Laurent Series and z-Transform - Geometric Series Reciprocity Properties (A)

20191030 Mon

2 formulas — Simple Pole Form

$$\frac{1}{z - p} \qquad \frac{1}{z^{-1} - p}$$

2 representations each — Geometric Series Form

$$\frac{1}{z-p} \;\rightarrow\; -\frac{p^{-1}}{1-p^{-1}z} \triangleq f(z) = X(z^{-1})$$

causal — anti- causal

$$\frac{1}{z-p} \;\rightarrow\; \frac{z^{-1}}{1-pz^{-1}} \triangleq Y(z) = g(z^{-1})$$

causal — anti- causal

($f(z) = Y(z)$, $X(z^{-1}) = g(z^{-1})$)

$$\frac{1}{z^{-1}-p} \;\rightarrow\; -\frac{p^{-1}}{1-p^{-1}z^{-1}} \triangleq X(z) = f(z^{-1})$$

causal — anti- causal

$$\frac{1}{z^{-1}-p} \;\rightarrow\; \frac{z}{1-pz} \triangleq g(z) = Y(z^{-1})$$

causal — anti- causal

($X(z) = g(z)$, $f(z^{-1}) = Y(z^{-1})$)

Simple Pole Form	Geometric Series Form

$$f(z) = f(a, z)$$

$$\Vert$$

$$g(z^{-1}) = g(a, z^{-1})$$

$$f(z^{-1}) = f(a, z^{-1})$$

$$\Vert$$

$$g(z) = g(a, z)$$

$$\bar{f}(z) = f(a^{-1}, z)$$

$$\Vert$$

$$\bar{g}(z^{-1}) = g(a^{-1}, z^{-1})$$

$$\bar{f}(z^{-1}) = f(a^{-1}, z^{-1})$$

$$\Vert$$

$$\bar{g}(z) = g(a^{-1}, z)$$

Geometric Series : f(z), g(z̄), f(z̄), g(z)

①

$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n+1}$	$(n \geq 0)$

②

$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

⑤

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$

⑥

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

Inverse(z)

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

① ⟷ ②: z^{-1}

① → ③: $\frac{1}{az}$; ③ → ①: az

② → ④: $\frac{1}{az^{-1}}$; ④ → ②: az^{-1}

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

③ ⟷ ④: z^{-1}

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

⑤ ⟷ ⑥: z^{-1}

⑤ → ⑦: $\frac{1}{az}$; ⑦ → ⑤: az

⑥ → ⑧: $\frac{1}{az^{-1}}$; ⑧ → ⑥: az^{-1}

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

⑦ ⟷ ⑧: z^{-1}

Inverse(a)

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

① ↕ a^{-1} ⑤ ② ↕ a^{-1} ⑥

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$
$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$

⑥

$a_n = -a^{n-1}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

⑤

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

⑥

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

⑤ ↕ a^{-1} ⑦ ⑥ ↕ a^{-1} ⑧

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

simple pole models with a unit nominator

$a^{-1} f(z),\ z^{-1} g(z),\ a\, \bar{f}(z),\ z\, \bar{g}(z)$

①

$a^{-1} f(z) = -\dfrac{1}{1 - az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^n$	$(n \geq 0)$

②

$a^{-1} f(z^{-1}) = -\dfrac{1}{1 - az^{-1}}$	$\lvert z\rvert > a$
$a_n = -(\frac{1}{a})^n$	$(n < 1)$

③

$z\, g(z^{-1}) = \dfrac{1}{1 - a^{-1} z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

④

$z^{-1} g(z) = \dfrac{1}{1 - a^{-1} z}$	$\lvert z\rvert < a$
$a_n = (\frac{1}{a})^n$	$(n \geq 0)$

⑤

$a \bar{f}(z) = -\dfrac{1}{1 - a^{-1} z}$	$\lvert z\rvert < a$
$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$

⑥

$a \bar{f}(z^{-1}) = -\dfrac{1}{1 - a^{-1} z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

⑦

$z\, \bar{g}(z^{-1}) = \dfrac{1}{1 - a z^{-1}}$	$\lvert z\rvert > a$
$a_n = (\frac{1}{a})^n$	$(n < 1)$

⑧

$z^{-1} \bar{g}(z) = \dfrac{1}{1 - a z}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

8 simple pole models with unit nominator

$$\pm\frac{1}{1-az}$$

$$\pm\frac{1}{1-az^{-1}}$$

$$\pm\frac{1}{1-a^{-1}z^{-1}}$$

$$\pm\frac{1}{1-a^{-1}z}$$

az	az^{-1}	$a^{-1}z^{-1}$	$a^{-1}z$
$\lvert z\rvert < a^{-1}$	$\lvert z\rvert > a$	$\lvert z\rvert > a^{-1}$	$\lvert z\rvert < a$

Geometric Series Expression $h(a, z)$

Region of Convergence Expression $R(a, z)$

$$\begin{pmatrix} +1 \\ -1 \end{pmatrix} \times \begin{pmatrix} a \\ a^{-1} \end{pmatrix} \times \begin{pmatrix} z \\ z^{-1} \end{pmatrix}$$

$h(a, z)$	$R(a, z)$	
$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$\lvert az\rvert < 1$
$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\lvert az^{-1}\rvert < 1$
$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\lvert a^{-1}z^{-1}\rvert < 1$
$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\lvert a^{-1}z\rvert < 1$
$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$	$\lvert az\rvert < 1$
$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$	$\lvert az^{-1}\rvert < 1$
$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\lvert a^{-1}z^{-1}\rvert < 1$
$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\lvert a^{-1}z\rvert < 1$

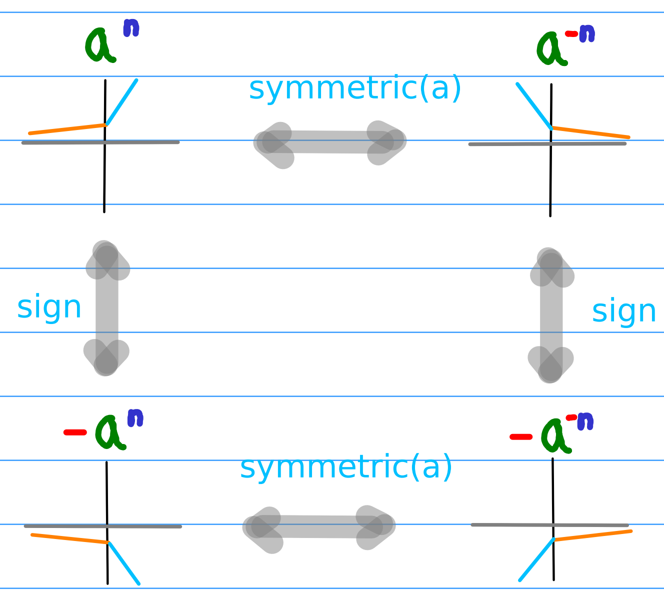
8 sequences

Power Selection — Range Selection

$$\left(\begin{array}{c} -a^{n} \\ a^{n} \end{array} \quad \begin{array}{c} -a^{-n} \\ a^{-n} \end{array} \right) \times \left(\begin{array}{c} u(n) \\ u(-n) \end{array} \quad \begin{array}{c} u(-n) \\ u(n) \end{array} \right)$$

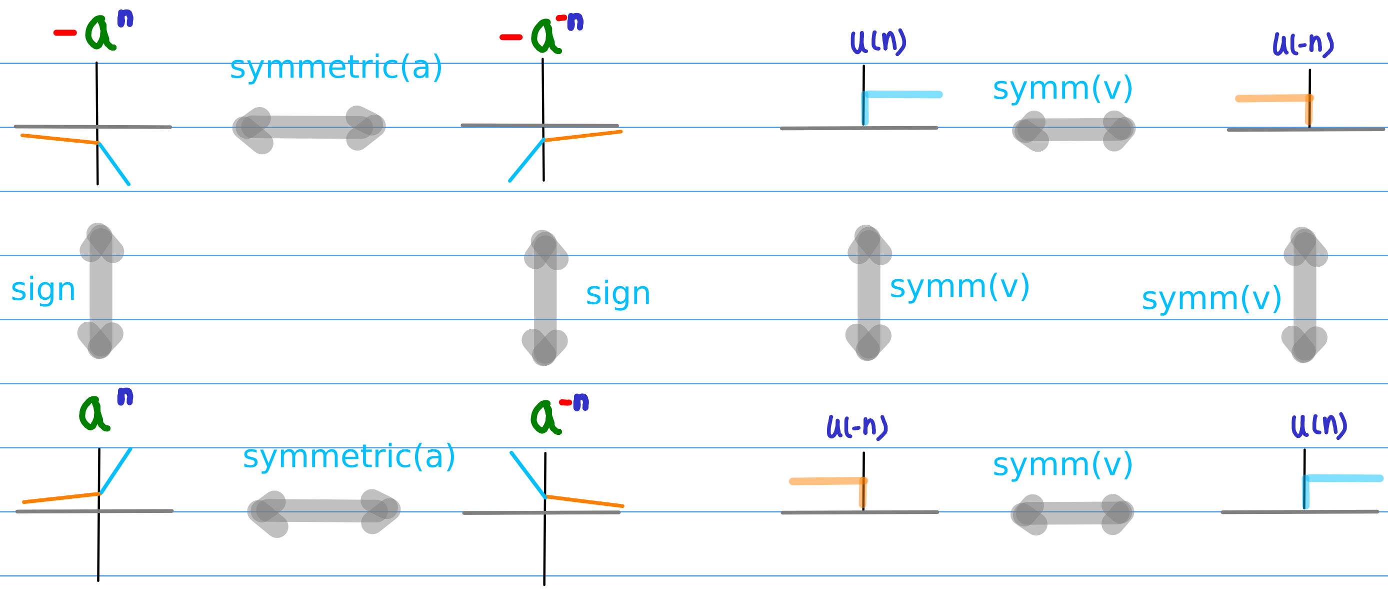
$$\begin{array}{c} -a^{n} \\ a^{n} \end{array} \cdot \begin{array}{c} u(n) \\ u(-n) \end{array} \qquad \begin{array}{c} -a^{n} \\ a^{n} \end{array} \cdot \begin{array}{c} u(-n) \\ u(n) \end{array}$$

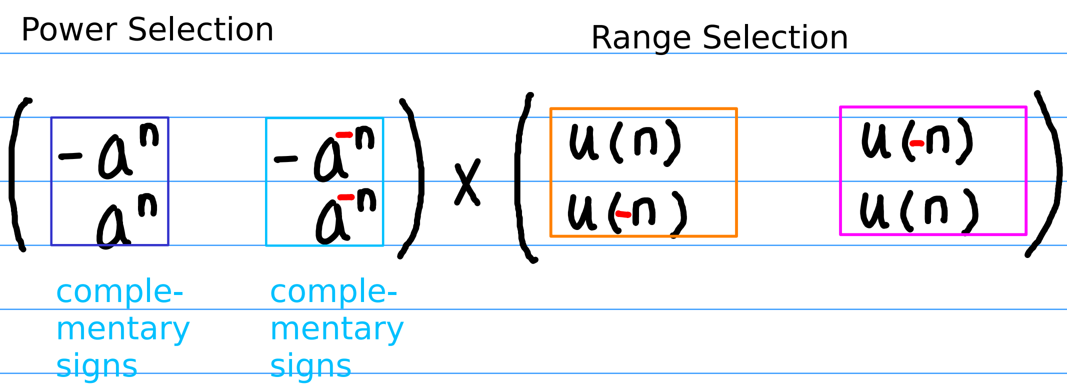
$$\begin{array}{c} -a^{-n} \\ a^{-n} \end{array} \cdot \begin{array}{c} u(n) \\ u(-n) \end{array} \qquad \begin{array}{c} -a^{-n} \\ a^{-n} \end{array} \cdot \begin{array}{c} u(-n) \\ u(n) \end{array}$$

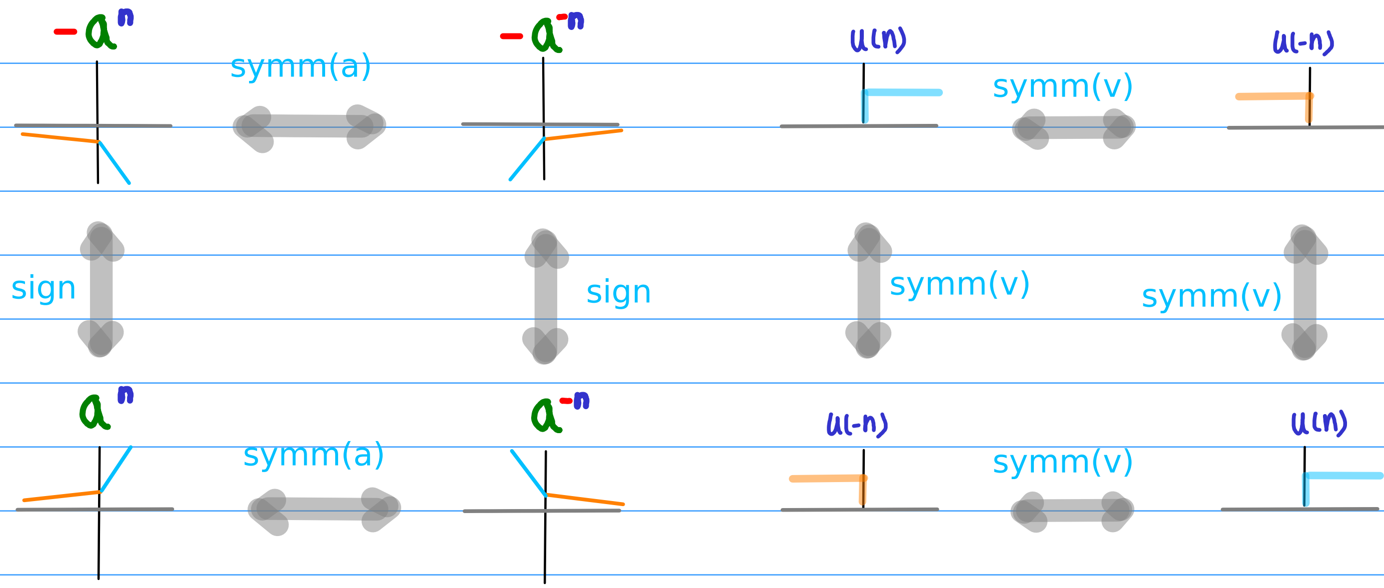


Power Selection

Range Selection





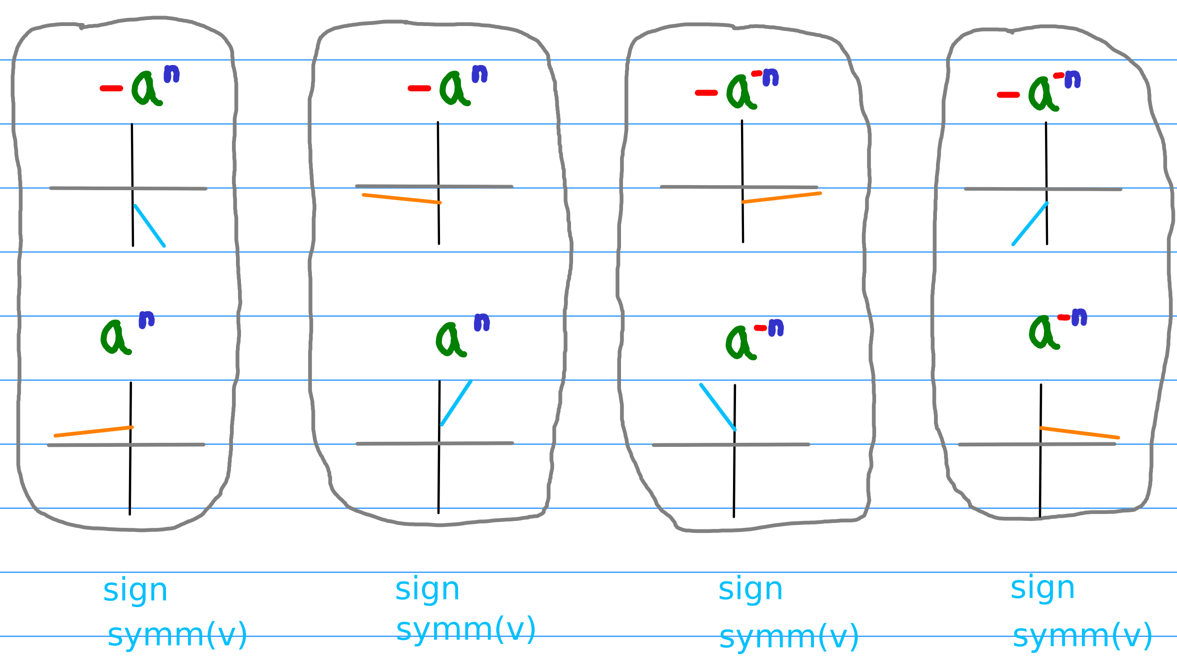


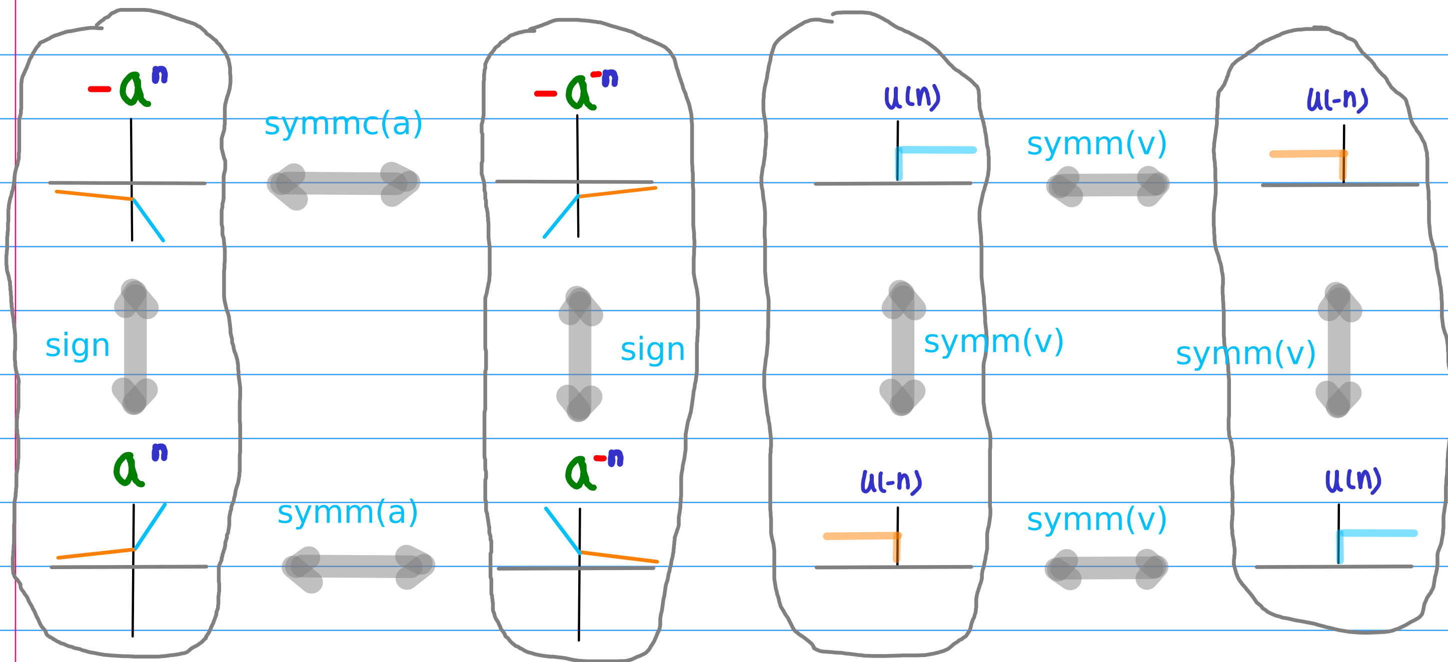
$-a^n \cdot u(n)$ $\quad$ $a^n \cdot u(-n)$

$-a^n \cdot u(-n)$ $\quad$ $a^n \cdot u(n)$

$-a^{-n} \cdot u(n)$ $\quad$ $a^{-n} \cdot u(-n)$

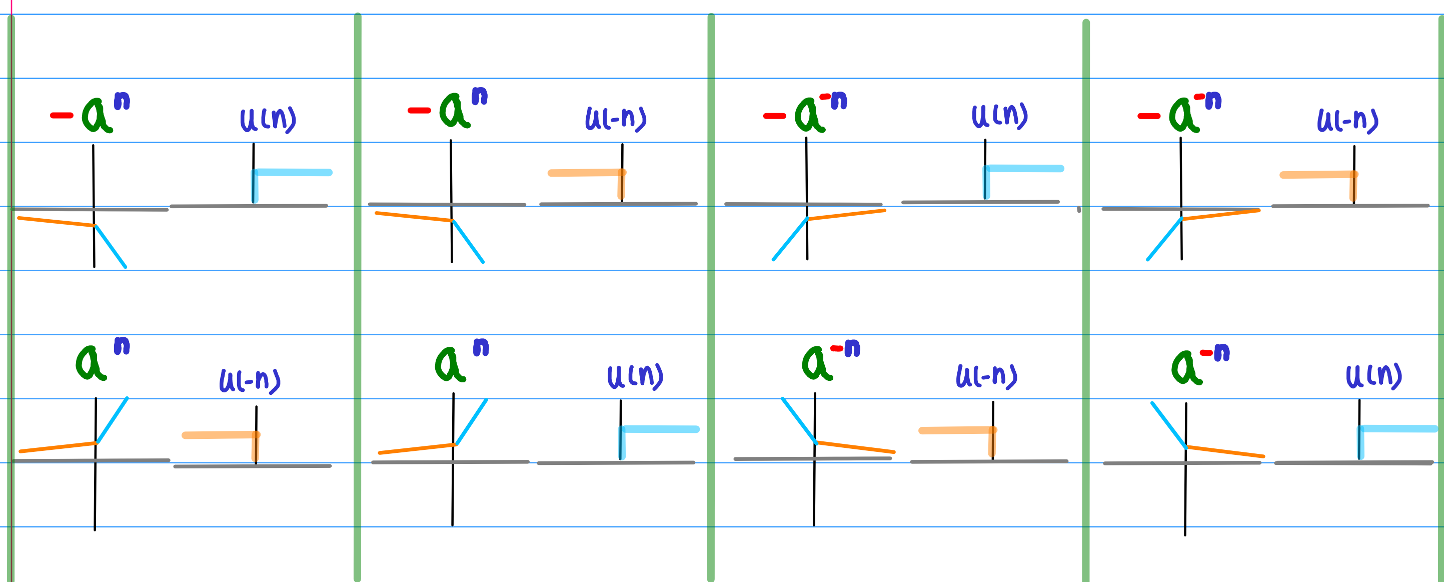
$-a^{-n} \cdot u(-n)$ $\quad$ $a^{-n} \cdot u(n)$

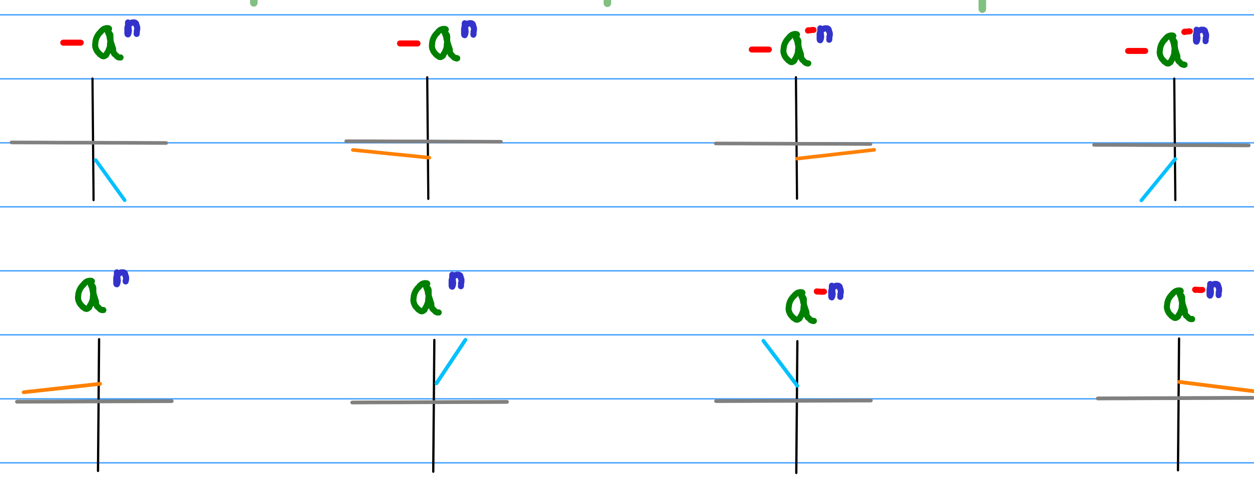




$-a^n \cdot u(n)$ | $-a^n \cdot u(-n)$ | $-a^{-n} \cdot u(n)$ | $-a^{-n} \cdot u(-n)$

$a^n \cdot u(-n)$ | $a^n \cdot u(n)$ | $a^{-n} \cdot u(-n)$ | $a^{-n} \cdot u(n)$





v(n) : a range selection expression

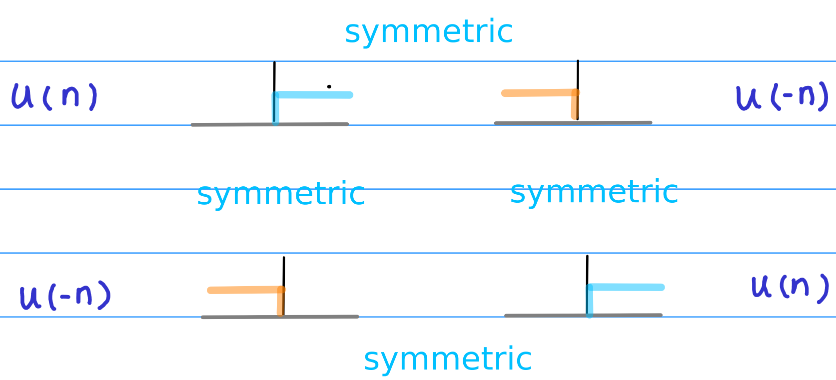
$v(n) \in \{u(n), u(-n)\}$

unit step function to denote a range

complementary and symmetric regions

$$\begin{pmatrix} u(n) & u(-n) \\ u(-n) & u(n) \end{pmatrix}$$

symmetric regions   symmetric regions



bipartite mappings
over 8 simple pole models

(1)
(3)
(5)
(7)

(2)
(4)
(6)
(8)

Inverse(a), Inverse(z), Inverse(a,z), Sign
determines such mappings

purpose: given a bipartite mapping over
8 simple pole models, which is determined by
Inverse(a), Inverse(z), Inverse(a,z), Sign

find out the corresponding mapping and
operations in the time domain

Domain & Range (Case A)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

①

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$-a^{n}$	$u(n)$

②

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-a^{-n}$	$u(-n)$

③

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^{n}	$u(-n)$

④

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
a^{-n}	$u(n)$

⑤

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$-a^{-n}$	$u(n)$

⑥

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-a^{n}$	$u(-n)$

⑦

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
a^{-n}	$u(-n)$

⑧

$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^{n}	$u(n)$

$-\dfrac{1}{1-az} = -(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$ $-a^n$ $\quad u(n)$	$-\dfrac{1}{1-az^{-1}} = -(a^0z^0 + a^1z^{-2} + a^2z^{-2} + \cdots)$ $-a^{-n}$ $\quad u(-n)$
$\dfrac{1}{1-a^{-1}z^{-1}} = +(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$ a^n $\quad u(-n)$	$\dfrac{1}{1-a^{-1}z} = +(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$ a^{-n} $\quad u(n)$
$-\dfrac{1}{1-a^{-1}z} = -(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$ $-a^{-n}$ $\quad u(n)$	$-\dfrac{1}{1-a^{-1}z^{-1}} = -(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$ $-a^n$ $\quad u(-n)$
$\dfrac{1}{1-az^{-1}} = +(a^0z^0 + a^1z^{-2} + a^2z^{-2} + \cdots)$ a^{-n} $\quad u(-n)$	$\dfrac{1}{1-az} = +(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$ a^n $\quad u(n)$

Domain & Range (Case B)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

(1)

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$-a^{n}$	$u(n)$

(2)

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-a^{-n}$	$u(-n)$

(3)

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^{n}	$u(-n)$

(4)

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
a^{-n}	$u(n)$

(5)

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$-a^{-n}$	$u(n)$

(6)

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-a^{n}$	$u(-n)$

(7)

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
a^{-n}	$u(-n)$

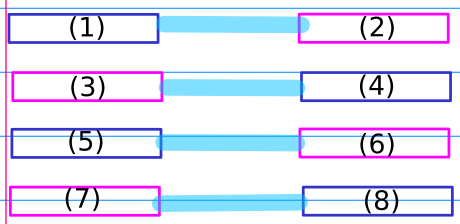
(8)

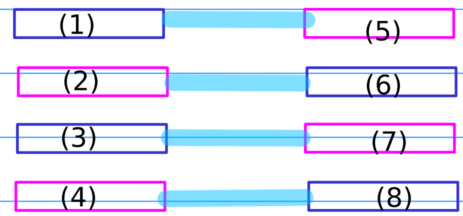
$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^{n}	$u(n)$

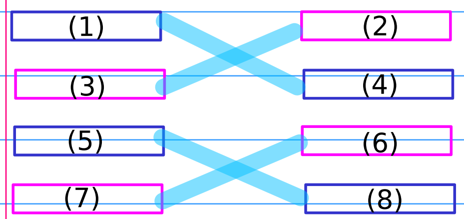
$-\frac{1}{1-az} = -(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$ $-a^n$ $u(n)$	$-\frac{1}{1-a^{-1}z} = -(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$ $-a^{-n}$ $u(n)$
$-\frac{1}{1-az^{-1}} = -(a^0z^0 + a^1z^{-2} + a^2z^{-2} + \cdots)$ $-a^{-n}$ $u(-n)$	$-\frac{1}{1-a^{-1}z^{-1}} = -(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$ $-a^n$ $u(-n)$
$\frac{1}{1-a^{-1}z^{-1}} = +(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$ a^n $u(-n)$	$\frac{1}{1-az^{-1}} = +(a^0z^0 + a^1z^{-2} + a^2z^{-2} + \cdots)$ a^{-n} $u(-n)$
$\frac{1}{1-a^{-1}z} = +(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$ a^{-n} $u(n)$	$\frac{1}{1-az} = +(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$ a^n $u(n)$

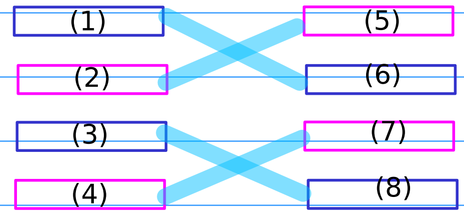
(1) (2)
(3) (4)
(5) (6)
(7) (8)

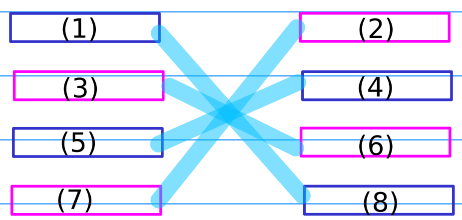
(1) (2)
(3) (4)
(5) (6)
(7) (8)

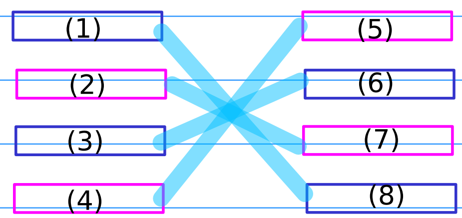


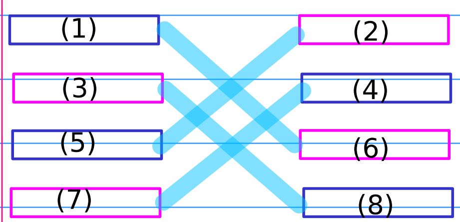


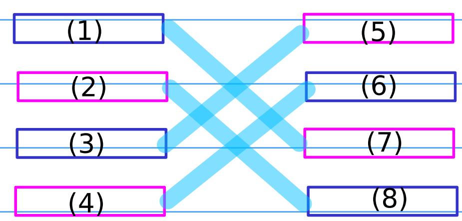












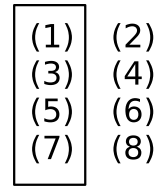
operations in z-domain

Inverse(a)	Inv(a)
Inverse(z)	Inv(z)
Inverse(a,z)	Inv(a,z)
Sign	Sign

operations in n-domain

Symmetric(a)	Symm(a)
Symmetric(v)	Symm(v)
Symmetric(a,v)	Symm(a,v)
Sign	Sign

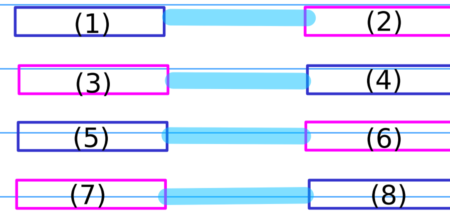
4 examples of such mappings



Permutation (I)

Inverse(z)

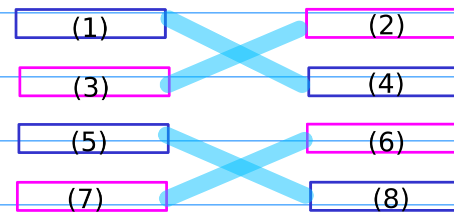
Symmetric(a, v)



Permutation (II)

Sign, Inverse(a)

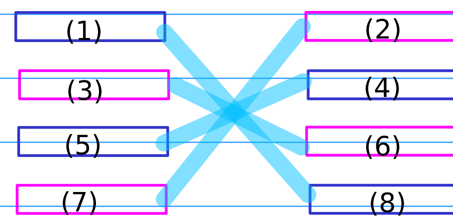
Sign, Symmetric(a)



Permutation (III)

Sign

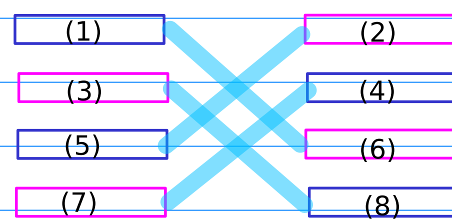
Sign



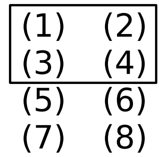
Permutation (IV)

Inverse(a,z)

Symmetric(v)



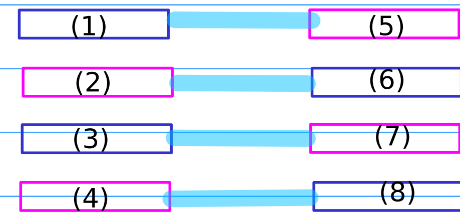
4 examples of such mappings



Permutation (I)

Inverse(z)

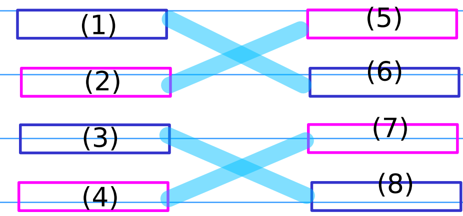
Symmetric(a)



Permutation (II)

Sign, Inverse(a)

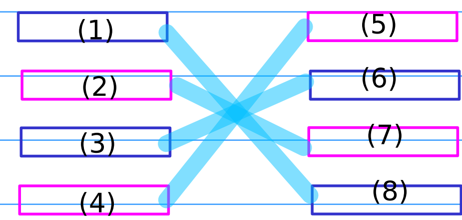
Symmetric(v)



Permutation (III)

Sign

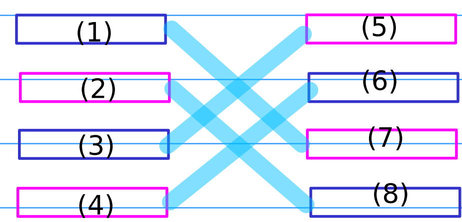
Sign



Permutation (IV)

Inverse(a,z)

Sign, Symmetric(a,v)



,

Transformation of h(a, z)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Inverse(z)

$h(a, z)$	$h(a, z^{-1})$
$h_1(a, z) = -(1-az)^{-1}$	$h_1(a, z^{-1}) = -(1-az^{-1})^{-1}$
$h_2(a, z) = (1-a^{-1}z^{-1})^{-1}$	$h_2(a, z^{-1}) = (1-a^{-1}z)^{-1}$
$h_3(a, z) = -(1-a^{-1}z)^{-1}$	$h_3(a, z^{-1}) = -(1-a^{-1}z^{-1})^{-1}$
$h_4(a, z) = (1-az^{-1})^{-1}$	$h_4(a, z^{-1}) = (1-az)^{-1}$

Sign, Inverse(a)

$h(a, z)$	$-h(a^{-1}, z)$
$h_1(a, z) = -(1-az)^{-1}$	$-h_1(a^{-1}, z) = (1-a^{-1}z)^{-1}$
$h_2(a, z) = (1-a^{-1}z^{-1})^{-1}$	$-h_2(a^{-1}, z) = -(1-az^{-1})^{-1}$
$h_3(a, z) = -(1-a^{-1}z)^{-1}$	$-h_3(a^{-1}, z) = (1-az)^{-1}$
$h_4(a, z) = (1-az^{-1})^{-1}$	$-h_4(a^{-1}, z) = -(1-a^{-1}z^{-1})^{-1}$

Sign

$h(a, z)$	$-h(a, z)$
$h_1(a, z) = -(1-az)^{-1}$	$-h_1(a, z) = (1-az)^{-1}$
$h_2(a, z) = (1-a^{-1}z^{-1})^{-1}$	$-h_2(a, z) = -(1-a^{-1}z^{-1})^{-1}$
$h_3(a, z) = -(1-a^{-1}z)^{-1}$	$-h_3(a, z) = (1-a^{-1}z)^{-1}$
$h_4(a, z) = (1-az^{-1})^{-1}$	$-h_4(a, z) = -(1-az^{-1})^{-1}$

Inverse(a,z)

$h(a, z)$	$h(a^{-1}, z^{-1})$
$h_1(a, z) = -(1-az)^{-1}$	$h_1(a^{-1}, z^{-1}) = -(1-a^{-1}z^{-1})^{-1}$
$h_2(a, z) = (1-a^{-1}z^{-1})^{-1}$	$h_2(a^{-1}, z^{-1}) = (1-az)^{-1}$
$h_3(a, z) = -(1-a^{-1}z)^{-1}$	$h_3(a^{-1}, z^{-1}) = -(1-az^{-1})^{-1}$
$h_4(a, z) = (1-az^{-1})^{-1}$	$h_4(a^{-1}, z^{-1}) = (1-a^{-1}z)^{-1}$

Transformation of h(a, z)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Inverse(a)

$h(a, z)$	$h(a^{-1}, z)$
$h_1(a, z) = -(1-az)^{-1}$	$h_1(a^{-1}, z) = -(1-a^{-1}z)^{-1}$
$h_2(a, z) = -(1-az^{-1})^{-1}$	$h_2(a^{-1}, z) = -(1-a^{-1}z^{-1})^{-1}$
$h_3(a, z) = (1-a^{-1}z^{-1})^{-1}$	$h_3(a^{-1}, z) = (1-az^{-1})^{-1}$
$h_4(a, z) = (1-a^{-1}z)^{-1}$	$h_4(a^{-1}, z) = (1-az)^{-1}$

Inverse(a,z)

$h(a, z)$	$h(a^{-1}, z^{-1})$
$h_1(a, z) = -(1-az)^{-1}$	$h_1(a^{-1}, z^{-1}) = -(1-a^{-1}z^{-1})^{-1}$
$h_2(a, z) = -(1-az^{-1})^{-1}$	$h_2(a^{-1}, z^{-1}) = -(1-a^{-1}z)^{-1}$
$h_3(a, z) = (1-a^{-1}z^{-1})^{-1}$	$h_3(a^{-1}, z^{-1}) = (1-az)^{-1}$
$h_4(a, z) = (1-a^{-1}z)^{-1}$	$h_4(a^{-1}, z^{-1}) = (1-az^{-1})^{-1}$

Sign

$h(a, z)$	$-h(a, z)$
$h_1(a, z) = -(1-az)^{-1}$	$-h_1(a, z) = (1-az)^{-1}$
$h_2(a, z) = -(1-az^{-1})^{-1}$	$-h_2(a, z) = (1-az^{-1})^{-1}$
$h_3(a, z) = (1-a^{-1}z^{-1})^{-1}$	$-h_3(a, z) = -(1-a^{-1}z^{-1})^{-1}$
$h_4(a, z) = (1-a^{-1}z)^{-1}$	$-h_4(a, z) = -(1-a^{-1}z)^{-1}$

Sign, Inverse(z)

$h(a, z)$	$-h(a, z^{-1})$
$h_1(a, z) = -(1-az)^{-1}$	$h_1(a, z^{-1}) = (1-az^{-1})^{-1}$
$h_2(a, z) = -(1-az^{-1})^{-1}$	$h_2(a, z^{-1}) = (1-az)^{-1}$
$h_3(a, z) = (1-a^{-1}z^{-1})^{-1}$	$h_3(a, z^{-1}) = -(1-a^{-1}z)^{-1}$
$h_4(a, z) = (1-a^{-1}z)^{-1}$	$h_4(a, z^{-1}) = -(1-a^{-1}z^{-1})^{-1}$

Transformation of R(a, z)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Inverse(z)

$R(a, z)$	$R(a, z^{-1})$
$R_1(a, z)$: $\lvert z\rvert < a^{-1}$	$R_1(a, z^{-1})$: $\lvert z\rvert > a$
$R_2(a, z)$: $\lvert z\rvert > a^{-1}$	$R_2(a, z^{-1})$: $\lvert z\rvert < a$
$R_3(a, z)$: $\lvert z\rvert < a$	$R_3(a, z^{-1})$: $\lvert z\rvert > a^{-1}$
$R_4(a, z)$: $\lvert z\rvert > a$	$R_4(a, z^{-1})$: $\lvert z\rvert < a^{-1}$

Inverse(a)

$R(a, z)$	$R(a^{-1}, z)$
$R_1(a, z)$: $\lvert z\rvert < a^{-1}$	$R_1(a^{-1}, z)$: $\lvert z\rvert < a$
$R_2(a, z)$: $\lvert z\rvert > a^{-1}$	$R_2(a^{-1}, z)$: $\lvert z\rvert > a$
$R_3(a, z)$: $\lvert z\rvert < a$	$R_3(a^{-1}, z)$: $\lvert z\rvert < a^{-1}$
$R_4(a, z)$: $\lvert z\rvert > a$	$R_4(a^{-1}, z)$: $\lvert z\rvert > a^{-1}$

Identity

$R(a, z)$	$R(a, z)$
$R_1(a, z)$: $\lvert z\rvert < a^{-1}$	$R_1(a, z)$: $\lvert z\rvert < a^{-1}$
$R_2(a, z)$: $\lvert z\rvert > a^{-1}$	$R_2(a, z)$: $\lvert z\rvert > a^{-1}$
$R_3(a, z)$: $\lvert z\rvert < a$	$R_3(a, z)$: $\lvert z\rvert < a$
$R_4(a, z)$: $\lvert z\rvert > a$	$R_4(a, z)$: $\lvert z\rvert > a$

Inverse(a,z)

$R(a, z)$	$R(a^{-1}, z^{-1})$
$R_1(a, z)$: $\lvert z\rvert < a^{-1}$	$R_1(a^{-1}, z^{-1})$: $\lvert z\rvert > a^{-1}$
$R_2(a, z)$: $\lvert z > a^{-1}$	$R_2(a^{-1}, z^{-1})$: $\lvert z\rvert < a^{-1}$
$R_3(a, z)$: $\lvert z\rvert < a$	$R_3(a^{-1}, z^{-1})$: $\lvert z\rvert > a$
$R_4(a, z)$: $\lvert z\rvert > a$	$R_4(a^{-1}, z^{-1})$: $\lvert z\rvert < a$

Transformation of R(a, z)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Inverse(a)

$R(a, z)$	$R(a^{-1}, z)$
$R_1(a, z) : \lvert z\rvert < a^{-1}$	$R_1(a^{-1}, z) : \lvert z\rvert < a$
$R_2(a, z) : \lvert z\rvert > a$	$R_2(a^{-1}, z) : \lvert z\rvert > a^{-1}$
$R_3(a, z) : \lvert z\rvert > a^{-1}$	$R_3(a^{-1}, z) : \lvert z\rvert > a$
$R_4(a, z) : \lvert z\rvert < a$	$R_4(a^{-1}, z) : \lvert z\rvert < a^{-1}$

Inverse(a,z)

$R(a, z)$	$R(a^{-1}, z^{-1})$
$R_1(a, z) : \lvert z\rvert < a^{-1}$	$R_1(a^{-1}, z^{-1}) : \lvert z\rvert > a^{-1}$
$R_2(a, z) : \lvert z\rvert > a$	$R_2(a^{-1}, z^{-1}) : \lvert z\rvert < a$
$R_3(a, z) : \lvert z\rvert > a^{-1}$	$R_3(a^{-1}, z^{-1}) : \lvert z\rvert < a^{-1}$
$R_4(a, z) : \lvert z\rvert < a$	$R_4(a^{-1}, z^{-1}) : \lvert z\rvert > a$

Identity

$R(a, z)$	$R(a, z)$
$R_1(a, z) : \lvert z\rvert < a^{-1}$	$R_1(a, z) : \lvert z\rvert < a^{-1}$
$R_2(a, z) : \lvert z\rvert > a$	$R_2(a, z) : \lvert z\rvert > a$
$R_3(a, z) : \lvert z\rvert > a^{-1}$	$R_3(a, z) : \lvert z\rvert > a^{-1}$
$R_4(a, z) : \lvert z\rvert < a$	$R_4(a, z) : \lvert z\rvert < a$

Inverse(z)

$R(a, z)$	$R(a^{-1}, z^{-1})$
$R_1(a, z) : \lvert z\rvert < a^{-1}$	$R_1(a^{-1}, z^{-1}) : \lvert z\rvert > a$
$R_2(a, z) : \lvert z\rvert > a$	$R_2(a^{-1}, z^{-1}) : \lvert z\rvert < a^{-1}$
$R_3(a, z) : \lvert z\rvert > a^{-1}$	$R_3(a^{-1}, z^{-1}) : \lvert z\rvert < a$
$R_4(a, z) : \lvert z\rvert < a$	$R_4(a^{-1}, z^{-1}) : \lvert z\rvert > a^{-1}$

Mappings of ROC and simple pole expressions

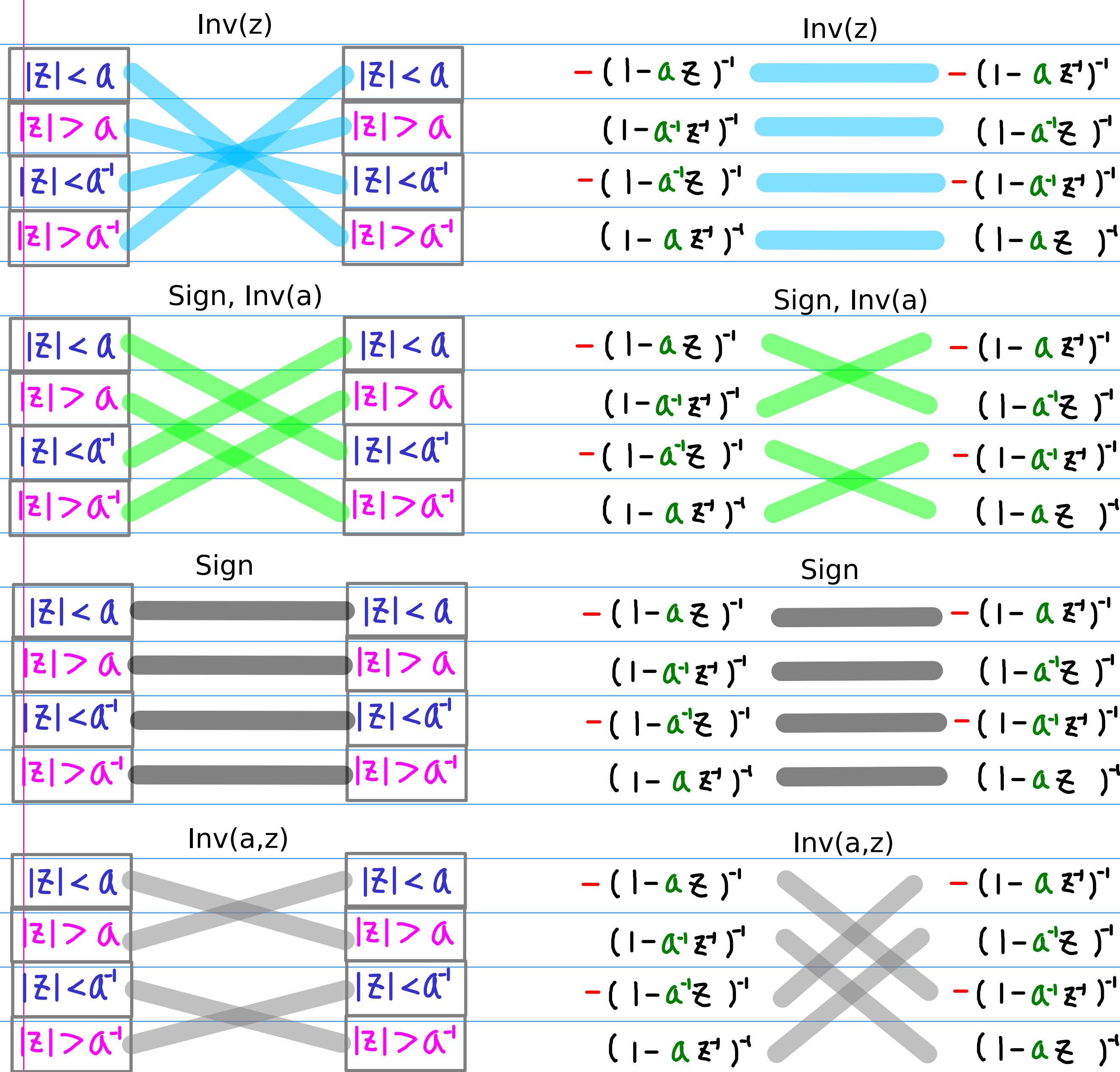
(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$\square \ \square^{-1}$ **Permutation 1** Inverse(z)

$(-1)\ \square^{-1}\ \square$ **Permutation 2** Sign, Inverse(a)

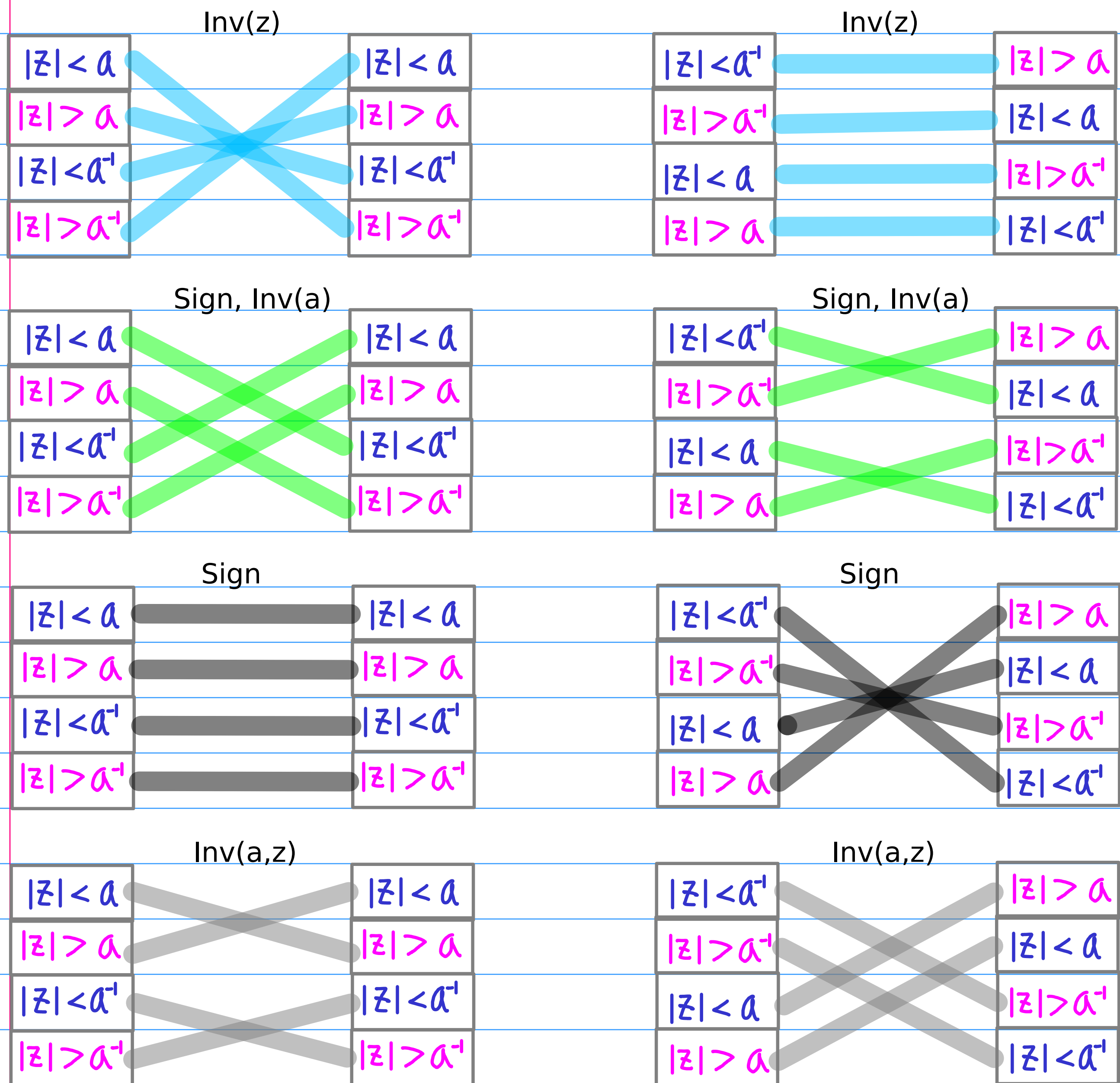
(-1) **Permutation 3** Sign

$\square^{-1}\ \square^{-1}$ **Permutation 4** Inverse(a,z)



Rearranging mappings of ROC expressions

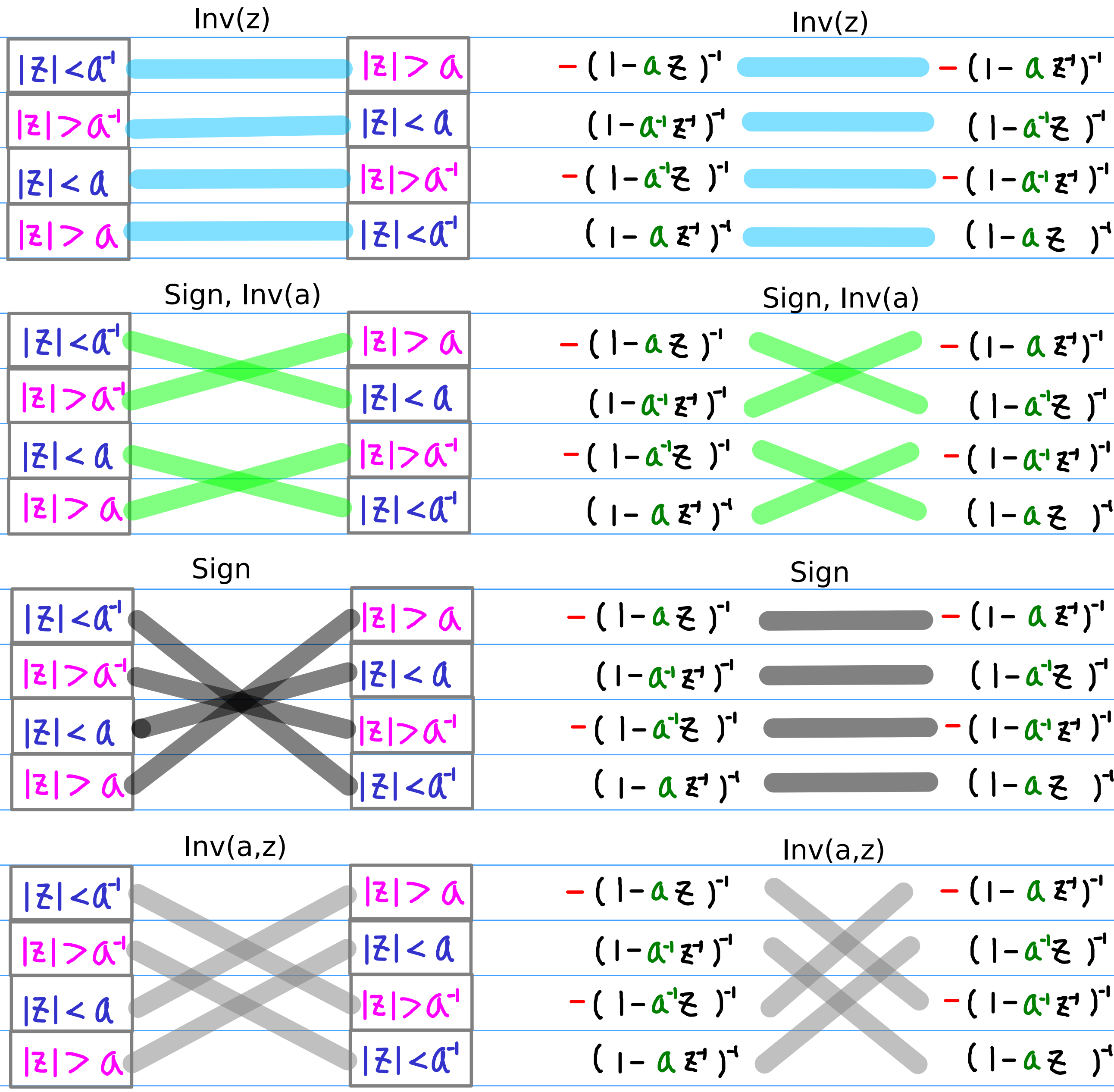
(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)



Mappings of ROC and simple pole expressions (Rearranged)

(1) (2)
(3) (4)
(5) (6)
(7) (8)

□ □$^{-1}$	**Permutation 1**	Inverse(z)
(−1) □$^{-1}$ □	**Permutation 2**	Sign, Inverse(a)
(−1)	**Permutation 3**	Sign
□$^{-1}$ □$^{-1}$	**Permutation 4**	Inverse(a,z)



Transformation of a(n)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Symmetric(a), Symmetric(v)

$a^{n} \cdot v(n)$	$a^{-n} \cdot v(-n)$
$a_n = -a^{n} \cdot u(n)$	$a_n = -a^{-n} \cdot u(-n)$
$a_n = a^{n} \cdot u(-n)$	$a_n = a^{-n} \cdot u(n)$
$a_n = -a^{-n} \cdot u(n)$	$a_n = -a^{n} \cdot u(-n)$
$a_n = a^{-n} \cdot u(-n)$	$c_1 = a^{n} \cdot u(n)$

Sign, Symmetric(a)

$a^{n} \cdot v(n)$	$-a^{-n} \cdot v(n)$
$a_n = -a^{n} \cdot u(n)$	$a_n = a^{-n} \cdot u(n)$
$a_n = a^{n} \cdot u(-n)$	$a_n = -a^{-n} \cdot u(-n)$
$a_n = -a^{-n} \cdot u(n)$	$a_n = a^{n} \cdot u(n)$
$a_n = a^{-n} \cdot u(-n)$	$a_n = -a^{n} \cdot u(-n)$

Sign

$a^{n} \cdot v(n)$	$-a^{n} \cdot v(n)$
$a_n = -a^{n} \cdot u(n)$	$a_n = a^{n} \cdot u(n)$
$a_n = a^{n} \cdot u(-n)$	$a_n = -a^{n} \cdot u(-n)$
$a_n = -a^{-n} \cdot u(n)$	$a_n = a^{-n} \cdot u(n)$
$a_n = a^{-n} \cdot u(-n)$	$a_n = -a^{-n} \cdot u(-n)$

Symmetric(v)

$a^{n} \cdot v(n)$	$a^{n} \cdot v(-n)$
$a_n = -a^{n} \cdot u(n)$	$a_n = -a^{n} \cdot u(-n)$
$a_n = a^{n} \cdot u(-n)$	$a_n = a^{n} \cdot u(n)$
$a_n = -a^{-n} \cdot u(n)$	$a_n = -a^{-n} \cdot u(-n)$
$a_n = a^{-n} \cdot u(-n)$	$a_n = a^{-n} \cdot u(n)$

Transformation of a(n)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Symmetric(a)

$a^n \cdot v(n)$	$a^{-n} \cdot v(n)$
$a_n = -a^n \cdot u(n)$	$a_n = -a^{-n} \cdot u(n)$
$a_n = -a^{-n} \cdot u(-n)$	$a_n = -a^n \cdot u(-n)$
$a_n = a^n \cdot u(-n)$	$a_n = a^{-n} \cdot u(-n)$
$a_n = a^{-n} \cdot u(n)$	$a_n = a^n \cdot u(n)$

Symmetric(v)

$a^n \cdot v(n)$	$a^n \cdot v(-n)$
$a_n = -a^n \cdot u(n)$	$a_n = -a^n \cdot u(-n)$
$a_n = -a^{-n} \cdot u(-n)$	$a_n = -a^{-n} \cdot u(n)$
$a_n = a^n \cdot u(-n)$	$a_n = a^n \cdot u(n)$
$a_n = a^{-n} \cdot u(n)$	$a_n = a^{-n} \cdot u(-n)$

Sign

$a^n \cdot v(n)$	$-a^n \cdot v(n)$
$a_n = -a^n \cdot u(n)$	$a_n = a^n \cdot u(n)$
$a_n = -a^{-n} \cdot u(-n)$	$a_n = a^{-n} \cdot u(-n)$
$a_n = a^n \cdot u(-n)$	$a_n = -a^n \cdot u(-n)$
$a_n = a^{-n} \cdot u(n)$	$a_n = -a^{-n} \cdot u(n)$

Sign, Symmetric(a), Symmetric(v)

$a^n \cdot v(n)$	$-a^{-n} \cdot v(-n)$
$a_n = -a^n \cdot u(n)$	$a_n = a^{-n} \cdot u(-n)$
$a_n = -a^{-n} \cdot u(-n)$	$a_n = a^n \cdot u(n)$
$a_n = a^n \cdot u(-n)$	$a_n = -a^{-n} \cdot u(n)$
$a_n = a^{-n} \cdot u(n)$	$a_n = -a^n \cdot u(-n)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Transformation of v(n)

Symmetric(v)

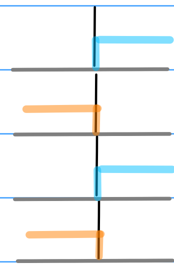
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



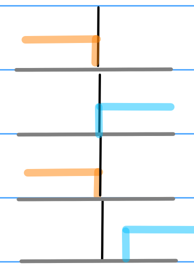
$v(-n)$

$v_1(-n) = u(-n)$

$v_2(-n) = u(n)$

$v_3(-n) = u(-n)$

$v_4(-n) = u(n)$



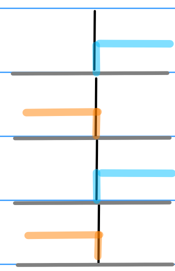
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



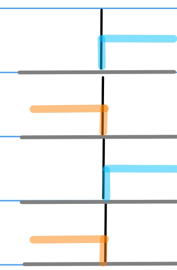
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



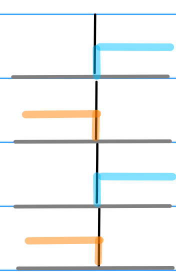
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



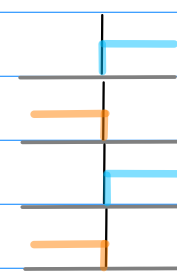
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



Symmetirc(v)

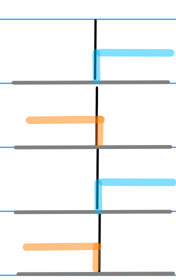
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(n)$

$v_4(n) = u(-n)$



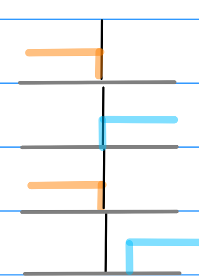
$v(-n)$

$v_1(-n) = u(-n)$

$v_2(-n) = u(n)$

$v_3(-n) = u(-n)$

$v_4(-n) = u(n)$



(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Transformation of v(n)

Identity(v)

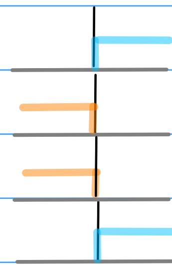
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



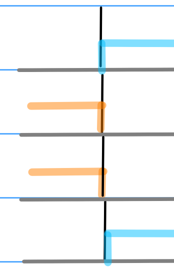
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



Symmetric(v)

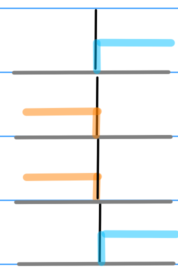
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



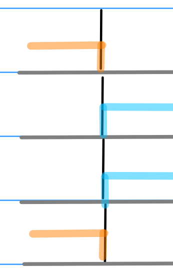
$v(-n)$

$v_1(-n) = u(-n)$

$v_2(-n) = u(n)$

$v_3(-n) = u(n)$

$v_4(-n) = u(-n)$



Identity(v)

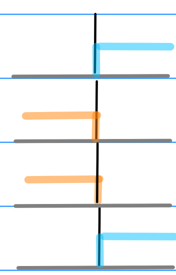
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



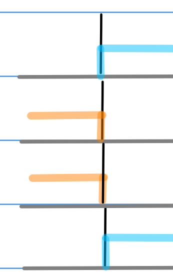
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



Symmetric(v)

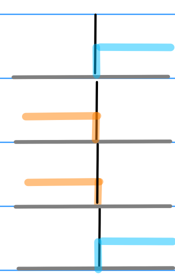
$v(n)$

$v_1(n) = u(n)$

$v_2(n) = u(-n)$

$v_3(n) = u(-n)$

$v_4(n) = u(n)$



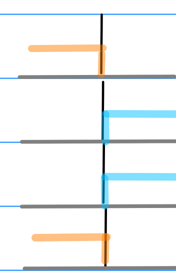
$v(-n)$

$v_1(-n) = u(-n)$

$v_2(-n) = u(n)$

$v_3(-n) = u(n)$

$v_4(-n) = u(-n)$



(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Permutation (I)

Inverse(z)	$h(a, z)$	$R(a, z)$	$h(a, z^{-1})$	$R(a, z^{-1})$
Symmetric(a, v)	a^{n}	$\cdot v(n)$	a^{-n}	$\cdot v(-n)$

Permutation (II)

Sign, Inverse(a)	$h(a, z)$	$R(a, z)$	$-h(a^{-1}, z)$	$R(a^{-1}, z)$
Sign, Symmetric(a)	a^{n}	$\cdot v(n)$	$-a^{n}$	$\cdot v(-n)$

Permutation (III)

Sign	$h(a, z)$	$R(a, z)$	$-h(a, z)$	$R(a, z)$
Sign	a^{n}	$\cdot v(n)$	$-a^{n}$	$\cdot v(n)$

Permutation (IV)

Inverse(a,z)	$h(a, z)$	$R(a, z)$	$h(a^{-1}, z^{-1})$	$R(a^{-1}, z^{-1})$
Symmetric(v)	a^{n}	$\cdot v(n)$	a^{n}	$\cdot v(-n)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Permutation (I)

Inverse(z)	$h(a, z)$	$R(a, z)$	$h(a, z^{-1})$	$R(a, z^{-1})$
Symmetric(a)	a^n	$\cdot v(n)$	a^{-n}	$\cdot v(n)$

Permutation (II)

Sign, Inverse(a)	$h(a, z)$	$R(a, z)$	$-h(a^{-1}, z)$	$R(a^{-1}, z)$
Symmetric(v)	a^n	$\cdot v(n)$	a^n	$\cdot v(-n)$

Permutation (III)

Sign	$h(a, z)$	$R(a, z)$	$-h(a, z)$	$R(a\ z)$
Sign	a^n	$\cdot v(n)$	$-a^n$	$\cdot v(n)$

Permutation (IV)

Inverse(a,z)	$h(a, z)$	$R(a, z)$	$h(a^{-1}, z^{-1})$	$R(a^{-1}, z^{-1})$
Sign, Symmetric(a,v)	a^n	$\cdot v(n)$	$-a^{-n}$	$\cdot v(-n)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Permutation (1) Inverse(z) Symm(a,v)	Permutation (2) Sign, Inverse(a) Sign, Symm(a)
Permutation (3) Sign Sign	Permutation (4) Inverse(a,z) Symm(v)

(2) = (1)+(3)+(4)
Inverse(z) (1),
Sign (3),
Inverse(a,z) (4)

Inverse(z,z) = iden

(2) = (1)+(3)+(4)
Symm(a,v) (1),
Sign (3),
Symm(v) (4)

Sign, Symm(a)

(1) = (2) + (3) + (4)
(2) = (1) + (3) + (4)
(3) = (1) + (2) + (4)
(4) = (1) + (2) + (3)

(1) + (2) = (3) + (4)
(2) + (1) = (3) + (4)
(3) + (1) = (2) + (4)
(4) + (1) = (2) + (3)

(1)	(2)
(3)	(4)

(5) (6)
(7) (8)

Permutation (1) Inverse(a) Symm(a)	Permutation (2) Inverse(a,z) Symm(v)
Permutation (3) Identity Sign	Permutation (4) Inverse(z) Sign, Symm(a,v) ,

(2) = (1)+(3)+(4)
Inverse(a) (1),
Identity (3),
Inverse(z) (4)

(2) = (1)+(3)+(4)
Symm(a) (1),
Sign (3),
Sign, Symm(a,v) (4)

Symm(v)

(1) = (2) + (3) + (4)
(2) = (1) + (3) + (4)
(3) = (1) + (2) + (4)
(4) = (1) + (2) + (3)

(1) + (2) = (3) + (4)
(2) + (1) = (3) + (4)
(3) + (1) = (2) + (4)
(4) + (1) = (2) + (3)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

z-domain

Permutation (1) Inverse(z)	Permutation (2) Sign, Inverse(a)
Permutation (3) Sign	Permutation (4) Inverse(a,z)

(2) = (1)+(3)+(4)
Inverse Z (1),
Sign (3),
Inverse a, Z (4)

Inverse Z + Z = iden

(1) Inverse(z)	(2)
(3)	(4) Inverse(z)

(1)	(2) Sign
(3) Sign	(4)

(1)	(2) Inverse(a)
(3)	(4) Inverse(a)

(1) (2)
(3) (4)
(5) (6)
(7) (8)

z-domain

Permutation (1) Inverse(a)	Permutation (2) Inverse(a,z)
Permutation (3) Identity	Permutation (4) Inverse(z)

(2) = (1)+(3)+(4)
Inverse(a) (1),
Identity (3),
Inverse(z) (4)

(1) Inverse(a)	(2) Inverse(a)
(3)	(4)

(1)	(2) Inverse(z)
(3)	(4) Inverse(z)

(1)	(2)
(3)	(4)

.

n-domain

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Symm(a,v)	Sign, Symm(a)
Sign	Symm(v)

(2) = (1)+(3)+(4)
Symm(a,v) (1),
Sign (3),
Symm(v) (4)

(1) a^{-n} Symm(a)	(2) a^{-n} Symm(a)
(3) a^{n}	(4) a^{n}

$a^{-n} \cdot v(-n)$

$-a^{-n} \cdot v(n)$

$-a^{n} \cdot v(n)$

$a^{n} \cdot v(-n)$

(1) $\cdot v(-n)$ Symm(v)	(2) $\cdot v(n)$
(3) $\cdot v(n)$ Sign	(4) $\cdot v(-n)$ Symm(v)

(1)	(2) $-$ Sign
(3) $-$ Sign	(4)

n-domain

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

Symm(a)	Symm(v)
Sign	Sign, Symm(a,v)

(2) = (1)+(3)+(4)
Symm(a) (1),
Sign (3),
Sign, Symm(a,v) (4)

(1) a^{-n} Symm(a)	(2) a^{n}
(3) a^{n}	(4) a^{-n} Symm(a)

$a^{-n} \cdot v(n)$

$a^{n} \cdot v(-n)$

$-a^{n} \cdot v(n)$

$-a^{-n} \cdot v(-n)$

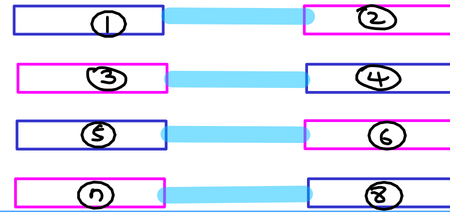
(1) $\cdot v(n)$	(2) $\cdot v(-n)$ Symm(v)
(3) $\cdot v(n)$	(4) $\cdot v(-n)$ Symm(v)

(1)	(2)
(3) $-$ Sign	(4) $-$ Sign

Permutation (1)

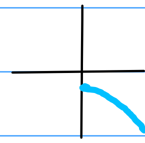
Inverse(z)

Symmetric(a, v)

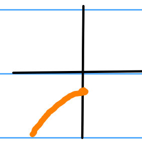


① $a_n = -2^n \cdot u(n)$ $(n \geq 0)$

-2^0	-2^1	-2^2	...
0	1	2	...



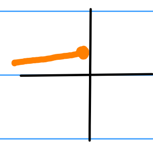
② $a_n = -(\frac{1}{2})^n \cdot u(-n)$ $(n \leq 0)$



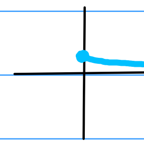
...	-2^2	-2^1	-2^0
...	-2	-1	0

③ $a_n = 2^n \cdot u(-n)$ $(n \leq 0)$

...	$(\frac{1}{2})^2$	$(\frac{1}{2})^1$	$(\frac{1}{2})^0$
...	-2	-1	0



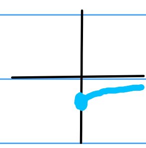
④ $a_n = (\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$



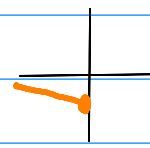
$(\frac{1}{2})^0$	$(\frac{1}{2})^1$	$(\frac{1}{2})^2$	...
0	1	2	...

⑤ $a_n = -(\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$

$-(\frac{1}{2})^0$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^2$	...
0	1	2	...



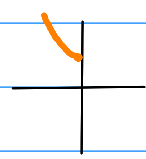
⑥ $a_n = -2^n \cdot u(-n)$ $(n \leq 0)$



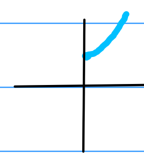
...	$-(\frac{1}{2})^2$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^0$
...	-2	-1	0

⑦ $a_n = (\frac{1}{2})^n \cdot u(-n-1)$ $(n \leq 0)$

...	2^2	2^1	2^0
...	-2	-1	0



⑧ $a_n = 2^n \cdot u(n-1)$ $(n \geq 0)$

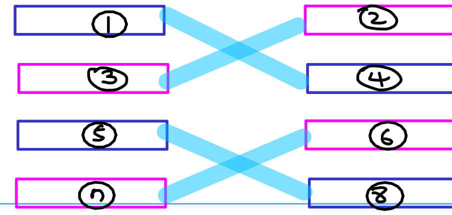


2^0	2^1	2^2	...
0	1	2	...

Permutation (2)

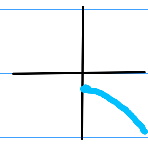
Sign, Inverse(a)

Sign,Shift(v),
Symmetric(a)

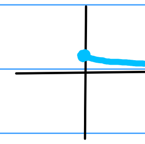


① $a_n = -2^n \cdot u(n)$ $(n \geq 0)$

-2^0	-2^1	-2^2	⋯
0	1	2	⋯



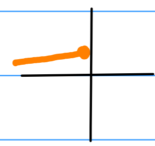
④ $a_n = \left(\frac{1}{2}\right)^n u(n)$ $(n \geq 0)$



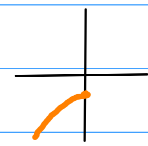
$\left(\frac{1}{2}\right)^0$	$\left(\frac{1}{2}\right)^1$	$\left(\frac{1}{2}\right)^2$	⋯
0	1	2	⋯

③ $a_n = 2^n \cdot u(-n)$ $(n \leq 0)$

⋯	$\left(\frac{1}{2}\right)^2$	$\left(\frac{1}{2}\right)^1$	$\left(\frac{1}{2}\right)^0$
⋯	-2	-1	0



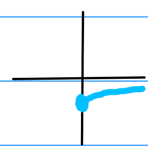
② $a_n = -\left(\frac{1}{2}\right)^n \cdot u(-n)$ $(n \leq 0)$



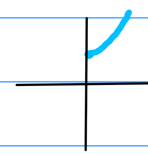
⋯	-2^2	-2^1	-2^0
⋯	-2	-1	0

⑤ $a_n = -\left(\frac{1}{2}\right)^n \cdot u(n)$ $(n \geq 0)$

$-\left(\frac{1}{2}\right)^0$	$-\left(\frac{1}{2}\right)^1$	$-\left(\frac{1}{2}\right)^2$	⋯
0	1	2	⋯



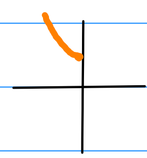
⑧ $a_n = 2^n \cdot u(n-1)$ $(n \geq 0)$



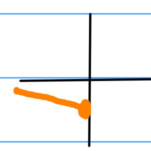
2^0	2^1	2^2	⋯
0	1	2	⋯

⑦ $a_n = \left(\frac{1}{2}\right)^n \cdot u(-n-1)$ $(n \leq 0)$

⋯	2^2	2^1	2^0
⋯	-2	-1	0



⑥ $a_n = -2^n \cdot u(-n)$ $(n \leq 0)$

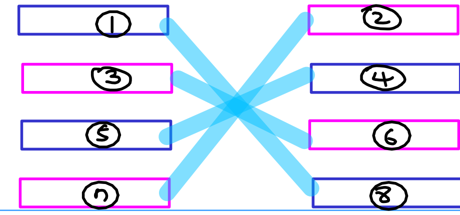


⋯	$-\left(\frac{1}{2}\right)^2$	$-\left(\frac{1}{2}\right)^1$	$-\left(\frac{1}{2}\right)^0$
⋯	-2	-1	0

Permutation (3)

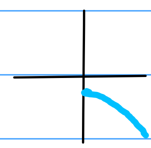
Sign

Sign, Shift(v)

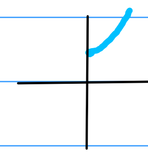


① $a_n = -2^n \cdot u(n)$ $(n \geq 0)$

-2^0	-2^1	-2^2	...
0	1	2	...



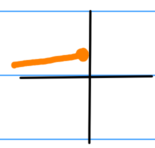
⑧ $a_n = 2^n \cdot u(n-1)$ $(n \geq 0)$



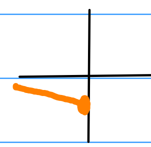
2^0	2^1	2^2	...
0	1	2	...

③ $a_n = 2^n \cdot u(-n)$ $(n \leq 0)$

...	$(\frac{1}{2})^2$	$(\frac{1}{2})^1$	$(\frac{1}{2})^0$
...	-2	-1	0



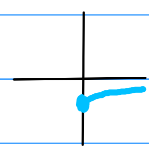
⑥ $a_n = -2^n \cdot u(-n)$ $(n \leq 0)$



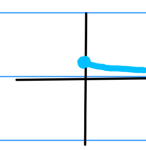
...	$-(\frac{1}{2})^2$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^0$
...	-2	-1	0

⑤ $a_n = -(\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$

$-(\frac{1}{2})^0$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^2$	...
0	1	2	...



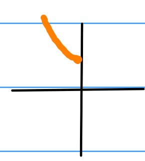
④ $a_n = (\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$



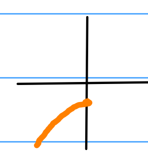
$(\frac{1}{2})^0$	$(\frac{1}{2})^1$	$(\frac{1}{2})^2$	...
0	1	2	...

⑦ $a_n = (\frac{1}{2})^n \cdot u(-n-1)$ $(n \leq 0)$

...	2^2	2^1	2^0
...	-2	-1	0



② $a_n = -(\frac{1}{2})^n \cdot u(-n)$ $(n \leq 0)$

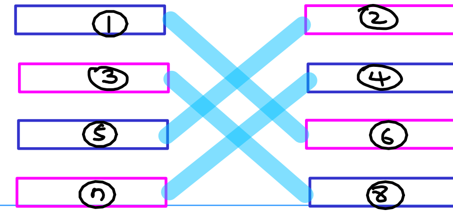


...	-2^2	-2^1	-2^0
...	-2	-1	0

Permutation (4)

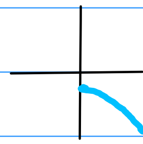
Inverse(a, z)

Symmetric(v)

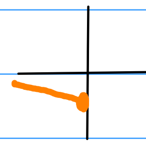


① $a_n = -2^n \cdot u(n)$ $(n \geq 0)$

-2^0	-2^1	-2^2	…
0	1	2	…



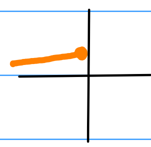
⑥ $a_n = -2^n \cdot u(-n)$ $(n \leq 0)$



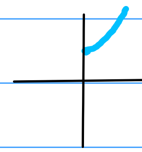
…	$-(\frac{1}{2})^2$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^0$
…	-2	-1	0

③ $a_n = 2^n \cdot u(-n)$ $(n \leq 0)$

…	$(\frac{1}{2})^2$	$(\frac{1}{2})^1$	$(\frac{1}{2})^0$
…	-2	-1	0



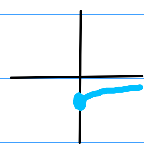
⑧ $a_n = 2^n \cdot u(n-1)$ $(n \geq 0)$



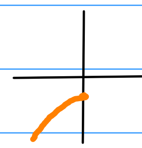
2^0	2^1	2^2	…
0	1	2	…

⑤ $a_n = -(\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$

$-(\frac{1}{2})^0$	$-(\frac{1}{2})^1$	$-(\frac{1}{2})^2$	…
0	1	2	…



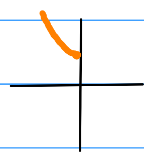
② $a_n = -(\frac{1}{2})^n \cdot u(-n)$ $(n \leq 0)$



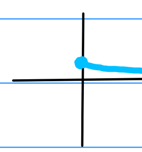
…	-2^2	-2^1	-2^0
…	-2	-1	0

⑦ $a_n = (\frac{1}{2})^n \cdot u(-n-1)$ $(n \leq 0)$

…	2^2	2^1	2^0
…	-2	-1	0



④ $a_n = (\frac{1}{2})^n \cdot u(n)$ $(n \geq 0)$



$(\frac{1}{2})^0$	$(\frac{1}{2})^1$	$(\frac{1}{2})^2$	…
0	1	2	…