CT Sinusoidal Function (0B)

- Continuous Time Sinusoidal Function

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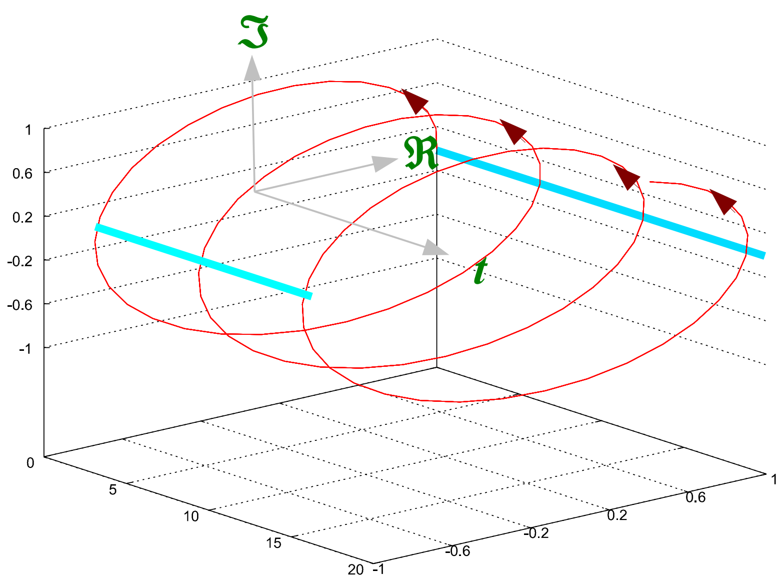
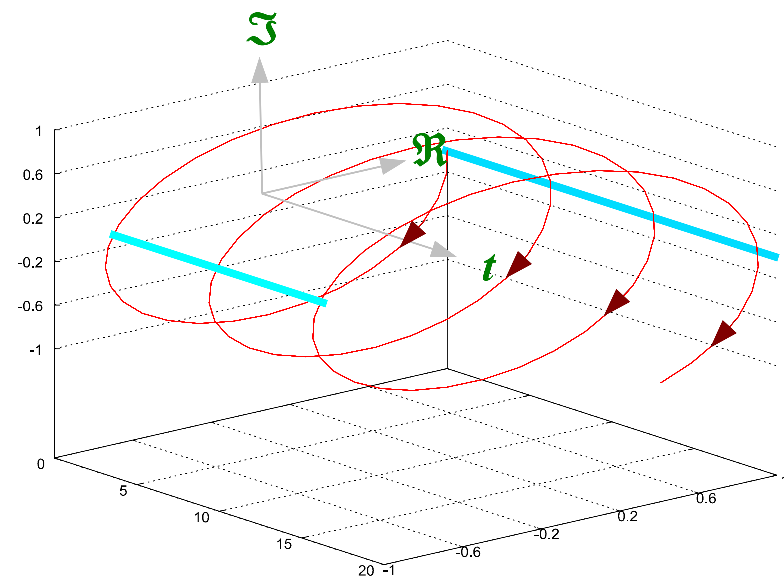
Complex Exponential





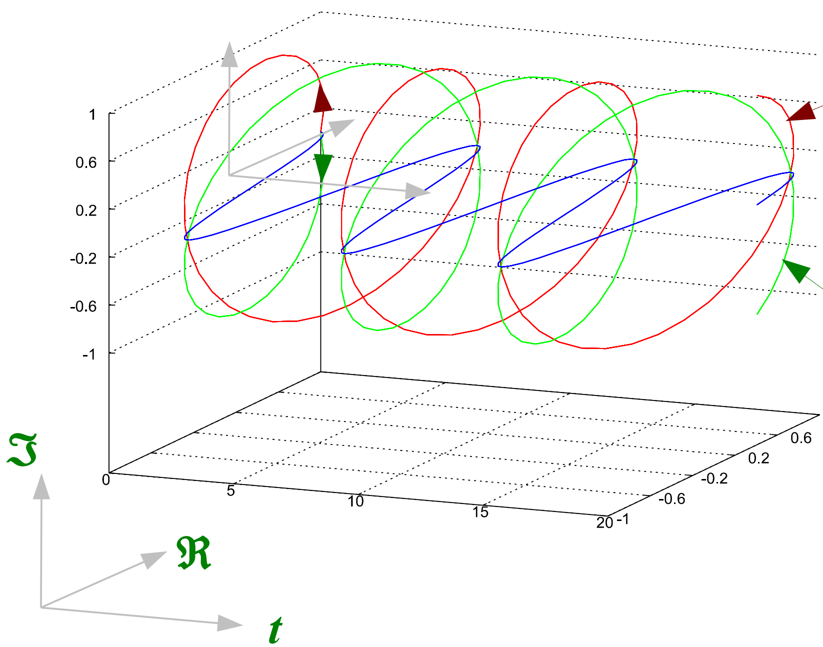
$$e^{+j\omega_0 t} = \cos(\omega_0 t) + j\sin(\omega_0 t)$$

$$e^{+jt} = \cos(t) + j\sin(t) \qquad (\omega_0 = 1)$$

$$e^{-j\omega t} = \cos(\omega_0 t) - j\sin(\omega_0 t)$$

$$e^{-jt} = \cos(t) - j\sin(t) \qquad (\omega_0 = 1)$$

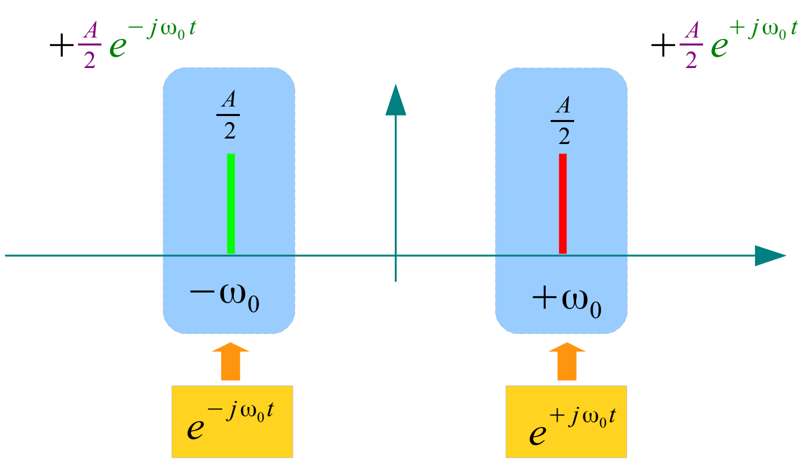
$\text{Cos}(\omega_0 t)$ Spectrum



$$e^{+jt} = \cos(t) + j\sin(t)$$

$$\left(e^{+jt} + e^{-jt}\right) = 2\cos(t)$$

$$e^{-jt} = \cos(t) - j\sin(t)$$

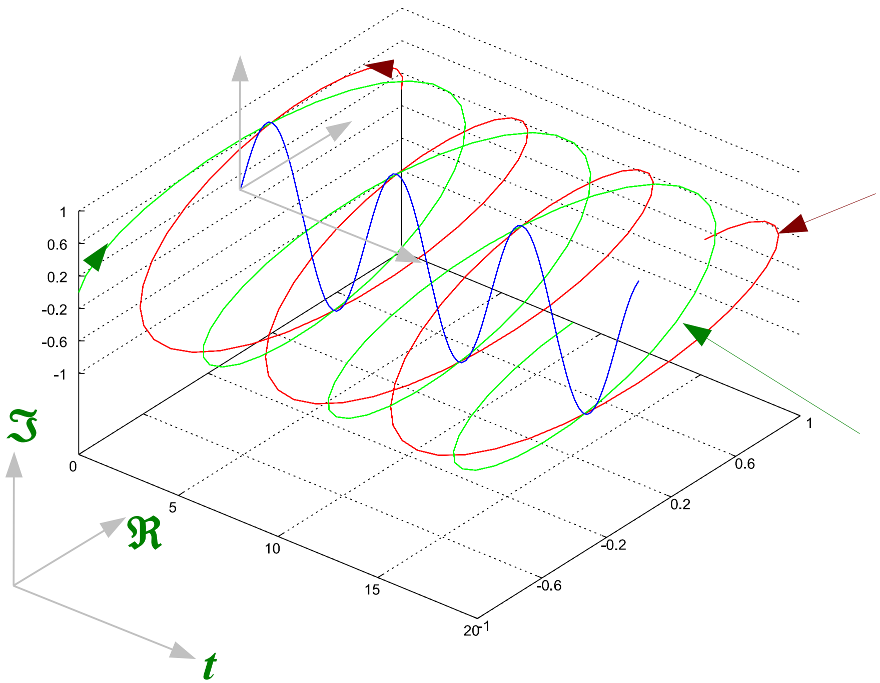


$$x(t) = A\cos(\omega_0 t)$$

$$= \frac{A}{2}e^{+j\omega_0 t} + \frac{A}{2}e^{-j\omega_0 t}$$

spectrum

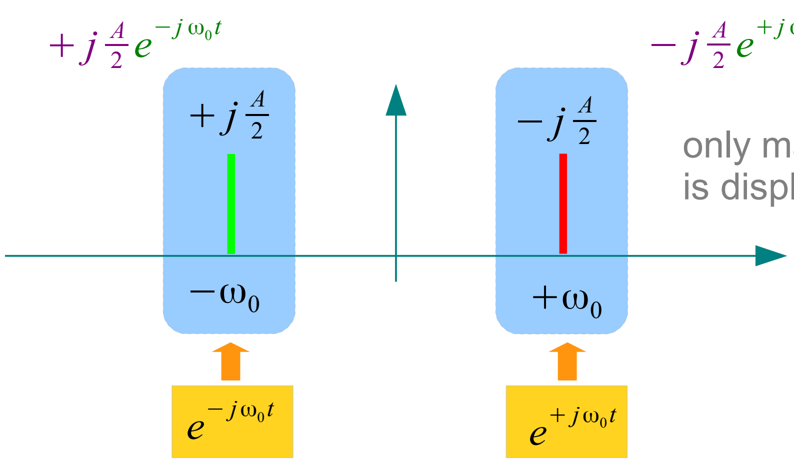
Sin(ω_0t) Spectrum



$$e^{+jt} = \cos(t) + j\sin(t)$$

$$(e^{+jt} - e^{-jt}) = 2j\sin(t)$$

$$-e^{-jt} = -\cos(t) + j\sin(t)$$



only magnitude is display

$$x(t) = A\sin(\omega_0 t)$$

$$= \frac{A}{2j}e^{+j\omega_0 t} - \frac{A}{2j}e^{-j\omega_0 t}$$

spectrum

Complex Exponential Signals

Complex Exponential Signal

$$z(t) = A\, e^{j(\omega_0 t + \phi)}$$

$$\begin{cases} |z(t)| = A & \textbf{\textit{real amplitude}} \\ arg(z(t)) = \omega_0 t + \phi \end{cases}$$

Real Cosine Signal

$$\begin{aligned} x(t) &= \Re\{A\, e^{j(\omega_0 t + \phi)}\} \\ &= A\cos(\omega_0 t + \phi) \end{aligned}$$

$$\begin{aligned} z(t) &= \cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi) \\ &= \cos(\omega_0 t + \phi) + j\sin(\omega_0 t + \phi - \pi/2) \end{aligned}$$

Always lag by $\dfrac{\pi}{2}$

$$\sin(\theta) = \cos(\theta - \frac{\pi}{2})$$

Phasor

Complex Exponential Signal

$$z(t) = A\, e^{j(\omega_0 t + \phi)}$$

$$\begin{cases} |z(t)| = A & \textbf{\textit{real amplitude}} \\ arg(z(t)) = \omega_0 t + \phi \end{cases}$$

Rotating Phasor Notation

$$z(t) = X\, e^{j\omega_0 t}$$

$$\begin{cases} |z(t)| = |X| = A & \textbf{\textit{real amplitude}} \\ arg(z(t)) = \omega_0 t + \phi \end{cases}$$

$$\begin{aligned} z(t) &= A\, e^{j(\omega_0 t + \phi)} \\ &= \boxed{A\, e^{j\phi}}\, e^{j\omega_0 t} \\ &= \boxed{X}\, e^{j\omega_0 t} \end{aligned}$$

$\boxed{X = A\, e^{j\phi}}$ ***complex amplitude***

phasor

$$z(t) = \underline{X}\; \underline{e^{j\omega_0 t}}$$

magnitude A

initial phase ϕ

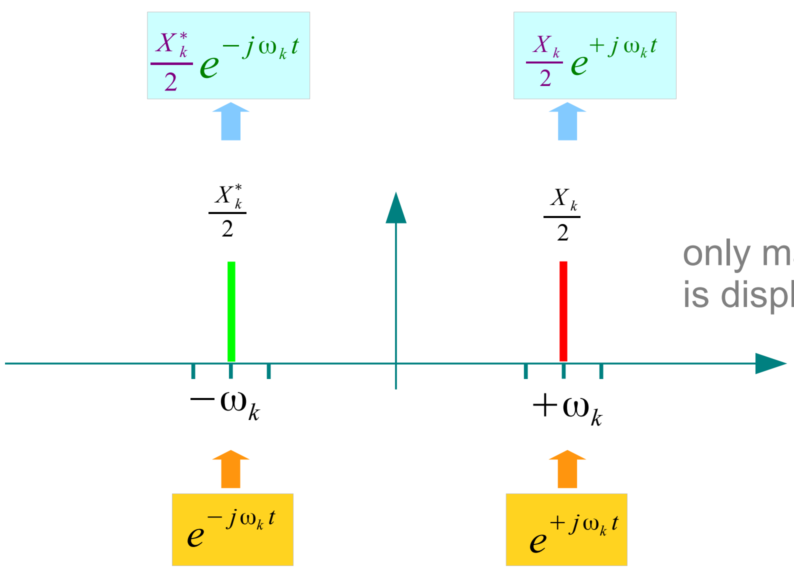
rotating part with the angular speed of $\omega_0 \; (rad/sec)$

Spectrum

Real Single-tone Sinusoidal Signal

$$x(t) = A\cos(\omega_0 t + \phi)$$

$$= \Re\{X e^{j\omega_0 t}\}$$



only magnitude is display

Real Multi-tone Sinusoidal Signal

$$x(t) = A_0 + \sum_{k=1}^{N} \cos(\omega_k t + \phi_k)$$

$$= X_0 + \Re\left\{ \sum_{k=1}^{N} X_k e^{j\omega_k t} \right\} \qquad (X_0 = A_0)$$

$$= X_0 + \sum_{k=1}^{N} \left\{ \boxed{\frac{X_k}{2}} e^{+j\omega_k t} + \boxed{\frac{X_k^*}{2}} e^{-j\omega_k t} \right\}$$

$X_k = A_k e^{j\phi_k}$ the phasor of angular frequency ω_k

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Complex Exponential Signals

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] G. Beale, http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html
[4] C. Langton, http://www.complextoreal.com/chapters/fft1.pdf