Laurent Series and z-Transform
- Geometric Series
Double Pole Examples B

20181113 Tue

2 formulas of z $\{1, 2\} \rightarrow \{1, 0.5\}$

① $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

$\Big\updownarrow z^{-1}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$$f(z) = g(z^{-1})$$
$$g(z) = f(z^{-1})$$

$$\{1,\ 2\} \rightarrow \{1,\ 0.5\}$$

$$\boxed{\frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}}$$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \left(\frac{1}{z^{-1}-1} - \frac{1}{z^{-1}-2}\right)$$
$$= \left(\frac{z}{1-z} - \frac{z}{1-2z}\right)$$
$$= \left(\frac{-z}{z-1} + \frac{0.5z}{z-0.5}\right)$$
$$= z\left(\frac{-1}{z-1} + \frac{0.5}{z-0.5}\right)$$
$$= z\left(\frac{-0.5z}{(z-1)(z-0.5)}\right)$$
$$= \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$$f(z) = f_1(z) = f_2(z^{-1}) \qquad g(z) = g_1(z) = g_2(z^{-1})$$

$$f(z) \quad \boxed{+\frac{1}{z-1} - \frac{1}{z-2}}$$

$$f_1(z) \quad +\frac{1}{z-1}\cdot\frac{(1/1)}{(1/1)} - \frac{1}{z-2}\cdot\frac{(1/2)}{(1/2)} \qquad \boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$$

$$f_2(z^{-1}) \quad +\frac{1}{z-1}\cdot\frac{(1/z)}{(1/z)} - \frac{1}{z-2}\cdot\frac{(1/z)}{(1/z)} \qquad \boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$$g(z) \quad \boxed{-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}}$$

$$g_1(z) \quad -\frac{z}{(z-1)}\cdot\frac{(1/1)}{(1/1)} + \frac{0.5z}{(z-0.5)}\cdot\frac{(1/0.5)}{(1/0.5)} \qquad \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$g_2(z^{-1}) \quad -\frac{z}{(z-1)}\cdot\frac{(1/z)}{(1/z)} + \frac{0.5z}{(z-0.5)}\cdot\frac{(1/z)}{(1/z)} \qquad \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$$

$$f(z) = f_1(z) = f_2(z^{-1})$$

3 representations $f(z),\ f_1(z),\ f_2(z^{-1})$

all equal formulas $f(z) = \boxed{\begin{matrix} f_1(z) \\ |z|<a \end{matrix}} = \boxed{\begin{matrix} f_2(z^{-1}) \\ |z|>a \end{matrix}}$

with complimentary ROC.

$$g(z) = g_1(z) = g_2(z^{-1})$$

3 representations $g(z),\ g_1(z),\ g_2(z^{-1})$

all equal formulas $g(z) = \boxed{\begin{matrix} g_1(z) \\ |z|<a \end{matrix}} = \boxed{\begin{matrix} g_2(z^{-1}) \\ |z|>a \end{matrix}}$

with complimentary ROC's

inverse relationship

$$f(z) = g(z^{-1})$$
$$g(z) = f(z^{-1})$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\dfrac{-1}{(z-1)(z-2)}$

$$\boxed{+\frac{1}{z-1} - \frac{1}{z-2}}$$

$$\boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<1}$$

causal $f(z)$

anti-causal $X(z)$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $f(z)$

causal $X(z)$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$$\boxed{-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>1}$$

anti-causal $f(z)$

causal $X(z)$

$$\boxed{+\frac{z}{1-z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

causal $f(z)$

anti-causal $X(z)$

$f(z) \Longleftrightarrow a_n$

$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<1}$ ①

$\boxed{-1^{n+1}+\left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>1}$ ②

$\boxed{-1^{n-1}+2^{n-1}}$ $(n<1)$

$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$ ④

$\boxed{+1^{n+1}-\left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$ ③

$\boxed{+1^{n-1}-2^{n-1}}$ $(n \geq 1)$

$-\frac{1}{1-z}+\frac{1}{1-0.5z}$ $|z|<1$ $\quad -1^{n}+\left(\frac{1}{2}\right)^{n}$ $(n \geq 0)$

① $-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$ $|z|<1$ $\quad -1^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

② $-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$ $|z|>1$ $\quad -1^{-n+1}+\left(\frac{1}{2}\right)^{-n+1} = -1^{n-1}+2^{n-1}$ $(n<1)$

$+\frac{1}{1-z}-\frac{1}{1-2z}$ $|z|<0.5$ $\quad +1^{n}-2^{n}$ $(n \geq 1)$

③ $+\frac{z}{1-z}-\frac{z}{1-2z}$ $|z|<0.5$ $\quad +1^{n-1}-2^{n-1}$ $(n \geq 1)$

④ $+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ $|z|>2$ $\quad +1^{-n-1}-\left(\frac{1}{2}\right)^{-n-1} = +1^{n+1}-2^{n+1}$ $(n<0)$

$$X(z) \longleftrightarrow x_n$$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z} \quad |z|<1 \quad ②$$

$$-1^{n-1}+2^{n-1} \quad (n<1)$$

$$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>1 \quad ①$$

$$-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \quad (n\geq 0)$$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2 \quad ③$$

$$+1^{n-1}-2^{n-1} \quad (n\geq 1)$$

$$+\frac{z}{1-z}-\frac{z}{1-2z} \quad |z|<0.5 \quad ④$$

$$+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}} \quad |z|>1 \qquad -1^{n}+\left(\tfrac{1}{2}\right)^{n} \quad (n\geq 0)$$

① $$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>1 \qquad -1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \quad (n\geq 0)$$

② $$-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|<1 \qquad -1^{-n+1}+\left(\tfrac{1}{2}\right)^{-n+1} = +2^{n-1}-1^{n-1} \quad (n<1)$$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}} \quad |z|>2 \qquad +1^{n}-2^{n} \quad (n\geq 0)$$

③ $$+\frac{z}{1-z^{-1}}-\frac{z}{1-2z^{-1}} \quad |z|>2 \qquad +1^{n-1}-2^{n-1} \quad (n\geq 1)$$

④ $$+\frac{z}{1-z}-\frac{z}{1-2z} \quad |z|<0.5 \qquad +1^{-n-1}-2^{-n-1} = +1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

$f(z) \longleftrightarrow a_n$

$X(z) \longleftrightarrow x_n$

① - Ⓐ

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<1}$$

$$a_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

② - Ⓐ

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>1}$$

$$a_n = \boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

$$x_n = \boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

① - Ⓑ

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

$$a_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

② - Ⓑ

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

$$a_n = \boxed{+1^{n-1}-2^{n-1}} \quad (n \geq 1)$$

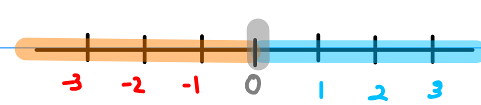
$$x_n = \boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = a_{-n}$$

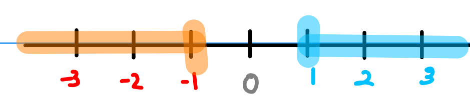
$$a_n = x_{-n}$$

$(n \geq 0) \longleftrightarrow (n<1)$

$(n \geq 1) \longleftrightarrow (n<0)$



$(n<1)$ $(n \geq 0)$



$(n<0)$ $(n \geq 1)$

$f(z)$

$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$ $|z|<1$

$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $|z|>2$

$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $|z|>1$

$+\frac{z}{1-z} - \frac{z}{1-2z}$ $|z|<0.5$

$\boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$

$|z|<1$

$\boxed{-1^{n-1} + 2^{n-1}}$ $(n<1)$

$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$

$|z|>1$

z^{-1} $-n$

$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$

$|z|<0.5$

$\boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$

$|z|>2$

z^{-1} $-n$

$+\frac{z}{1-z} - \frac{z}{1-2z}$

$|z|<0.5$

$\cdot z$ $n-1$

$+\frac{1}{1-z} - \frac{1}{1-2z}$

$|z|<0.5$

$\boxed{+1^{n-1} - 2^{n-1}}$ $(n \geq 1)$

$+\frac{z}{1-z} - \frac{z}{1-2z}$

$|z|<0.5$

$\cdot z$ $n-1$

$+\frac{1}{1-z} - \frac{1}{1-2z}$

$|z|<0.5$

$$X(z)\quad \begin{array}{|c|c|}\hline -\frac{1}{1-z}+\frac{0.5}{1-0.5z} & |z|<1 \\ \hline +\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} & |z|>2 \\ \hline \end{array} \qquad \begin{array}{|c|c|}\hline -\frac{1}{1-z^{-1}}+\frac{0.5}{1-0.5z^{-1}} & |z|>1 \\ \hline +\frac{z}{1-z}-\frac{z}{1-2z} & |z|<0.5 \\ \hline \end{array}$$

$$\boxed{-1^{n-1}+2^{n-1}} \quad (n<1)$$

$$\boxed{-\frac{1}{1-z}+\frac{0.5}{1-0.5z}}$$

$|z|<1$

$$\boxed{z^{-1} \quad -n}$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}}$$

$|z|>1$

$$\boxed{-1^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n\geq 0)$$

$$\boxed{-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}}$$

$|z|>1$

$$\boxed{+1^{n-1}-2^{n-1}} \quad (n\geq 1)$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}$$

$|z|>2$

$$\boxed{\cdot z^{-1} \quad n-1}$$

$$\boxed{+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}}$$

$|z|>2$

$$\boxed{+1^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$\boxed{+\frac{z}{1-z}-\frac{z}{1-2z}}$$

$|z|<0.5$

$$\boxed{z^{-1} \quad -n}$$

$$\boxed{+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}$$

$|z|>2$

$$\boxed{\cdot z^{-1} \quad n-1}$$

$$\boxed{+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}}$$

$|z|>2$

$f_c(z) \leftrightarrow a_n \quad g_a(z^{-1}) \leftrightarrow -b_n$	$f_a(z^{-1}) \leftrightarrow -a_n \quad g_c(z) \leftrightarrow b_n$
$Y_a(z^{-1}) \leftrightarrow -y_n \quad X_c(z) \leftrightarrow x_n$	$Y_c(z) \leftrightarrow y_n \quad X_a(z^{-1}) \leftrightarrow -x_n$

A ① $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ ② $-\dfrac{2}{1-2z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

A-① $f(z) =$	$-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$	$\lvert z\rvert < 1$	$-1^{n+1} + (\frac{1}{2})^{n+1}$	$(n \geq 0)$
A-② $g(z) =$	$-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$	$\lvert z\rvert > 1$	$-1^{n-1} + 2^{n-1}$	$(n < 1)$
A-① $Y(z) =$	$-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$	$\lvert z\rvert < 1$	$-1^{n-1} + 2^{n-1}$	$(n < 1)$
A-② $X(z) =$	$-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$	$\lvert z\rvert > 1$	$-1^{n+1} + (\frac{1}{2})^{n+1}$	$(n \geq 0)$

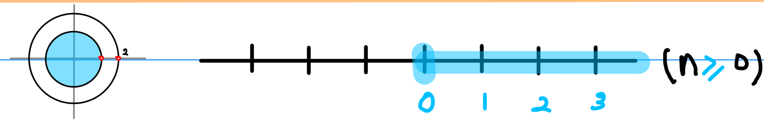
B ① $+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ ② $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

B-① $f(z) =$	$+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$	$\lvert z\rvert > 2$	$+1^{n+1} - (\frac{1}{2})^{n+1}$	$(n < 0)$
B-② $g(z) =$	$+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$	$\lvert z\rvert < 0.5$	$+1^{n-1} - 2^{n-1}$	$(n \geq 1)$
B-① $Y(z) =$	$+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$	$\lvert z\rvert > 2$	$+1^{n-1} - 2^{n-1}$	$(n \geq 1)$
B-② $X(z) =$	$+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$	$\lvert z\rvert < 0.5$	$+1^{n+1} - (\frac{1}{2})^{n+1}$	$(n < 0)$

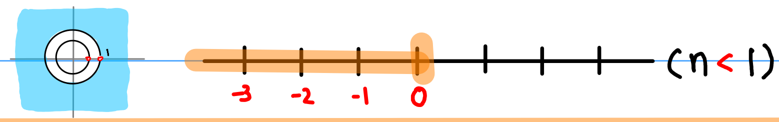
$f_c(z) \leftrightarrow a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$	$f_a(z^{-1}) \leftrightarrow -a_n$	$g_c(z) \leftrightarrow b_n$
$Y_a(z^{-1}) \leftrightarrow -y_n$	$X_c(z) \leftrightarrow x_n$	$Y_c(z) \leftrightarrow y_n$	$X_a(z^{-1}) \leftrightarrow -x_n$

A ① $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ ② $-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

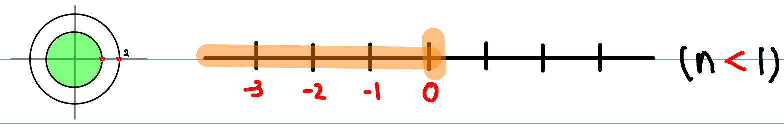
A-① $\boxed{-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$



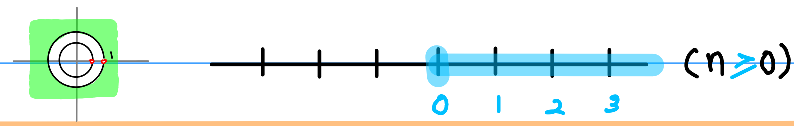
A-② $\boxed{-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$



A-① $\boxed{-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$

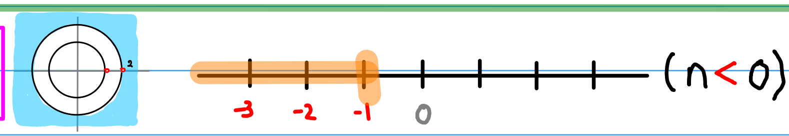


A-② $\boxed{-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$

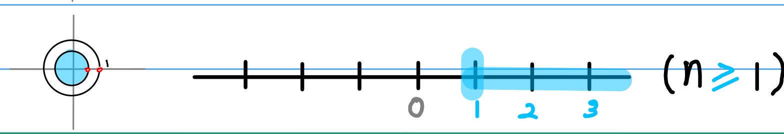


B ① $+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ ② $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$

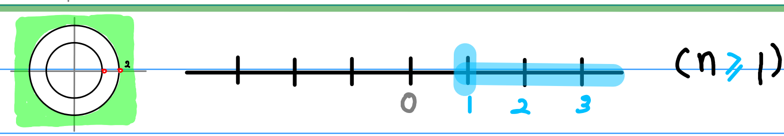
B-① $\boxed{+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$



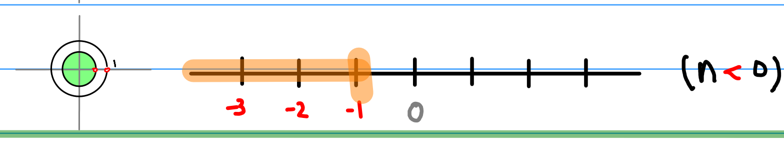
B-② $\boxed{+\dfrac{z}{1-z} - \dfrac{z}{1-2z}}$ $\boxed{|z|<0.5}$



B-① $\boxed{+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$



B-② $\boxed{+\dfrac{z}{1-z} - \dfrac{z}{1-2z}}$ $\boxed{|z|<0.5}$



$f_c(z) \leftrightarrow a_n$ $\quad$ $g_a(z^{-1}) \leftrightarrow -b_n$ $\quad$ $Y_a(z^{-1}) \leftrightarrow -y_n$ $\quad$ $X_c(z) \leftrightarrow x_n$

$f_a(z^{-1}) \leftrightarrow -a_n$ $\quad$ $g_c(z) \leftrightarrow b_n$ $\quad$ $Y_c(z) \leftrightarrow y_n$ $\quad$ $X_a(z^{-1}) \leftrightarrow -x_n$

① $$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

Ⓐ-① $f(z) = \boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$ $\boxed{-1^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$

Ⓑ-① $f(z) = \boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+1^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$

Ⓐ-① $Y(z) = \boxed{-\frac{1}{1-z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$ $\boxed{-1^{n-1} + 2^{n-1}}$ $(n<1)$

Ⓑ-① $Y(z) = \boxed{+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+1^{n-1} - 2^{n-1}}$ $(n \geq 1)$

② $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

Ⓐ-② $g(z) = \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$ $\boxed{-1^{n-1} + 2^{n-1}}$ $(n<1)$

Ⓑ-② $g(z) = \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+1^{n-1} - 2^{n-1}}$ $(n \geq 1)$

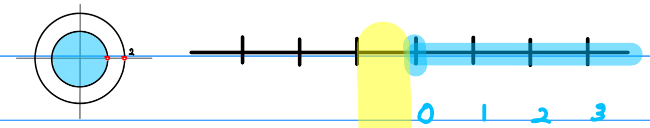
Ⓐ-② $X(z) = \boxed{-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$ $\boxed{-1^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$

Ⓑ-② $X(z) = \boxed{+\frac{z}{1-z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+1^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$

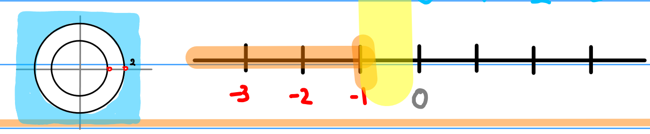
$f_c(z) \leftrightarrow a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$	$Y_a(z^{-1}) \leftrightarrow -y_n$	$X_c(z) \leftrightarrow x_n$
$f_a(z^{-1}) \leftrightarrow -a_n$	$g_c(z) \leftrightarrow b_n$	$Y_c(z) \leftrightarrow y_n$	$X_a(z^{-1}) \leftrightarrow -x_n$

① $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

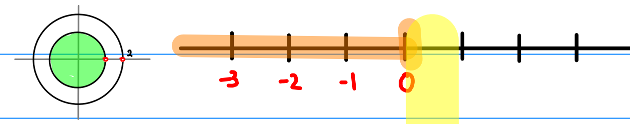
A-① $\boxed{-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$



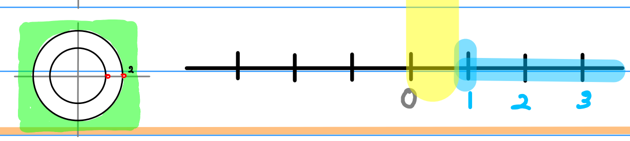
B-① $\boxed{+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$



A-① $\boxed{-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z|<1}$

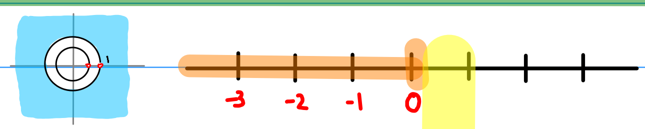


B-① $\boxed{+\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$

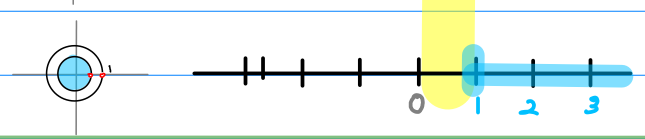


② $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

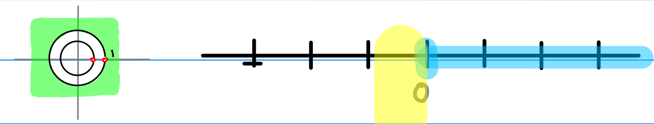
A-② $\boxed{-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$



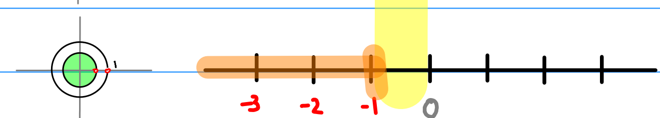
B-② $\boxed{+\dfrac{z}{1-z} - \dfrac{z}{1-2z}}$ $\boxed{|z|<0.5}$



A-② $\boxed{-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>1}$



B-② $\boxed{+\dfrac{z}{1-z} - \dfrac{z}{1-2z}}$ $\boxed{|z|<0.5}$

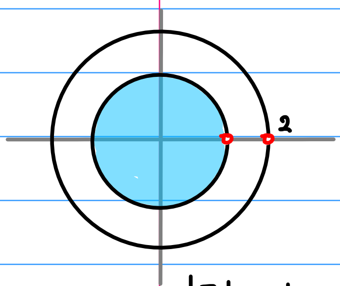


$$\frac{-1}{(z-1)(z-2)} = \boxed{f(z)} \qquad |z|<1 \quad |z|>2$$

causal　　anticausal

① - Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = \frac{-1}{1-z} + \frac{0.5}{1-0.5z}$$



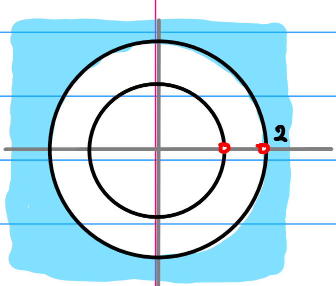
|z| < 1

$$\boxed{\begin{aligned} f(z) &= \frac{(-1)}{1-(1\cdot z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(1)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(1)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$(n \geq 0) \qquad a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

① - Ⓑ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



|z| > 2

$$\boxed{\begin{aligned} f(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{1}\right)^n\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\tfrac{1}{1}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}} \qquad \neq$$

$$= \sum_{n=-1}^{-\infty}(1)^{n+1} z^n - \sum_{n=-1}^{-\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n$$

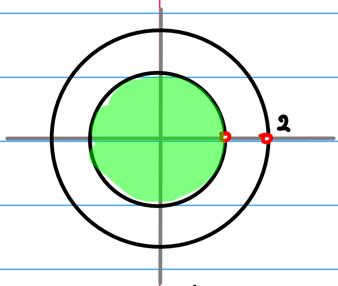
$$(n < 0) \qquad a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-1}{(z-1)(z-2)} = \boxed{X(z)}$$

$|z|<1$ anticausal $\qquad |z|>2$ causal

①-Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = \frac{-1}{1-z} + \frac{0.5}{1-0.5z}$$



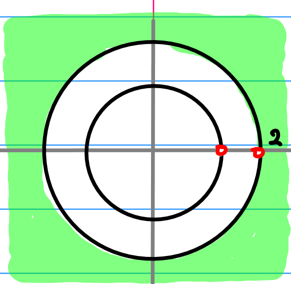
$|z|<1$

$$\begin{aligned} X(z) &= \frac{(-1)}{1-(1\cdot z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)} \\ &= -\sum_{n=0}^{\infty}(1)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(1)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^n \\ &= -\sum_{n=0}^{-\infty}\left(\tfrac{1}{1}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{-n} \end{aligned}$$

$(n<1) \qquad a_n = -1^{n-1} + 2^{n-1}$

①-Ⓑ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z|>2$

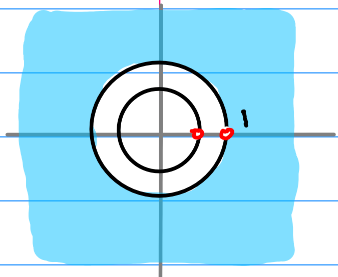
$$\begin{aligned} X(z) &= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)} \qquad \neq \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{1}\right)^n\left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\tfrac{1}{1}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}$$

$(n \geq 1) \qquad a_n = 1^{n-1} - (2)^{n-1}$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \boxed{f(z)}$$

$|z| > 1$ anticausal $|z| < 0.5$ causal

$$② - Ⓐ \quad \frac{-1}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = -\frac{1}{(1-z^{-1})} + \frac{0.5}{(1-0.5z^{-1})}$$



$|z| > 1$

$$f(z) = -\frac{\left(\frac{1}{1}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

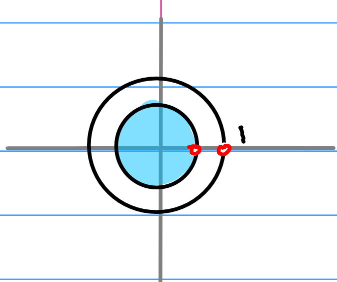
$$= -\sum_{n=0}^{\infty}\left(\frac{1}{1}\right)^{n+1}\left(\frac{1}{z}\right)^{n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^{n}$$

$$= -\sum_{n=0}^{\infty}\left(\frac{1}{1}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$$= -\sum_{n=0}^{-\infty}(1)^{n-1} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n}$$

$(n < 1)$ $a_n = -1^{n-1} + 2^{n-1}$

$$② - Ⓑ \quad \frac{-1}{(z-1)(z-2)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = +\frac{z}{(1-z)} - \frac{z}{(1-2z)}$$



$|z| < 0.5$

$$f(z) = +\frac{(z)}{1-(1\cdot z)} - \frac{(z)}{1-(2z)} \qquad \neq$$

$$= +\sum_{n=0}^{\infty}(1)^{n}(z)^{n+1} - \sum_{n=0}^{\infty}(2)^{n}(z)^{n+1}$$

$$= +\sum_{n=1}^{\infty}(1)^{n-1} z^{n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{n}$$

$(n \geq 1)$ $a_n = +1^{n-1} - 2^{n-1}$

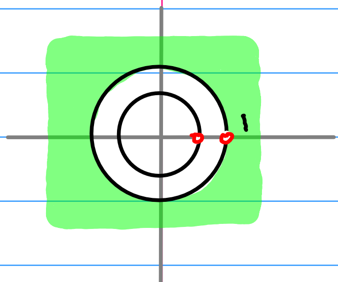
$$\frac{-z^2}{(z-1)(z-0.5)} = \boxed{X(z)}$$

$|z| > 1$ causal

$|z| < 0.5$ anticausal

②–Ⓐ

$$\frac{-1}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = -\frac{1}{(1-z^{-1})} + \frac{0.5}{(1-0.5z^{-1})}$$



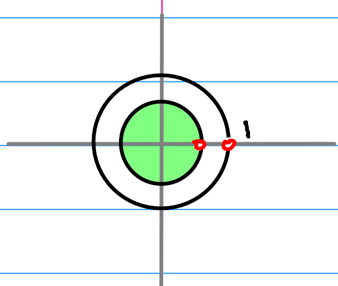
$|z| > 1$

$$\boxed{\begin{aligned} X(z) &= -\frac{\left(\frac{1}{1}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} \\ &= -\sum_{n=0}^{\infty}\left(\frac{1}{1}\right)^{n+1}\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n \\ &= -\sum_{n=0}^{\infty}\left(\frac{1}{1}\right)^{n+1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n} \end{aligned}}$$

$(n \geq 0)$ $\quad a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

②–Ⓑ

$$\frac{-1}{(z-1)(z-2)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right) = +\frac{z}{(1-z)} - \frac{z}{(1-2z)}$$



$|z| < 0.5$

$$\boxed{\begin{aligned} X(z) &= +\frac{(1z)}{1-(1z)} - \frac{(z)}{1-(2z)} \\ &= +\sum_{n=0}^{\infty}(1)^n(z)^{n+1} - \sum_{n=0}^{\infty}(2)^n(z)^{n+1} \\ &= +\sum_{n=1}^{\infty}(1)^{n-1} z^n - \sum_{n=1}^{\infty}(2)^{n-1} z^n \end{aligned}}$$

$$= +\sum_{n=-1}^{-\infty}\left(\frac{1}{1}\right)^{n+1} z^{-n} - \sum_{n=-1}^{-\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$(n < 0)$ $\quad a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1}$