```
-->      load (solve_rec);

(%i7)  v4: matrix([x[i+3]], [y[i+3]]);
```

$$(\%o7) \quad \begin{pmatrix} x_{i+3} \\ y_{i+3} \end{pmatrix}$$

```
(%i10) m3: matrix([1, -d[i+2]*2^-(i+2)], [ d[i+2]*2^-(i+2), 1]);
```

$$(\%o10) \quad \begin{pmatrix} 1 & -2^{-i-2}\,d_{i+2} \\ 2^{-i-2}\,d_{i+2} & 1 \end{pmatrix}$$

```
-->      mm: m2.m1;

-->      mmm: m3.m2.m1;

-->      ratsimp(matrix([-2^(-i-2)*(2^(-i-1)*d[i+1]+d[i]/2^i)*d[i+2]-2^(-2*i-1)*d[i]*d[i+1]+1,

-->      factor(matrix([-2^(-i-2)*(2^(-i-1)*d[i+1]+d[i]/2^i)*d[i+2]-2^(-2*i-1)*d[i]*d[i+1]+1,-

-->      expand(matrix([-2^(-2*i-3)*(d[i+1]*d[i+2]+2*d[i]*d[i+2]+4*d[i]*d[i+1]-2^(2*i+3)),2^(-

-->      radcan(matrix([-2^(-2*i-3)*d[i+1]*d[i+2]-2^(-2*i-2)*d[i]*d[i+2]-2^(-2*i-1)*d[i]*d[i+1

-->      mm;

-->      ratsimp(matrix([1-2^(-2*i-1)*d[i]*d[i+1],-2^(-i-1)*d[i+1]-d[i]/2^i],[2^(-i-1)*d[i+1]+

-->      2^(i+1) * mm;

-->      ratsimp(matrix([2^i*(1-2^(-2*i-1)*d[i]*d[i+1]),2^i*(-2^(-i-1)*d[i+1]-d[i]/2^i)],[2^i*

-->      ratsimp(matrix([2^(i-1)*(1-2^(-2*i-1)*d[i]*d[i+1]),2^(i-1)*(-2^(-i-1)*d[i+1]-d[i]/2^i

-->      ratsimp(matrix([2^(i+1)*(1-2^(-2*i-1)*d[i]*d[i+1]),2^(i+1)*(-2^(-i-1)*d[i+1]-d[i]/2^i

-->      radcan(matrix([-(d[i]*d[i+1]-2^(2*i+1))/2^i,-d[i+1]-2*d[i]],[d[i+1]+2*d[i],-(d[i]*d[i

-->      factor(matrix([-(d[i]*d[i+1]-2^(2*i+1))/2^i,-d[i+1]-2*d[i]],[d[i+1]+2*d[i],-(d[i]*d[i

-->      expand(matrix([-(d[i]*d[i+1]-2^(2*i+1))/2^i,-(d[i+1]+2*d[i])],[d[i+1]+2*d[i],-(d[i]*d

-->      v4 = mmm . v1;

-->      ratsimp(matrix([x[i+3]],[y[i+3]])=matrix([y[i]*(-2^(-i-2)*(1-2^(-2*i-1)*d[i]*d[i+1])*
```

1

```
-->     factor(matrix([x[i+3]],[y[i+3]])=matrix([2^(-3*i-3)*(((d[i]*y[i]-2^i*x[i])*d[i+1]-2^(
-->     gfactor(matrix([x[i+3]],[y[i+3]])=matrix([2^(-3*i-3)*(d[i]*y[i]*d[i+1]*d[i+2]-2^i*x[i
-->     expand(matrix([x[i+3]],[y[i+3]])=matrix([2^(-3*i-3)*(((d[i]*y[i]-2^i*x[i])*d[i+1]-2^(
-->     ratsimp(matrix([x[i+3]],[y[i+3]])=matrix([2^(-3*i-3)*d[i]*y[i]*d[i+1]*d[i+2]-2^(-2*i-
```

(%i3) v4;

(%o3) $v4$

(%i11)

      v1;

(%o11) $\begin{pmatrix} x_i \\ y_i \end{pmatrix}$

(%i12) v2;

(%o12) $\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix}$

(%i13) v3;

(%o13) $\begin{pmatrix} x_{i+2} \\ y_{i+2} \end{pmatrix}$

(%i14) v4;

(%o14) $\begin{pmatrix} x_{i+3} \\ y_{i+3} \end{pmatrix}$

(%i15) m1;

(%o15) $\begin{pmatrix} 1 & -\frac{d_i}{2^i} \\ \frac{d_i}{2^i} & 1 \end{pmatrix}$

(%i16) m2;

(%o16) $\begin{pmatrix} 1 & -2^{-i-1}\,d_{i+1} \\ 2^{-i-1}\,d_{i+1} & 1 \end{pmatrix}$

(%i17) m3;

(%o17) $\begin{pmatrix} 1 & -2^{-i-2}\,d_{i+2} \\ 2^{-i-2}\,d_{i+2} & 1 \end{pmatrix}$

(%i18) `v2 = m1. v1;`

(%o18) $\begin{pmatrix} x_{i+1} \\ y_{i+1} \end{pmatrix} = \begin{pmatrix} x_i - \frac{d_i\,y_i}{2^i} \\ y_i + \frac{d_i\,x_i}{2^i} \end{pmatrix}$

(%i19) `v3= m2. v2;`

(%o19) $\begin{pmatrix} x_{i+2} \\ y_{i+2} \end{pmatrix} = \begin{pmatrix} x_{i+1} - 2^{-i-1}\,d_{i+1}\,y_{i+1} \\ y_{i+1} + 2^{-i-1}\,d_{i+1}\,x_{i+1} \end{pmatrix}$

(%i20) `v4 = m3. v3;`

(%o20) $\begin{pmatrix} x_{i+3} \\ y_{i+3} \end{pmatrix} = \begin{pmatrix} x_{i+2} - 2^{-i-2}\,d_{i+2}\,y_{i+2} \\ y_{i+2} + 2^{-i-2}\,d_{i+2}\,x_{i+2} \end{pmatrix}$

(%i21) `v3 = m2. m1. v1;`

(%o21) $\begin{pmatrix} x_{i+2} \\ y_{i+2} \end{pmatrix} = \begin{pmatrix} -2^{-i-1}\left(y_i + \frac{d_i\,x_i}{2^i}\right)d_{i+1} - \frac{d_i\,y_i}{2^i} + x_i \\ 2^{-i-1}\left(x_i - \frac{d_i\,y_i}{2^i}\right)d_{i+1} + y_i + \frac{d_i\,x_i}{2^i} \end{pmatrix}$

(%i22) `mm = m2.m1;`

(%o22) $mm = \begin{pmatrix} 1 - 2^{-2\,i-1}\,d_i\,d_{i+1} & -2^{-i-1}\,d_{i+1} - \frac{d_i}{2^i} \\ 2^{-i-1}\,d_{i+1} + \frac{d_i}{2^i} & 1 - 2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$

(%i23) `v4 = m3.m2.m1.v1;`

(%o23) $\begin{pmatrix} x_{i+3} \\ y_{i+3} \end{pmatrix} = \begin{pmatrix} -2^{-i-2}\left(2^{-i-1}\left(x_i - \frac{d_i\,y_i}{2^i}\right)d_{i+1} + y_i + \frac{d_i\,x_i}{2^i}\right)d_{i+2} - 2^{-i-1}\left(y_i + \frac{d_i\,x_i}{2^i}\right)d_{i+1} - \frac{d_i\,y_i}{2^i} + x_i \\ 2^{-i-2}\left(-2^{-i-1}\left(y_i + \frac{d_i\,x_i}{2^i}\right)d_{i+1} - \frac{d_i\,y_i}{2^i} + x_i\right)d_{i+2} + 2^{-i-1}\left(x_i - \frac{d_i\,y_i}{2^i}\right)d_{i+1} + y_i + \frac{d_i\,x_i}{2^i} \end{pmatrix}$

(%i24) `mmm: m3.m2.m1;`

(%o24) $\begin{pmatrix} -2^{-i-2}\left(2^{-i-1}\,d_{i+1} + \frac{d_i}{2^i}\right)d_{i+2} - 2^{-2\,i-1}\,d_i\,d_{i+1} + 1 & -2^{-i-2}\left(1 - 2^{-2\,i-1}\,d_i\,d_{i+1}\right)d_{i+2} - 2^{-i-1}\,d_{i+1} \\ 2^{-i-2}\left(1 - 2^{-2\,i-1}\,d_i\,d_{i+1}\right)d_{i+2} + 2^{-i-1}\,d_{i+1} + \frac{d_i}{2^i} & 2^{-i-2}\left(-2^{-i-1}\,d_{i+1} - \frac{d_i}{2^i}\right)d_{i+2} - 2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$

(%i25) `mm: m2.m1;`

(%o25) $\begin{pmatrix} 1 - 2^{-2\,i-1}\,d_i\,d_{i+1} & -2^{-i-1}\,d_{i+1} - \frac{d_i}{2^i} \\ 2^{-i-1}\,d_{i+1} + \frac{d_i}{2^i} & 1 - 2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$

(%i26) `m2;`

(%o26) $\begin{pmatrix} 1 & -2^{-i-1}\,d_{i+1} \\ 2^{-i-1}\,d_{i+1} & 1 \end{pmatrix}$

(%i27) `m1;`

(%o27) $\begin{pmatrix} 1 & -\frac{d_i}{2^i} \\ \frac{d_i}{2^i} & 1 \end{pmatrix}$

(%i28) `mm;`

(%o28) $\begin{pmatrix} 1-2^{-2\,i-1}\,d_i\,d_{i+1} & -2^{-i-1}\,d_{i+1}-\frac{d_i}{2^i} \\ 2^{-i-1}\,d_{i+1}+\frac{d_i}{2^i} & 1-2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$

(%i29) `ratsimp(matrix([1-2^(-2*i-1)*d[i]*d[i+1],-2^(-i-1)*d[i+1]-d[i]/2^i],[2^(-i-1)*d[i+1]+`

(%o29) $\begin{pmatrix} -2^{-2\,i-1}\left(d_i\,d_{i+1}-2^{2\,i+1}\right) & -2^{-i-1}\left(d_{i+1}+2\,d_i\right) \\ 2^{-i-1}\left(d_{i+1}+2\,d_i\right) & -2^{-2\,i-1}\left(d_i\,d_{i+1}-2^{2\,i+1}\right) \end{pmatrix}$

(%i30) `2^(i+1) * mm;`

(%o30) $\begin{pmatrix} 2^{i+1}\left(1-2^{-2\,i-1}\,d_i\,d_{i+1}\right) & 2^{i+1}\left(-2^{-i-1}\,d_{i+1}-\frac{d_i}{2^i}\right) \\ 2^{i+1}\left(2^{-i-1}\,d_{i+1}+\frac{d_i}{2^i}\right) & 2^{i+1}\left(1-2^{-2\,i-1}\,d_i\,d_{i+1}\right) \end{pmatrix}$

(%i31) `expand(matrix([2^(i+1)*(1-2^(-2*i-1)*d[i]*d[i+1]),2^(i+1)*(-2^(-i-1)*d[i+1]-d[i]/2^i)`

(%o31) $\begin{pmatrix} 2^{i+1}-\frac{d_i\,d_{i+1}}{2^i} & -d_{i+1}-2\,d_i \\ d_{i+1}+2\,d_i & 2^{i+1}-\frac{d_i\,d_{i+1}}{2^i} \end{pmatrix}$