$$\mathit{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$

$$t \leftarrow \omega T/2\pi = f T$$

$$\mathit{sinc}(\omega T/2\pi) = \frac{\sin(\pi\omega T/2\pi)}{\pi\omega T/2\pi}$$

$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T\,\mathit{sinc}(\omega T/2\pi)$$

Zeros at $\omega T/2\pi = n$ $\Rightarrow$ $\omega = \frac{2\pi}{T}n$

n: integer $(n \neq 0)$

$$\frac{\sin(\omega T/2)}{\omega/2} = T\frac{\sin(\omega T/2)}{\omega T/2}$$

Unnormalized Sinc function

$$\mathit{sinc}(x) = \frac{\sin(x)}{x}$$

$$x \leftarrow \omega T/2$$

$$\mathit{sinc}(\omega T/2) = \frac{\sin(\omega T/2)}{\omega T/2}$$

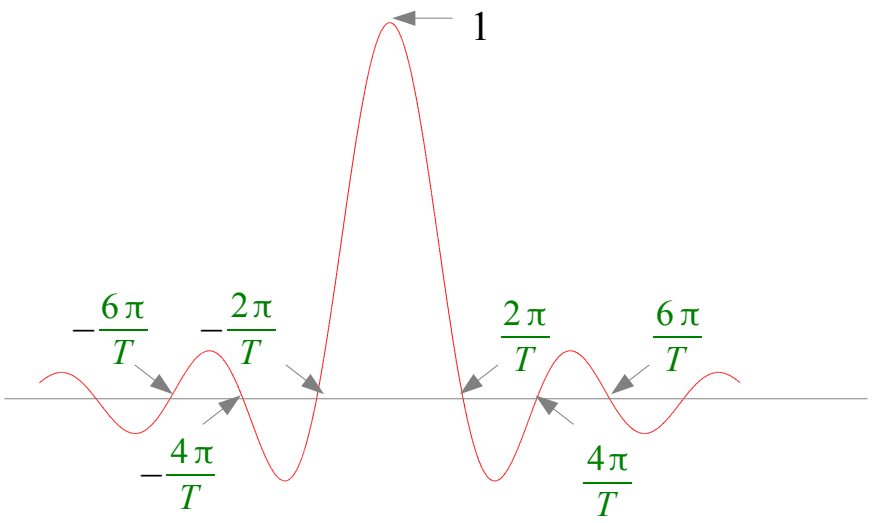
$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T\,\mathit{sinc}(\omega T/2)$$

Zeros at $\omega T/2 = \pi n$ $\Rightarrow$ $\omega = \frac{2\pi}{T}n$

n: integer $(n \neq 0)$

Example (2)

$$\frac{\sin(\omega T/2)}{\omega/2} = T\,\frac{\sin(\pi\omega T/2\pi)}{\pi\omega T/2\pi}$$

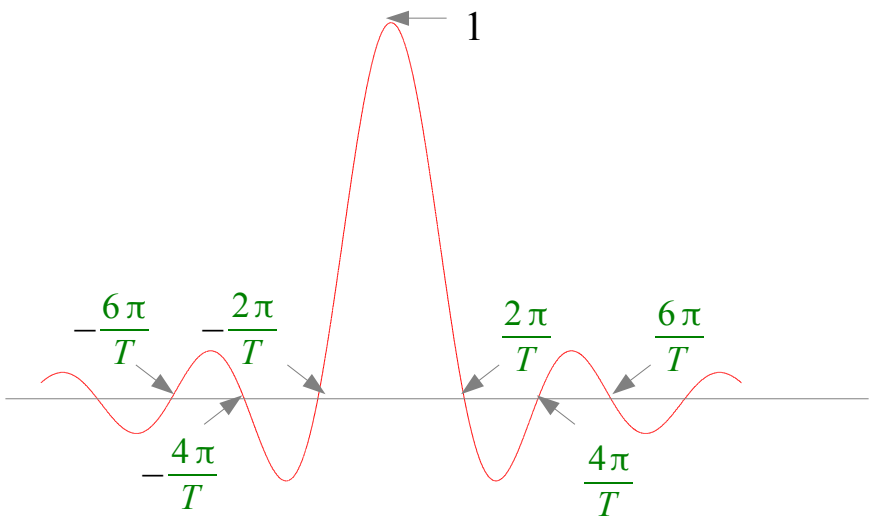


$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T\,\mathbf{sinc}(\omega T/2\pi)$$

Zeros at $\omega T/2\pi = n \Rightarrow \omega = \frac{2\pi}{T}n$

n: integer $(n \neq 0)$

$$\frac{\sin(\omega T/2)}{\omega/2} = T\,\frac{\sin(\omega T/2)}{\omega T/2}$$



$$\frac{\sin(\omega T/2)}{\omega/2} \Rightarrow T\,\mathbf{sinc}(\omega T/2)$$

Zeros at $\omega T/2 = \pi n \Rightarrow \omega = \frac{2\pi}{T}n$

n: integer $(n \neq 0)$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] G. Beale, http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html
[4] C. Langton, http://www.complextoreal.com/chapters/fft1.pdf