Density Functions

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Probability Density Function

Definition

$$f_X(x) = \frac{dF_x(x)}{dx}$$

- the *density* function of the random variable X
- the derivative of the *distribution* function $F_x(x)$

Probability Density Function - a discrete R.V.

Definition

$$F_X(x) = \sum_{i=1}^{N} P\{X = x_i\} u(x - x_i)$$

$$f_X(x) = \sum_{i=1}^{N} P\{X = x_i\} \delta(x - x_i) = \frac{dF_x(x)}{dx}$$

- a discrete random variable X
- $F_x(x)$: the *cumulative distribution* function
- $f_X(x)$: the *proabability density* function

Unit impulse function

Definition

- **unit impulse** function $\delta(x) = \dfrac{du(x)}{dx}$
- **unit step** function $u(x) = \int_{-\infty}^{x} \delta(\xi) d\xi$
- any continuous function $\phi(x)$, $\phi(x_0) = \int_{-\infty}^{\infty} \phi(x)\delta(x - x_0)dx$

The properties of a density function

- $0 \leq f_X(x)$ for all x
- $\int_{-\infty}^{+\infty} f_X(x)dx = 1$
- $F_X(x) = \int_{-\infty}^{x} f_X(\xi)d\xi$
- $x_1 < x_2 \implies F_X(x_1) \leq F_X(x_2)$
- $\int_{x_1}^{x_2} f_X(x)dx = P\{x_1 < X \leq x_2\}$