Second Order ODE's (2B)

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$$\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 1y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$m^2 + 3m + 2 = 0$$

$$m^2 + 2m + 1 = 0$$

$$m^2 + 2m + 2 = 0$$

$$(m+2)(m+1) = 0$$

$$(m+1)^2 = 0$$

$$(m+1-i)(m+1+i) = 0$$

$$m = -1, \ -2$$

$$m = -1$$

$$m = -1+i, \ -1-i$$

$$e^{-1x}, \ e^{-2x}$$

$$e^{-1x}, \ xe^{-1x}$$

$$e^{(-1+i)x}, \ e^{(-1-i)x}$$

$$y = c_1 e^{-1x} + c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$

$$y = c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}$$

$$y = e^{-x}\left(c_1 e^{+ix} + c_2 e^{-ix}\right)$$

$$y = e^{-x}\left(c_3 \cos x + c_4 \sin x\right)$$

First Order ODE examples – verification (1)

$$\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 1y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$y = c_1 e^{-1x} + c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$

$$y = c_1 e^{(-1+i)x} + c_2 e^{(-1-i)x}$$

$$2y = 2c_1 e^{-1x} + 2c_2 e^{-2x}$$
$$y' = -c_1 e^{-1x} - 2c_2 e^{-2x}$$
$$3y' = -3c_1 e^{-1x} - 6c_2 e^{-2x}$$
$$y'' = +c_1 e^{-1x} + 4c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$
$$y' = -c_1 e^{-1x} + c_2 e^{-1x}$$
$$\qquad - c_2 x e^{-1x}$$
$$2y' = -2c_1 e^{-1x} + 2c_2 e^{-1x}$$
$$\qquad - 2c_2 x e^{-1x}$$
$$y'' = +c_1 e^{-1x} - c_2 e^{-1x}$$
$$\qquad - c_2 e^{-1x} + c_2 x e^{-1x}$$

$$2y = 2c_1 e^{(-1+i)x} + 2c_2 e^{(-1-i)x}$$
$$y' = (-1+i)c_1 e^{(-1+i)x}$$
$$\qquad + (-1-i)c_2 e^{(-1-i)x}$$
$$2y' = (-2+2i)c_1 e^{(-1+i)x}$$
$$\qquad + (-2-2i)c_2 e^{(-1-i)x}$$
$$y'' = (-1+i)^2 c_1 e^{(-1+i)x}$$
$$\qquad + (-1-i)^2 c_2 e^{(-1-i)x}$$
$$= -2ic_1 e^{(-1+i)x} + 2ic_2 e^{(-1-i)x}$$

First Order ODE examples – verification (2)

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2\,y \;=\; 0$$

$$y = e^{-x}\left(c_1 e^{+ix} + c_2 e^{-ix}\right)$$

$$y = e^{-x}\left(c_3 \cos x + c_4 \sin x\right)$$

$$y = e^{-x}\left(c_3 \cos x + c_4 \sin x\right)$$

$$2\,y = 2\,c_1 e^{(-1+i)x} + 2\,c_2 e^{(-1-i)x}$$

$$y' = (-1+i)\,c_1 e^{(-1+i)x} + (-1-i)\,c_2 e^{(-1-i)x}$$

$$2\,y' = (-2+2i)\,c_1 e^{(-1+i)x} + (-2-2i)\,c_2 e^{(-1-i)x}$$

$$y'' = (-1+i)^2 c_1 e^{(-1+i)x} + (-1-i)^2 c_2 e^{(-1-i)x}$$

$$\quad = -2i\,c_1 e^{(-1+i)x} + 2i\,c_2 e^{(-1-i)x}$$

$$y'' + 2\,y' + 2\,y \;=\; 0$$

First Order ODE examples – verification (3)

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$y = e^{-x}(c_1 e^{+ix} + c_2 e^{-ix})$$

$$y = e^{-x}(c_3 \cos x + c_4 \sin x)$$

$$\boxed{y = e^{-x}(c_3 \cos x + c_4 \sin x)}$$

$$\underline{2y = e^{-x}(2c_3 \cos x + 2c_4 \sin x)}$$

$$y' = -e^{-x}(c_3 \cos x + c_4 \sin x) + e^{-x}(-c_3 \sin x + c_4 \cos x)$$

$$= e^{-x}((-c_3 + c_4)\cos x - (c_3 + c_4)\sin x)$$
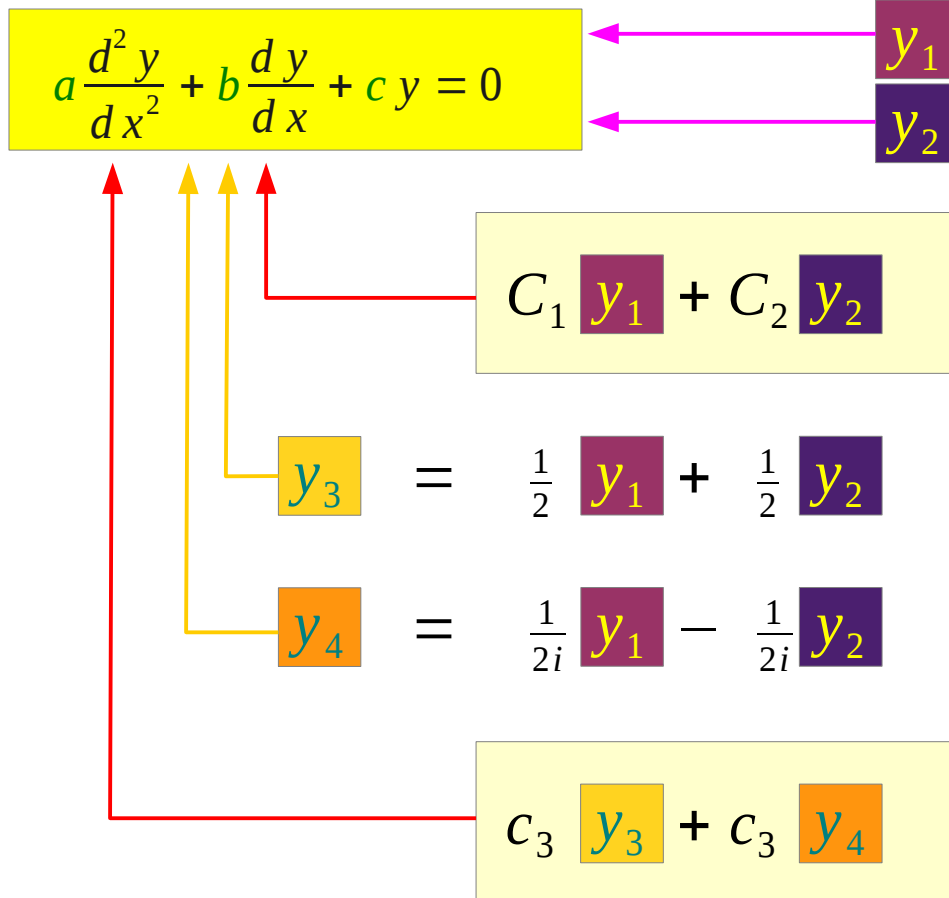
$$\underline{2y' = e^{-x}(2(-c_3 + c_4)\cos x - 2(c_3 + c_4)\sin x)}$$

$$y'' = -e^{-x}((-c_3 + c_4)\cos x - (c_3 + c_4)\sin x) + e^{-x}((c_3 - c_4)\sin x - (c_3 + c_4)\cos x)$$

$$= e^{-x}((c_3 - c_4 - c_3 - c_4)\cos x + (c_3 + c_4 + c_3 - c_4)\sin x)$$

$$\underline{y'' = e^{-x}(-2c_4 \cos x + 2c_3 \sin x)}$$

$$y'' + 2y' + 2y = 0$$

Fundamental Set Examples

Second Order EQ

$$a\frac{d^2 y}{d x^2} + b\frac{d y}{d x} + c\,y = 0$$

y_1

y_2

$e^{(\alpha + i\beta)x}$

$e^{(\alpha - i\beta)x}$

$$C_1\,\boxed{y_1} + C_2\,\boxed{y_2}$$

$$C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x}$$

$$\boxed{y_3} = \tfrac{1}{2}\,\boxed{y_1} + \tfrac{1}{2}\,\boxed{y_2}$$

$$\{e^{(\alpha + i\beta)x} + e^{(\alpha + i\beta)x}\}/2 = e^{\alpha x}\cos(\beta x)$$

$$\boxed{y_4} = \tfrac{1}{2i}\,\boxed{y_1} - \tfrac{1}{2i}\,\boxed{y_2}$$

$$\{e^{(\alpha + i\beta)x} + i e^{(\alpha + i\beta)x}\}/2i = e^{\alpha x}\sin(\beta x)$$

$$c_3\,\boxed{y_3} + c_3\,\boxed{y_4}$$

$$c_3 e^{\alpha x}\cos(\beta x) + c_4 e^{\alpha x}\sin(\beta x)$$

$$= e^{\alpha x}\left(c_3\cos(\beta x) + c_4\sin(\beta x)\right)$$

Complex Exponentials
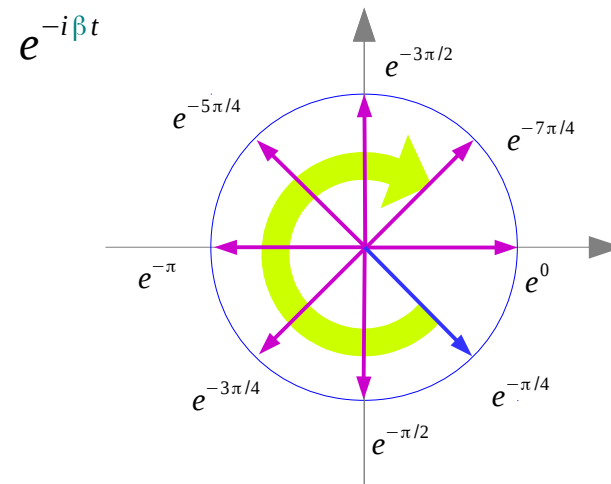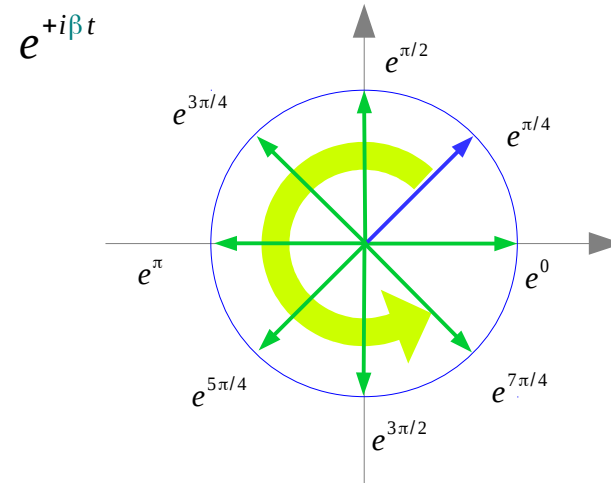
$e^{+i\beta t}$

$e^{\pi/2}$

$e^{3\pi/4}$ $e^{\pi/4}$

e^{π} e^{0}

$e^{5\pi/4}$ $e^{7\pi/4}$

$e^{3\pi/2}$

$e^{-i\beta t}$

$e^{-3\pi/2}$

$e^{-5\pi/4}$ $e^{-7\pi/4}$

$e^{-\pi}$ e^{0}

$e^{-3\pi/4}$ $e^{-\pi/4}$

$e^{-\pi/2}$

First Order ODE examples (1)

$$\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 1y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$

$$y = c_1 e^{-1x} + c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$

$$y = e^{-x}(c_3 \cos x + c_4 \sin x)$$

```
(%i45) wxplot2d([2*(%e^(-x)+%e^(-2*x)), 3*(-%e^(-x)-2*%e^(-2*x)),(%e^(-x)+4*%e^(-2*x))], [x,-2,2])$
```

(%t45)

Legend:
- 2*(%e^-x+%e^-(2*x))
- 3*(-%e^-x-2*%e^-(2*x))
- %e^-x+4*%e^-(2*x)

$$y = (e^{-1x} + e^{-2x})$$

$$2y = 2(e^{-1x} + e^{-2x})$$

$$3y' = 3(-e^{-1x} - 2e^{-2x})$$

$$y'' = +e^{-1x} + 4e^{-2x}$$

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First Order ODE examples (1)

$$\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 1y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$$
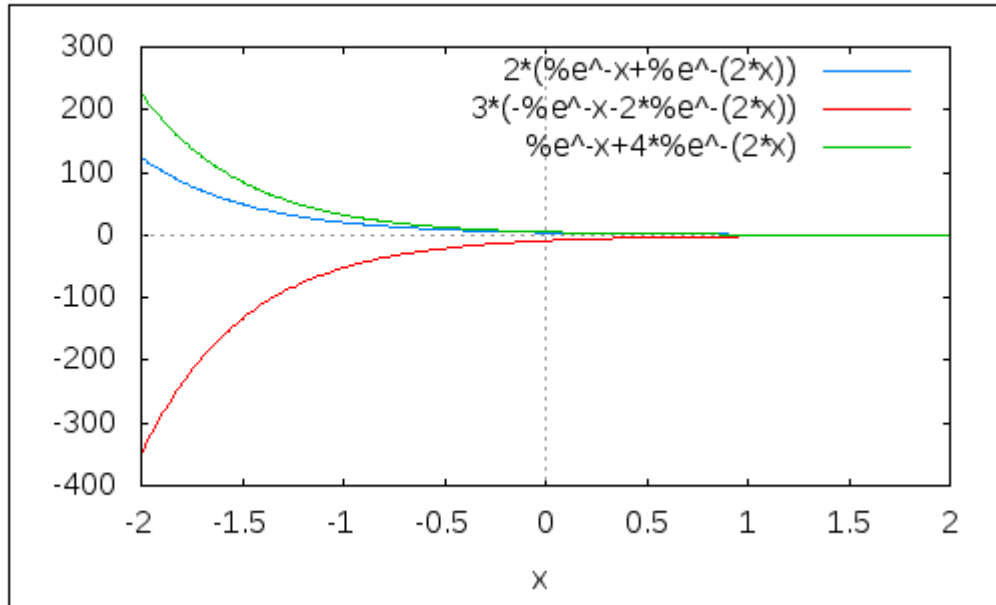
$$y = c_1 e^{-1x} + c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$

$$y = e^{-x}(c_3 \cos x + c_4 \sin x)$$

(%i47) wxplot2d([%e^(-x)+x*%e^(-x),-2*x*%e^(-x), -%e^(-x) +x*%e^(-x)], [x,-2,2])$
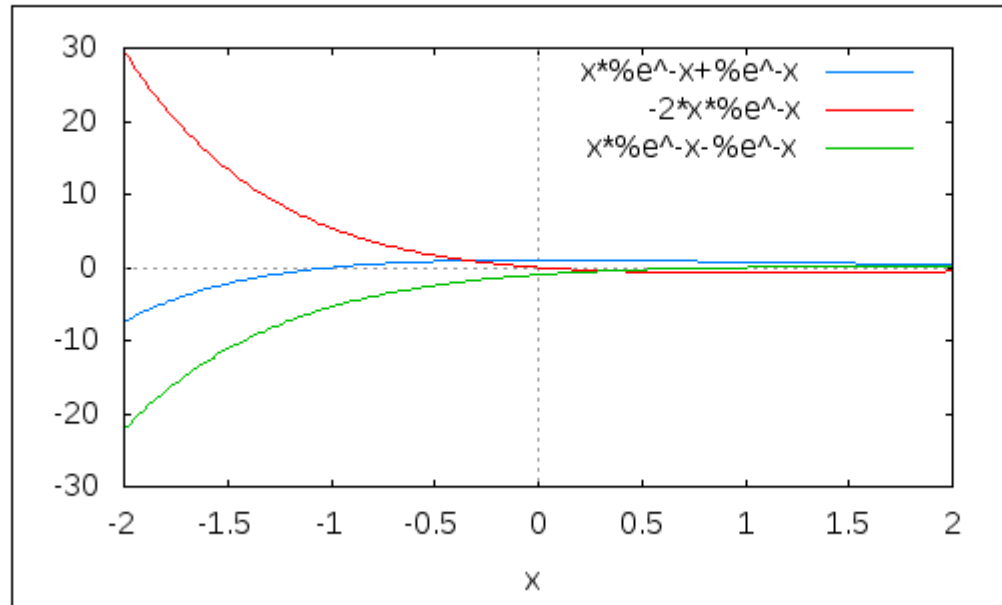
(%t47)

$$y = (e^{-x} + x e^{-x})$$

$$2y' = 2(-x e^{-x})$$

$$y'' = -e^{-x} + x e^{-x}$$

First Order ODE examples (1)

$$\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + 2\,y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 1\,y = 0$$

$$\frac{d^2 y}{dx^2} + 2\frac{dy}{dx} + 2\,y = 0$$

$$y = c_1 e^{-1x} + c_2 e^{-2x}$$

$$y = c_1 e^{-1x} + c_2 x e^{-1x}$$

$$y = e^{-x}(c_3 \cos x + c_4 \sin x)$$

(%i50) wxplot2d([%e^(-x)*(cos(x)+sin(x)), 4*%e^(-4*x)*cos(x), -2*%e^(-x)*(cos(x) + sin(x))], [x,-5, 5])$

(%t50)
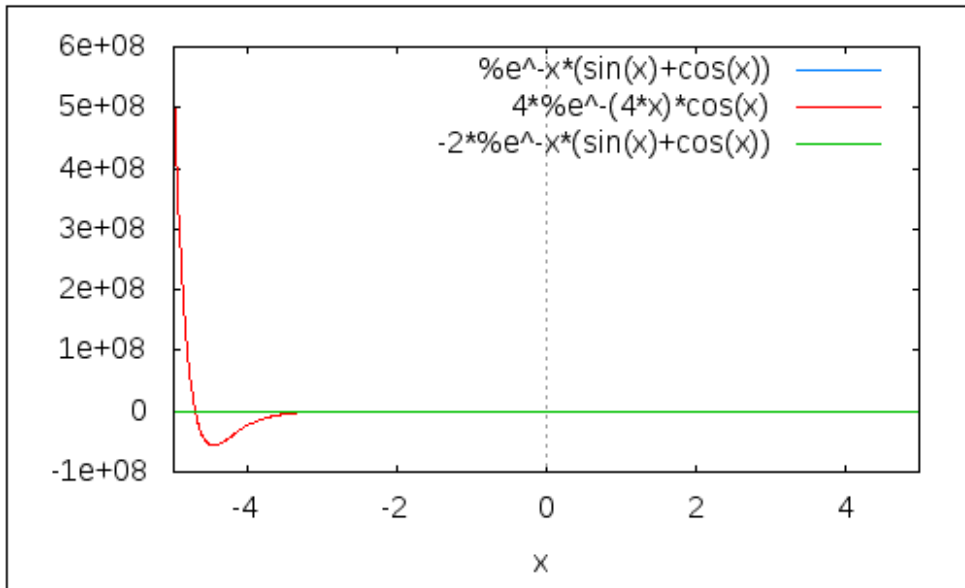
$$y = e^{-x}(\cos x + \sin x)$$

$$2\,y = 2 e^{-x}(\cos x + \sin x)$$

$$2\,y' = 2 e^{-x}(\cos x + \sin x)$$

$$\qquad + 2 e^{-x}(-\sin x + \cos x)$$

$$\qquad = 2\cdot 2 e^{-x}\cos x$$

$$y'' = 2(-e^{-x}\cos x - e^{-x}\sin x)$$

cos(x) + sin(x)

(%i52) wxplot2d([cos(x), sin(x), (cos(x)+sin(x))], [x,-10, 10])$

(%t52)
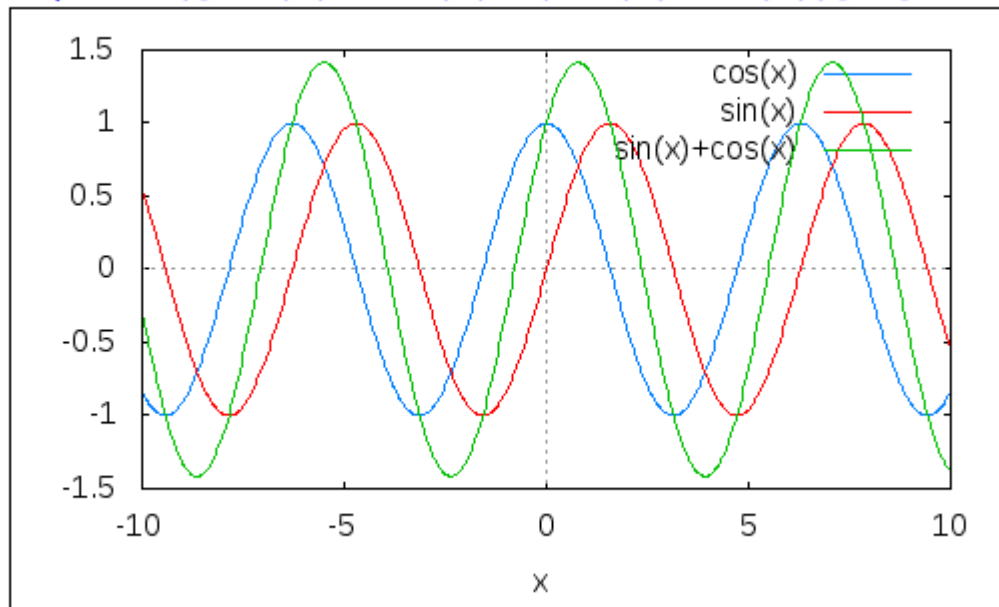
e^{-x} (cos(x) + sin(x))


```
(%i55) wxplot2d([%e^(-x), (cos(x)+sin(x)), %e^(-x)*(cos(x)+sin(x))], [x,-10, 10], [y, -10, +10])$
plot2d: some values were clipped.
plot2d: some values were clipped.
```

(%t55)
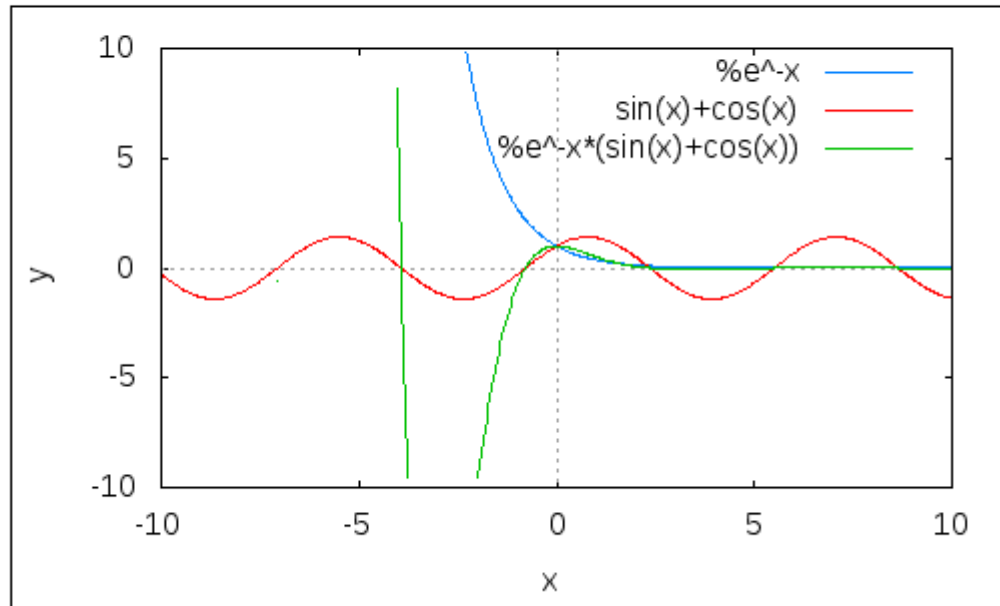
e^{-x} (cos(x) + sin(x))

$e^{-x}\,(\cos(x) + \sin(x))$

```
(%i57) wxplot2d([%e^(-x), (cos(x)+sin(x)), %e^(-x)*(cos(x)+sin(x))], [x,-10, 10], [y, -20, +20])$
plot2d: some values were clipped.
plot2d: some values were clipped.
```

(%t57)

e^{-x} (cos(x) + sin(x))

x e^{-x}

$$x\, e^{-x}$$


```
(%i60) wxplot2d([%e^(-x), x, %e^(-x)*x], [x,-10, 10], [y, -10, 10])$
plot2d: some values were clipped.
plot2d: some values were clipped.

(%t60)
```
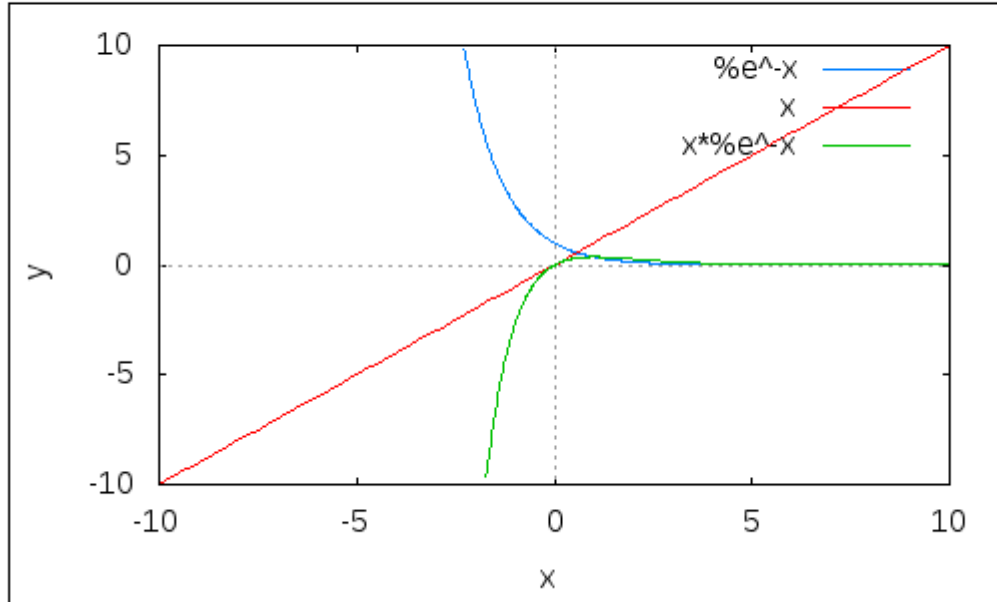
x e⁻ˣ

```
(%i61) wxplot2d([%e^(-x), x, %e^(-x)*x], [x,-10, 10], [y, -50, 50])$
plot2d: some values were clipped.
plot2d: some values were clipped.
```
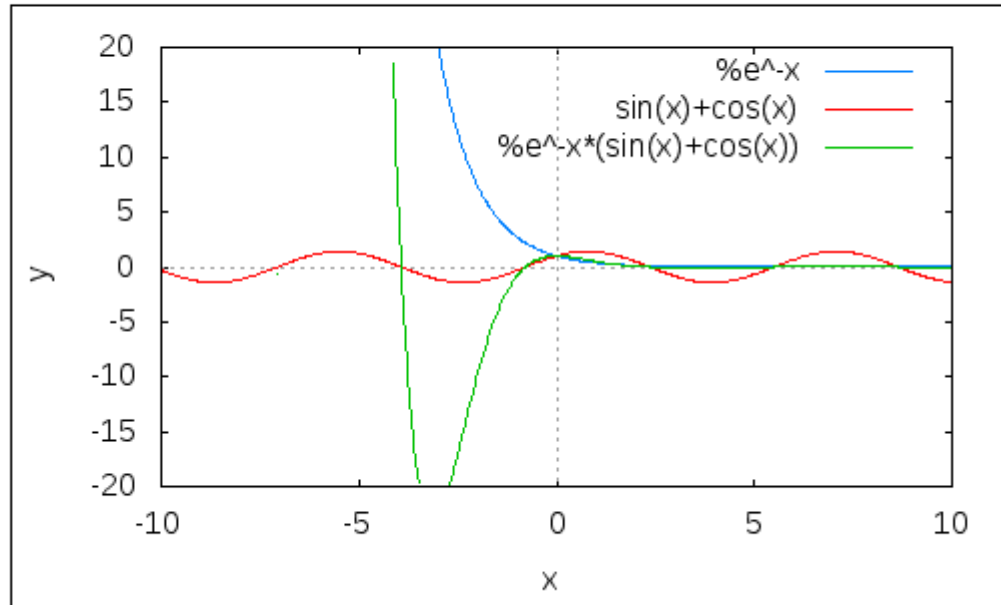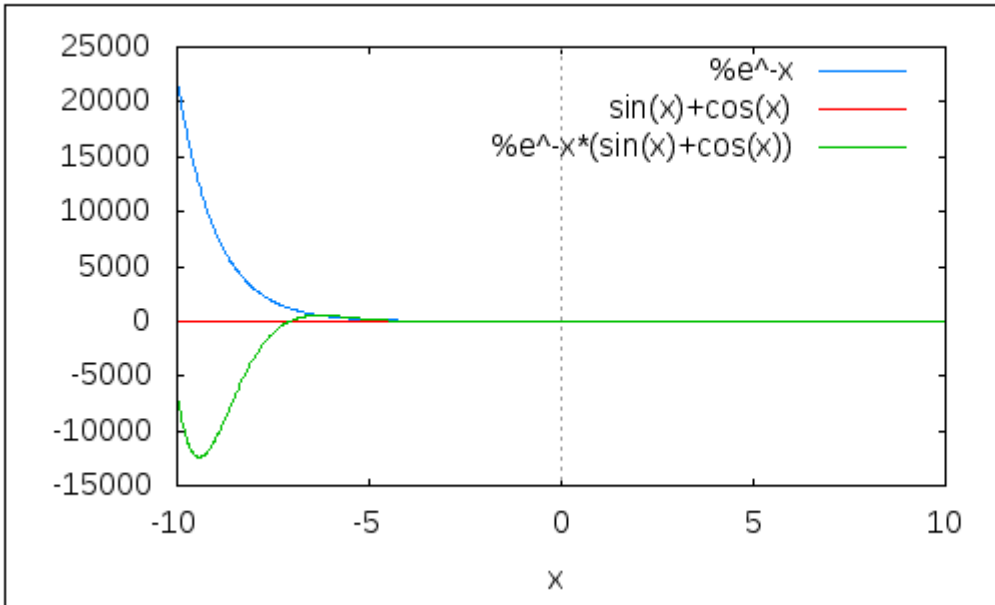
(%t61)
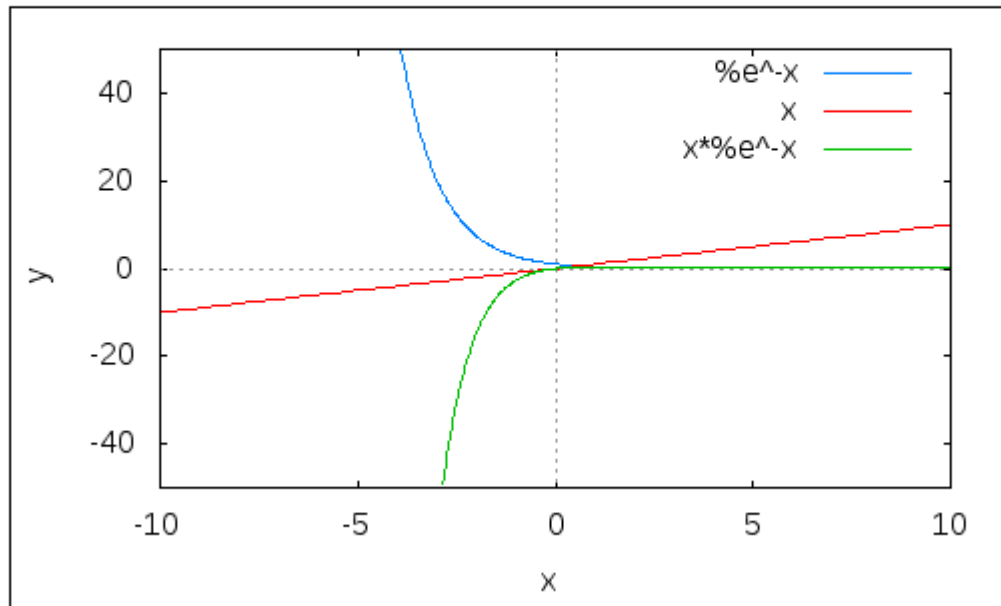
x e^{-x}

(%i62) wxplot2d([%e^(-x), x, %e^(-x)*x], [x,-10, 10])$

(%t62)
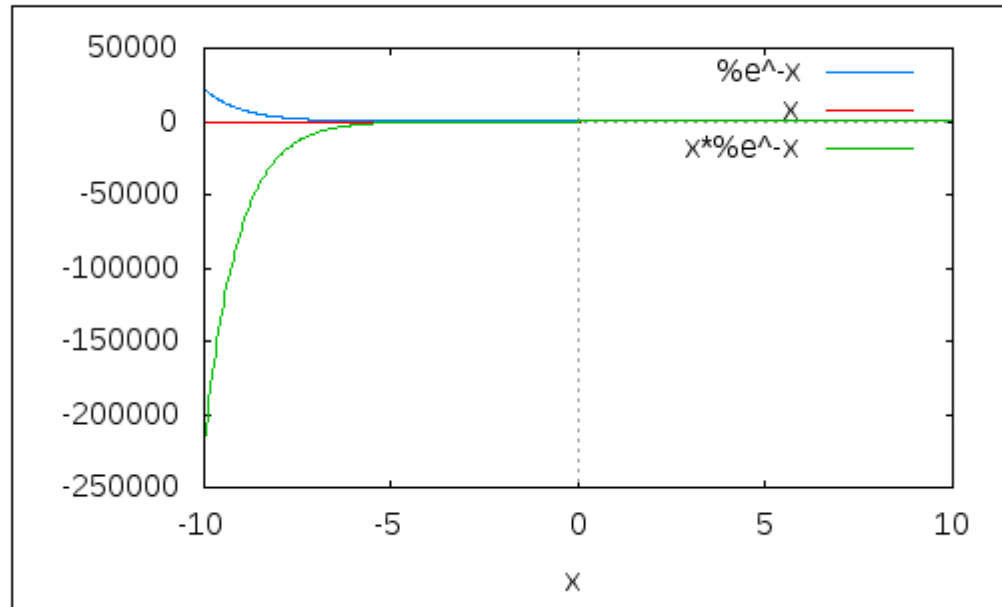
Diff & Integrate xe^x

```
(%i29) a: x^1*%e^x;
```
$(\%o29)\ x\,\%e^x$
```
(%i30) for i: 1 thru 10 do (a: diff(a, x), display(a) );
```
$a = x\,\%e^x + \%e^x$

$a = x\,\%e^x + 2\,\%e^x$

$a = x\,\%e^x + 3\,\%e^x$

$a = x\,\%e^x + 4\,\%e^x$

$a = x\,\%e^x + 5\,\%e^x$

$a = x\,\%e^x + 6\,\%e^x$

$a = x\,\%e^x + 7\,\%e^x$

$a = x\,\%e^x + 8\,\%e^x$

$a = x\,\%e^x + 9\,\%e^x$

$a = x\,\%e^x + 10\,\%e^x$

$(\%o30)\ done$

```
(%i52) a: x^1*%e^x;
```
$(\%o52)\ x\,\%e^x$
```
(%i53) for i: 1 thru 10 do (a: ratsimp(integrate(a, x)), display(a) );
```
$a = \left(x - 1\right)\%e^x$

$a = \left(x - 2\right)\%e^x$

$a = \left(x - 3\right)\%e^x$

$a = \left(x - 4\right)\%e^x$

$a = \left(x - 5\right)\%e^x$

$a = \left(x - 6\right)\%e^x$

$a = \left(x - 7\right)\%e^x$

$a = \left(x - 8\right)\%e^x$

$a = \left(x - 9\right)\%e^x$

$a = \left(x - 10\right)\%e^x$

$(\%o53)\ done$

Diff & Integrate $x^2 e^x$

```
(%i31) a: x^2*%e^x;
```
(%o31) $x^2 \%e^x$

```
(%i32) for i: 1 thru 10 do (a: diff(a, x), display(a) );
```
$a = x^2 \%e^x + 2x \%e^x$

$a = x^2 \%e^x + 4x \%e^x + 2 \%e^x$

$a = x^2 \%e^x + 6x \%e^x + 6 \%e^x$

$a = x^2 \%e^x + 8x \%e^x + 12 \%e^x$

$a = x^2 \%e^x + 10x \%e^x + 20 \%e^x$

$a = x^2 \%e^x + 12x \%e^x + 30 \%e^x$

$a = x^2 \%e^x + 14x \%e^x + 42 \%e^x$

$a = x^2 \%e^x + 16x \%e^x + 56 \%e^x$

$a = x^2 \%e^x + 18x \%e^x + 72 \%e^x$

$a = x^2 \%e^x + 20x \%e^x + 90 \%e^x$

(%o32) *done*

```
(%i54) a: x^2*%e^x;
```
(%o54) $x^2 \%e^x$

```
(%i55) for i: 1 thru 10 do (a: ratsimp(integrate(a, x)), display(a) );
```
$a = \left(x^2 - 2x + 2\right) \%e^x$

$a = \left(x^2 - 4x + 6\right) \%e^x$

$a = \left(x^2 - 6x + 12\right) \%e^x$

$a = \left(x^2 - 8x + 20\right) \%e^x$

$a = \left(x^2 - 10x + 30\right) \%e^x$

$a = \left(x^2 - 12x + 42\right) \%e^x$

$a = \left(x^2 - 14x + 56\right) \%e^x$

$a = \left(x^2 - 16x + 72\right) \%e^x$

$a = \left(x^2 - 18x + 90\right) \%e^x$

$a = \left(x^2 - 20x + 110\right) \%e^x$

(%o55) *done*

Second Order ODEs (2B)

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Diff & Integrate $x^3 e^x$

(%i33) a: x^3*%e^x;

(%o33) $x^3\%e^x$

(%i34) for i: 1 thru 10 do (a: diff(a, x), display(a));

$a = x^3\%e^x + 3x^2\%e^x$

$a = x^3\%e^x + 6x^2\%e^x + 6x\%e^x$

$a = x^3\%e^x + 9x^2\%e^x + 18x\%e^x + 6\%e^x$

$a = x^3\%e^x + 12x^2\%e^x + 36x\%e^x + 24\%e^x$

$a = x^3\%e^x + 15x^2\%e^x + 60x\%e^x + 60\%e^x$

$a = x^3\%e^x + 18x^2\%e^x + 90x\%e^x + 120\%e^x$

$a = x^3\%e^x + 21x^2\%e^x + 126x\%e^x + 210\%e^x$

$a = x^3\%e^x + 24x^2\%e^x + 168x\%e^x + 336\%e^x$

$a = x^3\%e^x + 27x^2\%e^x + 216x\%e^x + 504\%e^x$

$a = x^3\%e^x + 30x^2\%e^x + 270x\%e^x + 720\%e^x$

(%o34) *done*

(%i56) a: x^3*%e^x;

(%o56) $x^3\%e^x$

(%i57) for i: 1 thru 10 do (a: ratsimp(integrate(a, x)), d

$a = \left(x^3 - 3x^2 + 6x - 6\right)\%e^x$

$a = \left(x^3 - 6x^2 + 18x - 24\right)\%e^x$

$a = \left(x^3 - 9x^2 + 36x - 60\right)\%e^x$

$a = \left(x^3 - 12x^2 + 60x - 120\right)\%e^x$

$a = \left(x^3 - 15x^2 + 90x - 210\right)\%e^x$

$a = \left(x^3 - 18x^2 + 126x - 336\right)\%e^x$

$a = \left(x^3 - 21x^2 + 168x - 504\right)\%e^x$

$a = \left(x^3 - 24x^2 + 216x - 720\right)\%e^x$

$a = \left(x^3 - 27x^2 + 270x - 990\right)\%e^x$

$a = \left(x^3 - 30x^2 + 330x - 1320\right)\%e^x$

(%o57) *done*

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```
(%i35)  a: x^10*%e^x;
```
$(\%o35)\quad x^{10}\,\%e^x$

```
(%i36)  for i: 1 thru 10 do (a: diff(a, x), display(a) );
```

$a = x^{10}\%e^x + 10\,x^9\%e^x$

$a = x^{10}\%e^x + 20\,x^9\%e^x + 90\,x^8\%e^x$

$a = x^{10}\%e^x + 30\,x^9\%e^x + 270\,x^8\%e^x + 720\,x^7\%e^x$

$a = x^{10}\%e^x + 40\,x^9\%e^x + 540\,x^8\%e^x + 2880\,x^7\%e^x + 5040\,x^6\%e^x$

$a = x^{10}\%e^x + 50\,x^9\%e^x + 900\,x^8\%e^x + 7200\,x^7\%e^x + 25200\,x^6\%e^x + 30240\,x^5\%e^x$

$a = x^{10}\%e^x + 60\,x^9\%e^x + 1350\,x^8\%e^x + 14400\,x^7\%e^x + 75600\,x^6\%e^x + 181440\,x^5\%e^x + 151200\,x^4\%e^x$

$a = x^{10}\%e^x + 70\,x^9\%e^x + 1890\,x^8\%e^x + 25200\,x^7\%e^x + 176400\,x^6\%e^x + 635040\,x^5\%e^x + 1058400\,x^4\%e^x + 604800\,x^3\%e^x$

$a = x^{10}\%e^x + 80\,x^9\%e^x + 2520\,x^8\%e^x + 40320\,x^7\%e^x + 352800\,x^6\%e^x + 1693440\,x^5\%e^x + 4233600\,x^4\%e^x + 4838400\,x^3\%e^x + 181\ldots$
$x^2\%e^x$

$a = x^{10}\%e^x + 90\,x^9\%e^x + 3240\,x^8\%e^x + 60480\,x^7\%e^x + 635040\,x^6\%e^x + 3810240\,x^5\%e^x + 12700800\,x^4\%e^x + 21772800\,x^3\%e^x +$
$16329600\,x^2\%e^x + 3628800\,x\,\%e^x$

$a = x^{10}\%e^x + 100\,x^9\%e^x + 4050\,x^8\%e^x + 86400\,x^7\%e^x + 1058400\,x^6\%e^x + 7620480\,x^5\%e^x + 31752000\,x^4\%e^x + 72576000\,x^3\%e^x$
$81648000\,x^2\%e^x + 36288000\,x\,\%e^x + 3628800\,\%e^x$

$(\%o36)\quad done$

Integrate $x^{10}e^x$

```
(%i58) a: x^10*%e^x;
```
$(\%o58)\quad x^{10}\,\%e^x$

```
(%i59) for i: 1 thru 10 do (a: ratsimp(integrate(a, x)), display(a) );
```

$a = \left(x^{10} - 10\,x^9 + 90\,x^8 - 720\,x^7 + 5040\,x^6 - 30240\,x^5 + 151200\,x^4 - 604800\,x^3 + 1814400\,x^2 - 3628800\,x + 3628800 \right)\%e^x$

$a = \left(x^{10} - 20\,x^9 + 270\,x^8 - 2880\,x^7 + 25200\,x^6 - 181440\,x^5 + 1058400\,x^4 - 4838400\,x^3 + 16329600\,x^2 - 36288000\,x + 39916800 \right)\%e^x$

$a = \left(x^{10} - 30\,x^9 + 540\,x^8 - 7200\,x^7 + 75600\,x^6 - 635040\,x^5 + 4233600\,x^4 - 21772800\,x^3 + 81648000\,x^2 - 199584000\,x + 239500800 \right)\%e^x$

$a = \left(x^{10} - 40\,x^9 + 900\,x^8 - 14400\,x^7 + 176400\,x^6 - 1693440\,x^5 + 12700800\,x^4 - 72576000\,x^3 + 299376000\,x^2 - 798336000\,x + 1037836800 \right)\%e^x$

$a = \left(x^{10} - 50\,x^9 + 1350\,x^8 - 25200\,x^7 + 352800\,x^6 - 3810240\,x^5 + 31752000\,x^4 - 199584000\,x^3 + 898128000\,x^2 - 2594592000\,x + 3632428800 \right)\%e^x$

$a = \left(x^{10} - 60\,x^9 + 1890\,x^8 - 40320\,x^7 + 635040\,x^6 - 7620480\,x^5 + 69854400\,x^4 - 479001600\,x^3 + 2335132800\,x^2 - 7264857600\,x + 10897286400 \right)\%e^x$

$a = \left(x^{10} - 70\,x^9 + 2520\,x^8 - 60480\,x^7 + 1058400\,x^6 - 13970880\,x^5 + 139708800\,x^4 - 1037836800\,x^3 + 5448643200\,x^2 - 18162144000\,x + 29059430400 \right)\%e^x$

$a = \left(x^{10} - 80\,x^9 + 3240\,x^8 - 86400\,x^7 + 1663200\,x^6 - 23950080\,x^5 + 259459200\,x^4 - 2075673600\,x^3 + 11675664000\,x^2 - 41513472000\,x + 70572902400 \right)\%e^x$

$a = \left(x^{10} - 90\,x^9 + 4050\,x^8 - 118800\,x^7 + 2494800\,x^6 - 38918880\,x^5 + 454053600\,x^4 - 3891888000\,x^3 + 23351328000\,x^2 - 88216128000\,x + 158789030400 \right)\%e^x$

$a = \left(x^{10} - 100\,x^9 + 4950\,x^8 - 158400\,x^7 + 3603600\,x^6 - 60540480\,x^5 + 756756000\,x^4 - 6918912000\,x^3 + 44108064000\,x^2 - 176432256000\,x + 335221286400 \right)\%e^x$

$(\%o59)\quad done$

ODE y" -2y' +y = 0

(%i109) y: %e^x;

(%o109) $%e^x$

(%i110) diff(y, x, 2);

(%o110) $%e^x$

(%i111) diff(y, x, 1);

(%o111) $%e^x$

(%i126) 'diff(y, x, 2) - 2*'diff(y, x, 1) + y;

(%o126) $\dfrac{d^2}{dx^2}%e^x - 2\left(\dfrac{d}{dx}%e^x\right)+%e^x$

(%i127) diff(y, x, 2) - 2*diff(y, x, 1) + y;;

(%o127) 0

(%i1) y: x*%e^x;

(%o1) $x\,%e^x$

(%i2) diff(y, x, 2);

(%o2) $x\,%e^x+2\,%e^x$

(%i3) diff(y, x, 1);

(%o3) $x\,%e^x+%e^x$

(%i4) 'diff(y, x, 2) - 2*'diff(y, x, 1) + y;

(%o4) $\dfrac{d^2}{dx^2}\left(x\,%e^x\right)-2\left(\dfrac{d}{dx}\left(x\,%e^x\right)\right)+x\,%e^x$

(%i6) diff(y, x, 2) - 2*diff(y, x, 1) + y;

(%o6) $-2\left(x\,%e^x+%e^x\right)+2\,x\,%e^x+2\,%e^x$

(%i8) ratsimp(%);

(%o8) 0

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(%i9) y: x^2*%e^x;

(%o9) $x^2 \%e^x$

(%i10) diff(y, x, 2);

(%o10) $x^2 \%e^x + 4x \%e^x + 2 \%e^x$

(%i11) diff(y, x, 1);

(%o11) $x^2 \%e^x + 2x \%e^x$

(%i12) 'diff(y, x, 2) - 2*'diff(y, x, 1) + y;

(%o12) $\dfrac{d^2}{dx^2}\left(x^2 \%e^x\right) - 2\left(\dfrac{d}{dx}\left(x^2 \%e^x\right)\right) + x^2 \%e^x$

(%i13) diff(y, x, 2) - 2*diff(y, x, 1) + y;

(%o13) $-2\left(x^2 \%e^x + 2x \%e^x\right) + 2x^2 \%e^x + 4x \%e^x + 2 \%e^x$

(%i14) ratsimp(%);

(%o14) $2 \%e^x$

$x^i e^x$ => y'' -2y' +y

```
(%i8)  for i: 1 thru 10 do (
         y: x^i*%e^x,
         a: diff(y, x, 2) - 2*diff(y, x, 1)+y,
         a: ratsimp(a), print("y=", y, "    y\"-2y'+y=", a) );
```

$y = x \, \%e^x$ $y'' - 2y' + y = 0$

$y = x^2 \, \%e^x$ $y'' - 2y' + y = 2 \, \%e^x$

$y = x^3 \, \%e^x$ $y'' - 2y' + y = 6 \, x \, \%e^x$

$y = x^4 \, \%e^x$ $y'' - 2y' + y = 12 \, x^2 \, \%e^x$

$y = x^5 \, \%e^x$ $y'' - 2y' + y = 20 \, x^3 \, \%e^x$

$y = x^6 \, \%e^x$ $y'' - 2y' + y = 30 \, x^4 \, \%e^x$

$y = x^7 \, \%e^x$ $y'' - 2y' + y = 42 \, x^5 \, \%e^x$

$y = x^8 \, \%e^x$ $y'' - 2y' + y = 56 \, x^6 \, \%e^x$

$y = x^9 \, \%e^x$ $y'' - 2y' + y = 72 \, x^7 \, \%e^x$

$y = x^{10} \, \%e^x$ $y'' - 2y' + y = 90 \, x^8 \, \%e^x$

(%o8) *done*

```
(%i87) for i:0 thru 9 do (
         a: 'diff(y,x, 2) -2*'diff(y,x) + y = x^i*%e^x,
         b: ode2(a, y, x), display(b) );
```

$$y = \frac{x^2\,\%e^x}{2} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^3\,\%e^x}{6} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^4\,\%e^x}{12} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^5\,\%e^x}{20} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^6\,\%e^x}{30} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^7\,\%e^x}{42} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^8\,\%e^x}{56} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^9\,\%e^x}{72} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^{10}\,\%e^x}{90} + \left(\%k2\,x + \%k1\right)\%e^x$$

$$y = \frac{x^{11}\,\%e^x}{110} + \left(\%k2\,x + \%k1\right)\%e^x$$

(%o87) done

ODE y" -2y' +y = cos^i(x)e^x

```
(%i90)  for i:1 thru 5 do (
        a: 'diff(y,x, 2) -2*'diff(y,x) + y = sin(x)^i,
        b: ode2(a, y, x), display(b) );
```

$$y=\frac{\cos(x)}{2}+\left(\%k2\,x+\%k1\right)\%e^x$$

$$y=\frac{4\sin(2x)+3\cos(2x)+25}{50}+\left(\%k2\,x+\%k1\right)\%e^x$$

$$y=\frac{4\sin(3x)-3\cos(3x)+75\cos(x)}{200}+\left(\%k2\,x+\%k1\right)\%e^x$$

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{200\sin(4x)+375\cos(4x)-4624\sin(2x)-3468\cos(2x)-21675}{57800}$$

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{60\sin(5x)-25\cos(5x)-676\sin(3x)+507\cos(3x)-8450\cos(x}{27040}$$

(%o90) done

```
(%i91)  for i:1 thru 4 do (
        a: 'diff(y,x, 2) -2*'diff(y,x) + y = cos(x)^i,
        b: ode2(a, y, x), display(b) );
```

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{\sin(x)}{2}$$

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{4\sin(2x)+3\cos(2x)-25}{50}$$

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{3\sin(3x)+4\cos(3x)+75\sin(x)}{200}$$

$$y=\left(\%k2\,x+\%k1\right)\%e^x-\frac{200\sin(4x)+375\cos(4x)+4624\sin(2x)+3468\cos(2x)-2167}{57800}$$

(%o91) done

ODE $y'' - 2y' + y = e^{ix}$, x^i

```
(%i95) for i:1 thru 5 do (
        a: 'diff(y,x, 2) -2*'diff(y,x) + y = %e^(i*x),
        b: ode2(a, y, x), display(b) );
```

$$y = \frac{x^2\,\%e^x}{2} + (\%k2\,x + \%k1)\,\%e^x$$

$$y = \%e^{2x} + (\%k2\,x + \%k1)\,\%e^x$$

$$y = \frac{\%e^{3x}}{4} + (\%k2\,x + \%k1)\,\%e^x$$

$$y = \frac{\%e^{4x}}{9} + (\%k2\,x + \%k1)\,\%e^x$$

$$y = \frac{\%e^{5x}}{16} + (\%k2\,x + \%k1)\,\%e^x$$

(%o95) done

```
(%i96) for i:1 thru 5 do (
        a: 'diff(y,x, 2) -2*'diff(y,x) + y = x^(i),
        b: ode2(a, y, x), display(b) );
```

$$y = (\%k2\,x + \%k1)\,\%e^x + x + 2$$

$$y = (\%k2\,x + \%k1)\,\%e^x + x^2 + 4x + 6$$

$$y = (\%k2\,x + \%k1)\,\%e^x + x^3 + 6x^2 + 18x + 24$$

$$y = (\%k2\,x + \%k1)\,\%e^x + x^4 + 8x^3 + 36x^2 + 96x + 120$$

$$y = (\%k2\,x + \%k1)\,\%e^x + x^5 + 10x^4 + 60x^3 + 240x^2 + 600x + 72$$

(%o96) done

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