Moments (2B)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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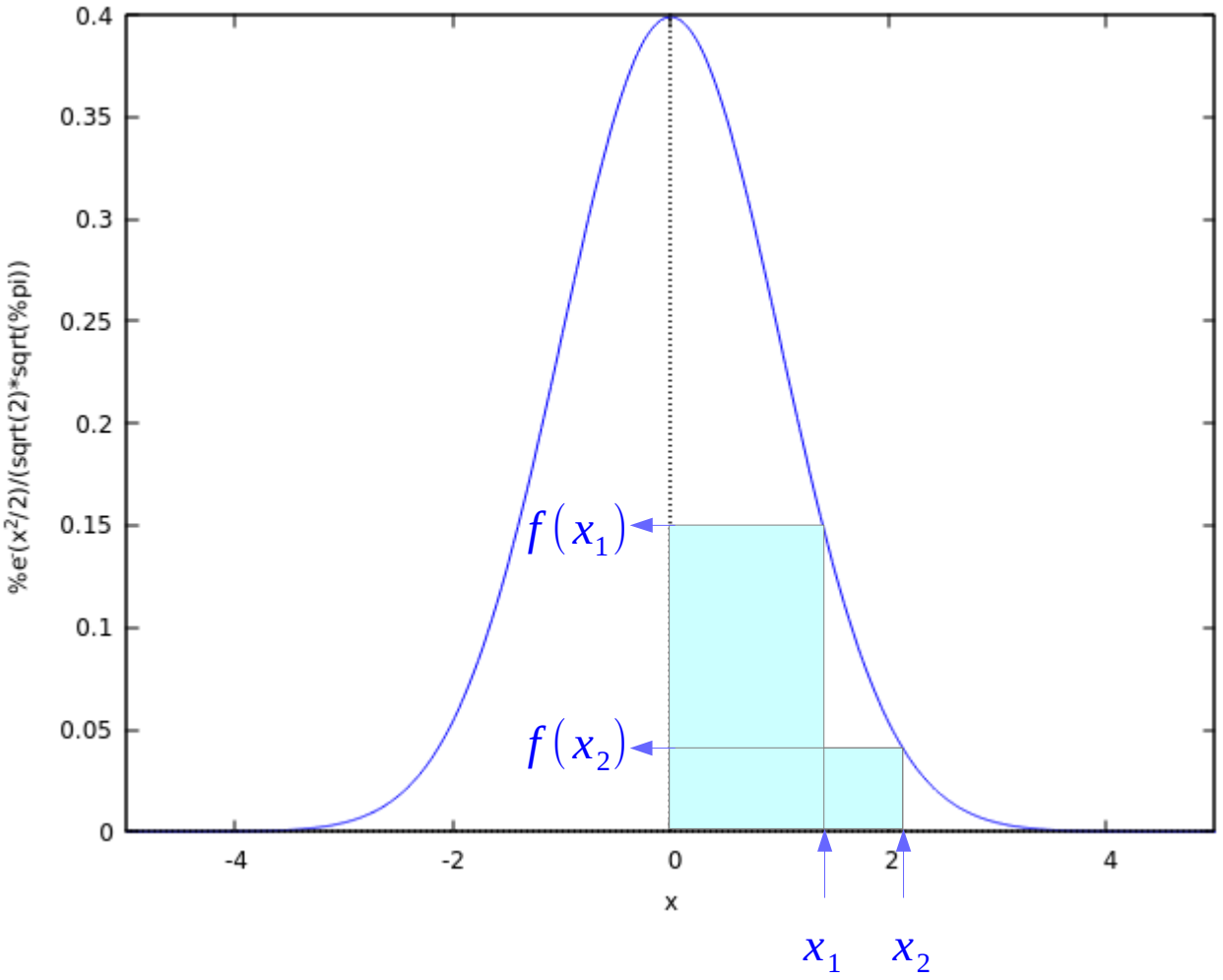
Gaussian Distribution



$$\int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\sum_{k=-\infty}^{\infty} x_k\, f(x_k)\, \Delta_k$$

$$\sum_{k=-\infty}^{\infty} x_k\, y_k\, \Delta_k$$

The Probability Density Function (pdf)

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

$$(x_1y_1+x_2y_2+x_3y_3)^2 = x_1^2y_1^2+x_2^2y_2^2+x_3^2y_3^2+2x_1x_2y_1y_2+2x_2x_3y_2y_3+2x_3x_1y_3y_1$$

$$E[X] = \sum x_i y_i = x_1y_1+x_2y_2+x_3y_3$$
$$E[X^2] = \sum x_i^2 y_i = x_1^2y_1+x_2^2y_2+x_3^2y_3$$

$$\begin{aligned} E[X^2]-E^2[X] &= (x_1^2y_1+x_2^2y_2+x_3^2y_3) - (x_1y_1+x_2y_2+x_3y_3)^2 \\ &= (x_1^2y_1+x_2^2y_2+x_3^2y_3) - (x_1^2y_1^2+x_2^2y_2^2+x_3^2y_3^2) \\ &\quad -(2x_1y_1x_2y_2+2x_2y_2x_3y_3+2x_3y_3x_1y_1) \end{aligned}$$

The Probability Density Function (pdf)

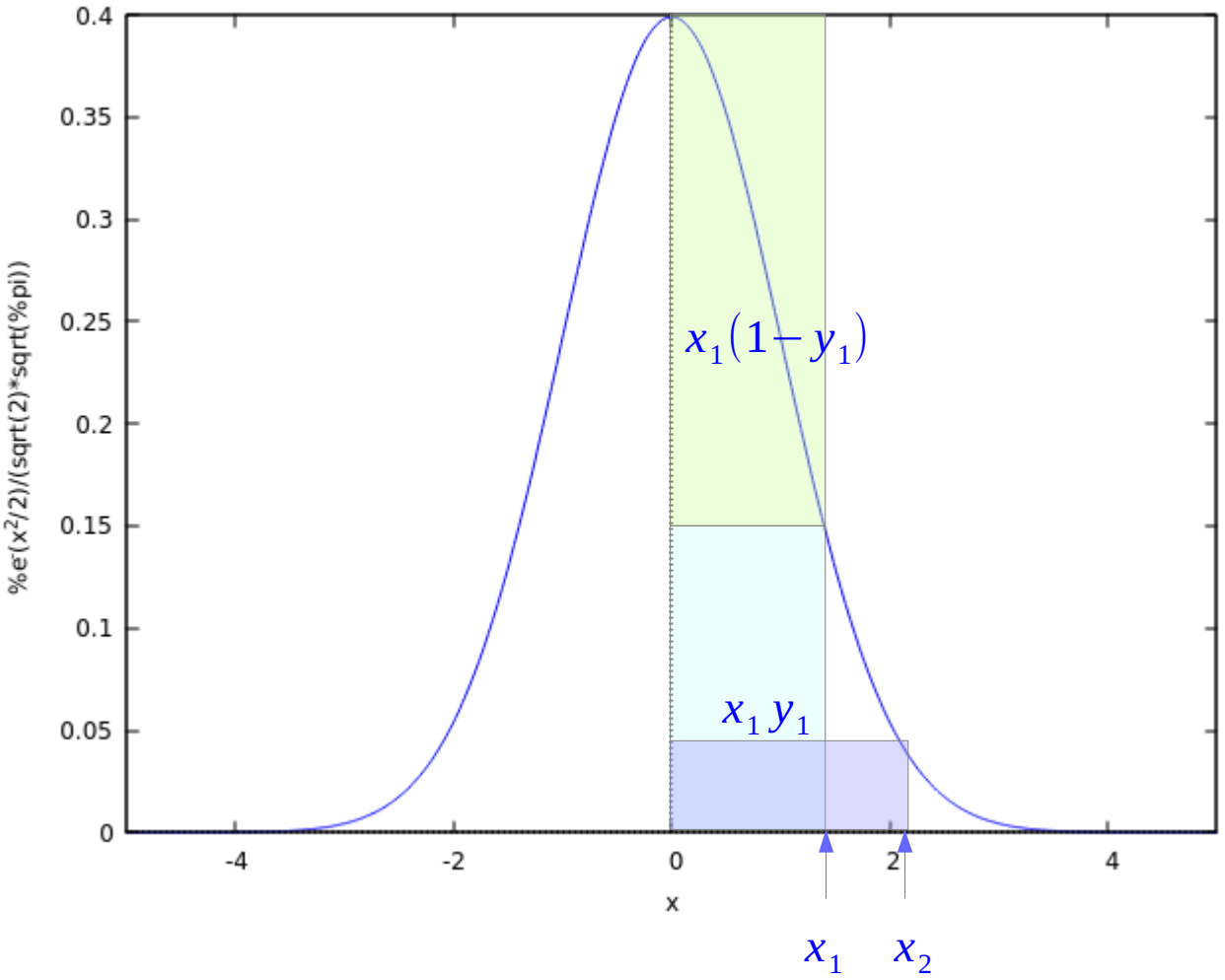
$$E[X^2]-E^2[X] = (x_1^2 y_1 + x_2^2 y_2 + x_3^2 y_3) - (x_1 y_1 + x_2 y_2 + x_3 y_3)^2$$

$$= (x_1^2 y_1 + x_2^2 y_2 + x_3^2 y_3) - (x_1^2 y_1^2 + x_2^2 y_2^2 + x_3^2 y_3^2)$$
$$-(2 x_1 y_1 x_2 y_2 + 2 x_2 y_2 x_3 y_3 + 2 x_3 y_3 x_1 y_1)$$

$$\begin{array}{lll} + x_1^2 y_1 & + x_2^2 y_2 & + x_3^2 y_3 \\ - x_1^2 y_1^2 & - x_2^2 y_2^2 & - x_3^2 y_3^2 \\ - x_1 y_1 x_2 y_2 & - x_2 y_2 x_1 y_1 & - x_3 y_3 x_1 y_1 \\ - x_1 y_1 x_3 y_3 & - x_2 y_2 x_3 y_3 & - x_3 y_3 x_2 y_2 \end{array}$$

$$\begin{aligned} x_1 y_1 (x_1 - x_1 y_1 - x_2 y_2 - x_3 y_3) &= x_1 y_1 (x_1 - E[X]) \\ x_2 y_2 (x_2 - x_1 y_1 - x_2 y_2 - x_3 y_3) &= x_2 y_2 (x_2 - E[X]) \\ x_3 y_3 (x_3 - x_1 y_1 - x_2 y_2 - x_3 y_3) &= x_3 y_3 (x_3 - E[X]) \end{aligned}$$

Gaussian Distribution

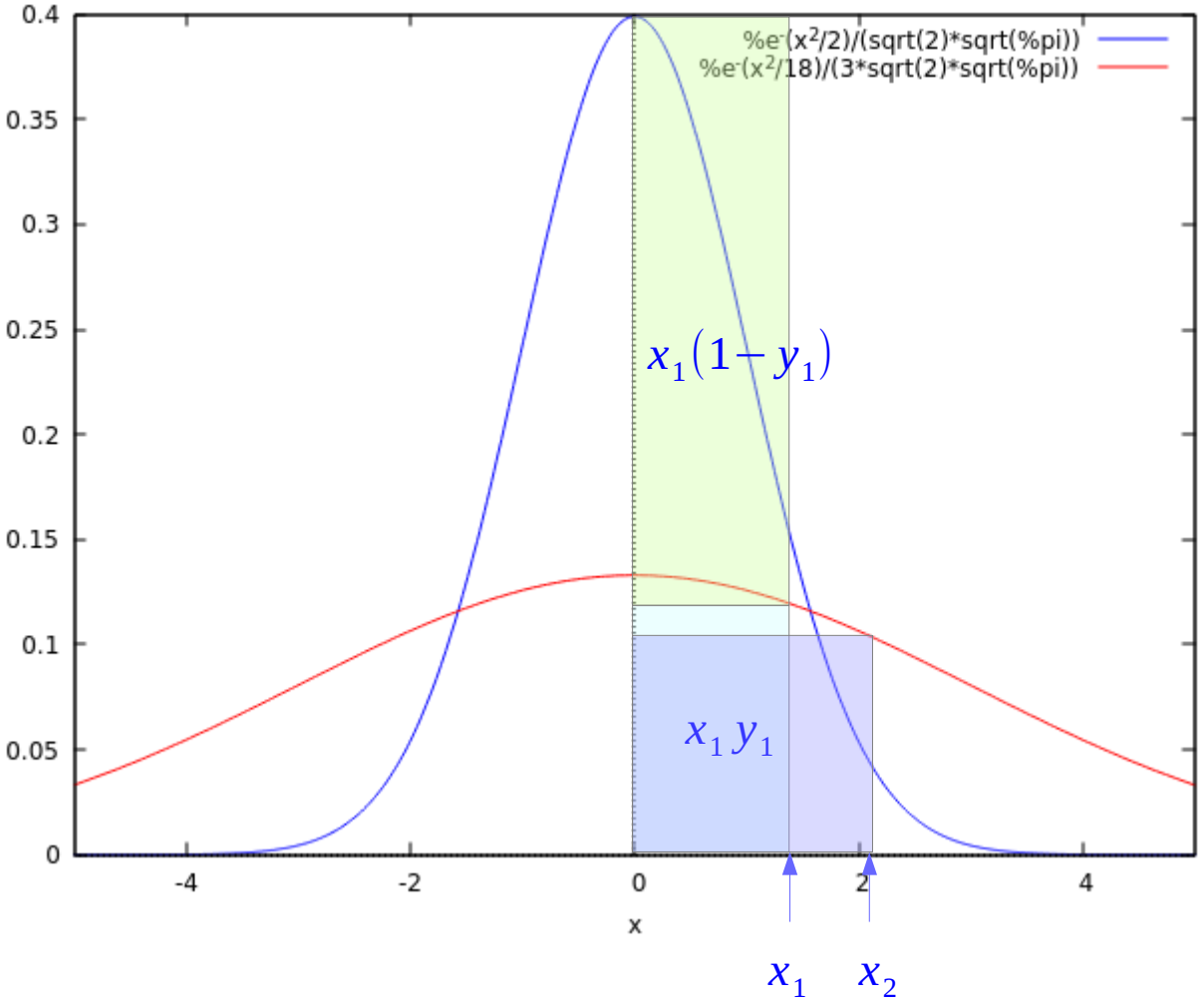


$$\int_{-\infty}^{\infty} x f(x)\,dx$$

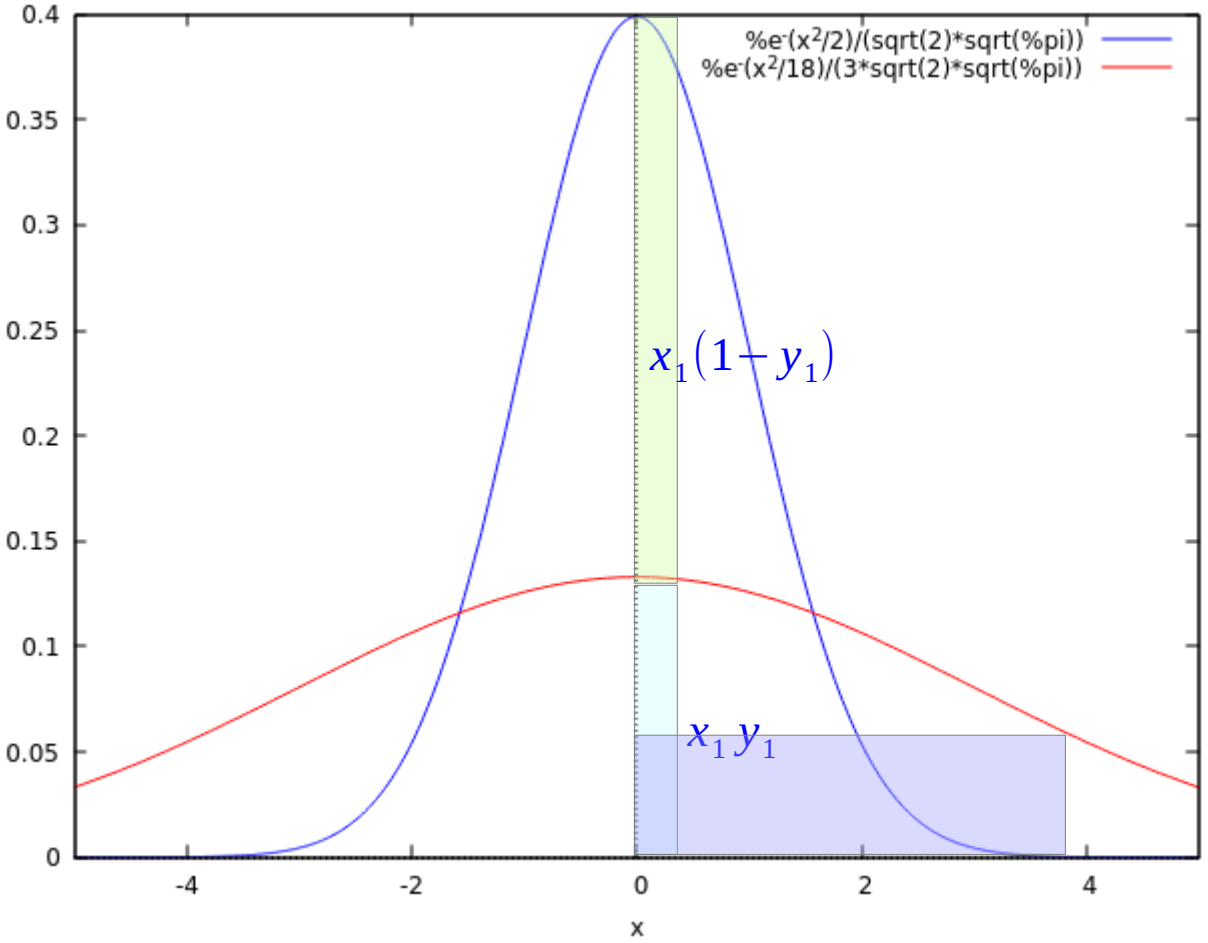
$$\sum_{k=-\infty}^{\infty} x_k f(x_k)\,\Delta_k$$

$$\sum_{k=-\infty}^{\infty} x_k y_k \Delta_k$$

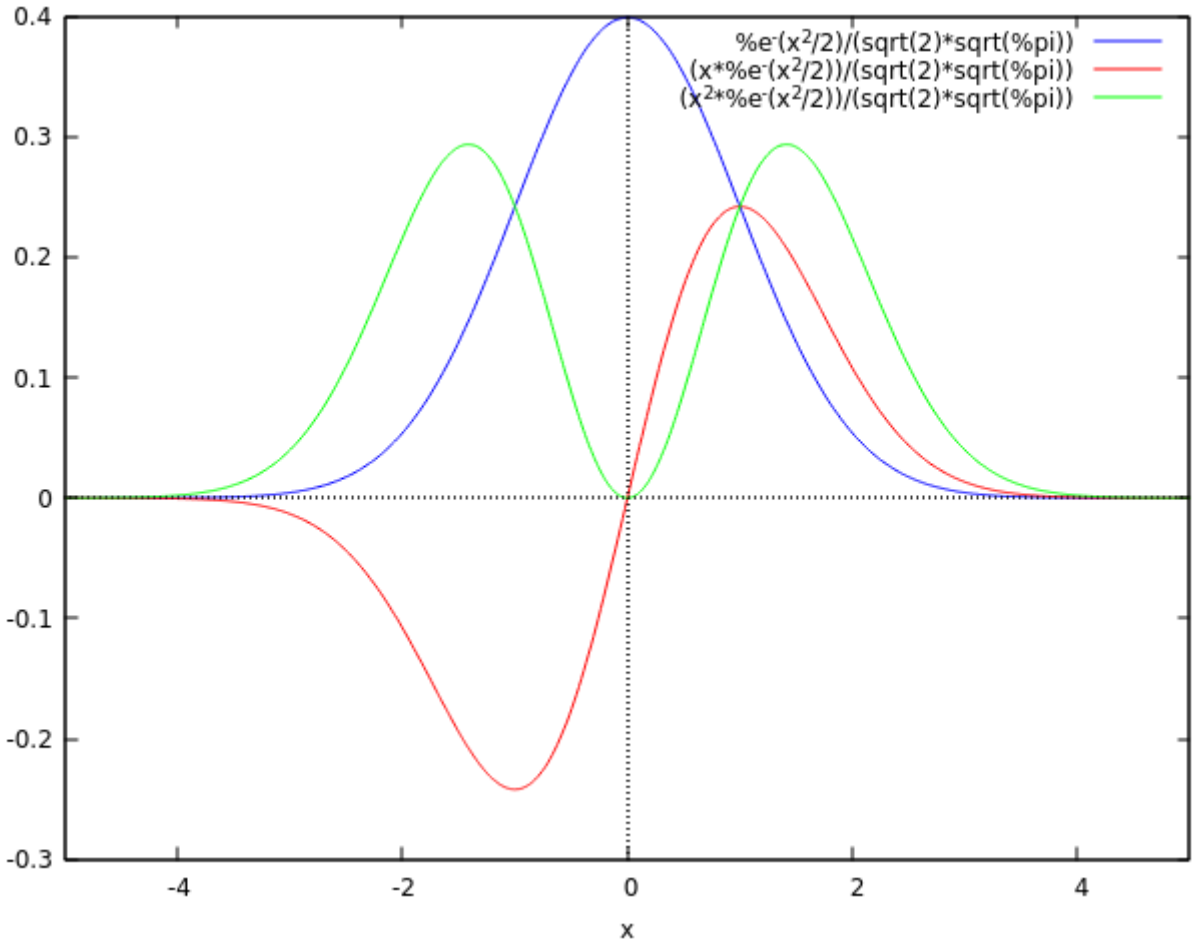
Moment Functions



Moment Functions



Moment Function Plots



Numerical Integration Results

```
(%i16) float(integrate(f(x,0,1), x, -10, +10));
(%o16) 1.0

(%i15) float(integrate(g(x,0,1), x, -10, +10));
(%o15) 0.0

(%i27) float(integrate(h(x,0,1), x, -10, +10));
(%o27) 0.9999999999999998
```

The core function test

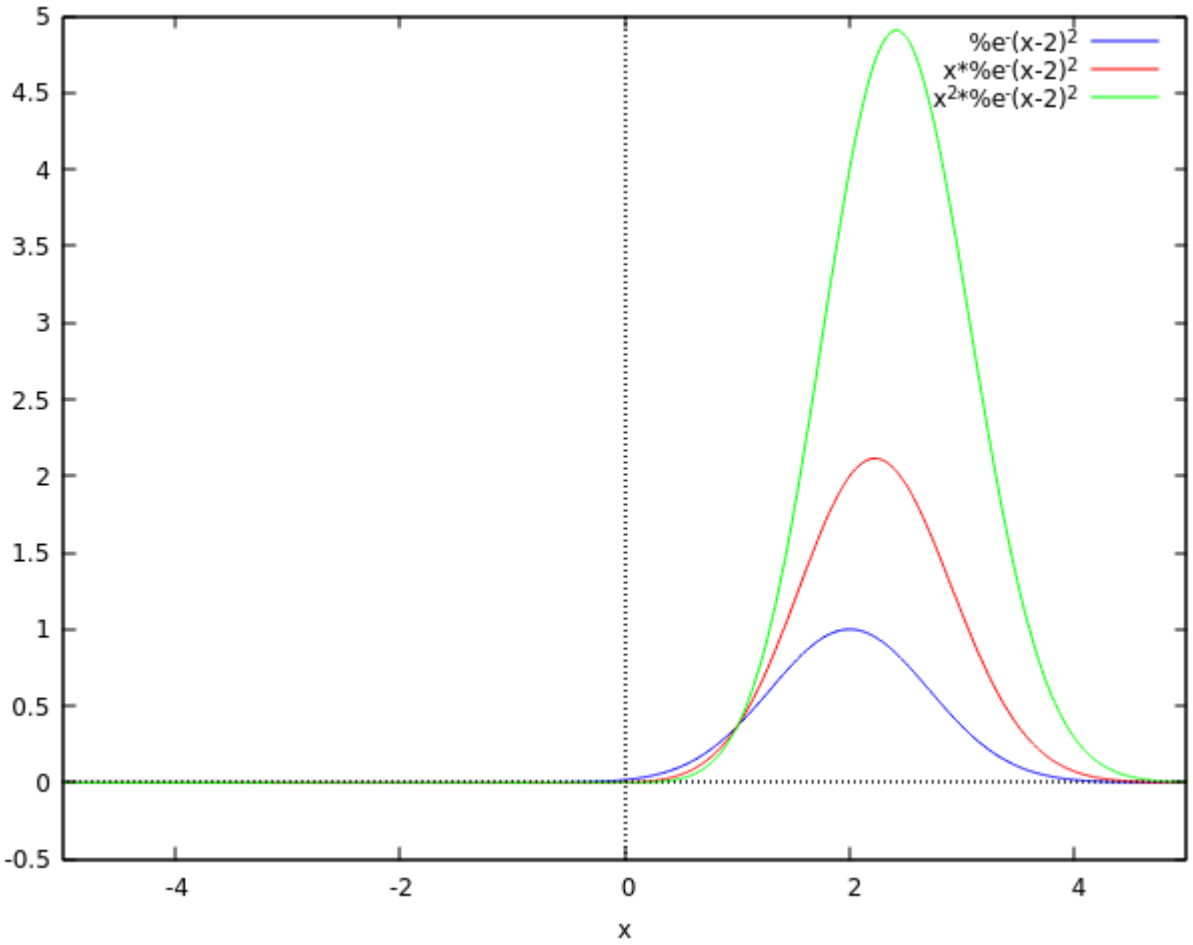
```
(%i28) m1(x) := %e^(-(x-2)^2);
(%o28) m1(x):=%e^-(x-2)^2

(%i29) m2(x) := m1(x) * x;
(%o29) m2(x):=m1(x) x

(%i30) m3(x) := m1(x) * x^2;
(%o30) m3(x):=m1(x) x^2

(%i31) plot2d([m1(x), m2(x), m3(x)], [x, -5, 5], [plot_format, gnuplot]);
(%o31) [/home/young/maxout.gnuplot]
```

The core function plots



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann