Laurent Series and z-Transform - Geometric Series Double Pole Examples A

20181109 Fri

2 formulas of z $\{0.5, 2\} \to \{2, 0.5\}$

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = g(z^{-1})$$
$$g(z) = f(z^{-1})$$

$$\{0.5,\ 2\} \rightarrow \{2,\ 0.5\}$$

$$\boxed{\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\frac{2}{3}\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$
$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\frac{2}{3}\left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$
$$= \left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1}\right)$$
$$= \left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z}\right)$$
$$= \left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5}\right)$$
$$= z\left(\frac{-2}{z-2} + \frac{0.5}{z-0.5}\right)$$
$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$
$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\frac{2}{3}\left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = f_1(z) = f_2(z^{-1}) \qquad g(z) = g_1(z) = g_2(z^{-1})$$

$$f(z) \quad \boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$$

$$f_1(z) \quad +\frac{1}{z-0.5}\cdot\frac{(1/0.5)}{(1/0.5)} - \frac{1}{z-2}\cdot\frac{(1/2)}{(1/2)} \quad \boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \; \boxed{|z|<0.5}$$

$$\|$$

$$f_2(z^{-1}) \quad +\frac{1}{z-0.5}\cdot\frac{(1/z)}{(1/z)} - \frac{1}{z-2}\cdot\frac{(1/z)}{(1/z)} \quad \boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \; \boxed{|z|>2}$$

$$g(z) \quad \boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$g_1(z) \quad -\frac{2z}{(z-2)}\cdot\frac{(1/2)}{(1/2)} + \frac{0.5z}{(z-0.5)}\cdot\frac{(1/0.5)}{(1/0.5)} \quad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \; \boxed{|z|<0.5}$$

$$\|$$

$$g_2(z^{-1}) \quad -\frac{2z}{(z-2)}\cdot\frac{(1/z)}{(1/z)} + \frac{0.5z}{(z-0.5)}\cdot\frac{(1/z)}{(1/z)} \quad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \; \boxed{|z|>2}$$

$$f(z) = f_1(z) = f_2(z^{-1})$$

3 representations $\quad f(z),\ f_1(z),\ f_2(z^{-1})$

all equal formulas $\quad f(z) = f_1(z) = f_2(z^{-1})$

with complimentary ROC. $\quad |z| < a \quad |z| > a$

$$g(z) = g_1(z) = g_2(z^{-1})$$

3 representations $\quad g(z),\ g_1(z),\ g_2(z^{-1})$

all equal formulas $\quad g(z) = g_1(z) = g_2(z^{-1})$

with complimentary ROC's $\quad |z| < a \quad |z| > a$

inverse relationship

$$f(z) = g(z^{-1})$$

$$g(z) = f(z^{-1})$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

causal $f_1(z)$

anti-causal $X_1(z)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $f_2(z)$

causal $X_2(z)$

② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$$\boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

anti-causal $g_1(z)$

causal $Y_1(z)$

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

causal $g_2(z)$

anti-causal $Y_2(z)$

$f(z) \longleftrightarrow a_n$

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}\quad ①$$

$$\boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}}\quad (n\geq 0)$$

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}\quad ②$$

$$\boxed{+2^{n-1}-\left(\tfrac{1}{2}\right)^{n-1}}\quad (n<1)$$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}\quad ④$$

$$\boxed{-\left(\tfrac{1}{2}\right)^{n+1}+2^{n+1}}\quad (n<0)$$

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}\quad ③$$

$$\boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}}\quad (n\geq 1)$$

$$-\frac{1}{1-2z}+\frac{1}{1-0.5z}\quad |z|<0.5\qquad -2^{n}+\left(\tfrac{1}{2}\right)^{n}\quad (n\geq 0)$$

① $$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}\quad |z|<0.5\qquad -2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}\quad (n\geq 0)$$

② $$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}\quad |z|>2\qquad -2^{-n+1}+\left(\tfrac{1}{2}\right)^{-n+1}=-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}\quad (n<0)$$

$$-\frac{1}{1-2z}+\frac{1}{1-0.5z}\quad |z|<0.5\qquad -2^{n}+\left(\tfrac{1}{2}\right)^{n}\quad (n\geq 0)$$

③ $$-\frac{z}{1-2z}+\frac{z}{1-0.5z}\quad |z|<0.5\qquad -2^{n-1}+\left(\tfrac{1}{2}\right)^{n-1}\quad (n\geq 1)$$

④ $$-\frac{z^{-1}}{1-2z^{-1}}+\frac{z^{-1}}{1-0.5z^{-1}}\quad |z|>2\qquad -2^{-n-1}+\left(\tfrac{1}{2}\right)^{-n-1}=-\left(\tfrac{1}{2}\right)^{n+1}+2^{n+1}\quad (n<0)$$

$X(z) \leftrightarrow x_n$

$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$ ②　　$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$ ①

$\boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}$ $(n<1)$　　$\boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}$ $(n\geq 0)$

$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$ ③　　$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$ ④

$\boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}$ $(n\geq 1)$　　$\boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

$-\frac{1}{1-2z^{-1}}+\frac{1}{1-0.5z^{-1}}$　$|z|>2$　$-2^{n}+\left(\frac{1}{2}\right)^{n}$　$(n\geq 0)$

① $-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$　$|z|>2$　$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$　$(n\geq 0)$

② $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$　$|z|<0.5$　$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}=-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$　$(n<1)$

$+\frac{1}{1-0.5z^{-1}}-\frac{1}{1-2z^{-1}}$　$|z|>2$　$+\left(\frac{1}{2}\right)^{n}-2^{n}$　$(n\geq 0)$

③ $+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$　$|z|>2$　$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$　$(n\geq 1)$

④ $+\frac{z}{1-0.5z}-\frac{z}{1-2z}$　$|z|<0.5$　$+\left(\frac{1}{2}\right)^{-n-1}-2^{-n-1}=+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$　$(n<0)$

$f(z) \Longleftrightarrow a_n$

$X(z) \Longleftrightarrow x_n$

① − Ⓐ

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

$a_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}}$ $(n<1)$

② − Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}}$ $(n<1)$

$x_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

① − Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$a_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}}$ $(n<0)$

$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}}$ $(n \geq 1)$

② − Ⓑ

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}}$ $(n \geq 1)$

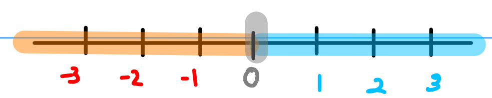
$x_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}}$ $(n<0)$

$$x_n = a_{-n}$$

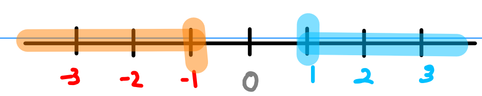
$$a_n = x_{-n}$$

$(n \geq 0) \longleftrightarrow (n<1)$

$(n \geq 1) \longleftrightarrow (n<0)$



$(n<1)$ $(n \geq 0)$



$(n<0)$ $(n \geq 1)$

$$f(z)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$|z|<0.5$$

$$-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$$

$$|z|>2$$

$$z^{-1} \qquad -n$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$|z|<0.5$$

$$+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$|z|>2$$

$$z^{-1} \qquad -n$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$$

$$|z|<0.5$$

$$\cdot z \qquad n-1$$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$$|z|<0.5$$

$$+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$$

$$|z|<0.5$$

$$\cdot z \qquad n-1$$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$$|z|<0.5$$

$$X(z)$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$|z|<0.5$

z^{-1} $\quad -n$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$|z|>2$

$$-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$|z|>2$

$$+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

$|z|<0.5$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$|z|>2$

$\cdot z^{-1}$ $\quad n-1$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$|z|>2$

$$+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$|z|<0.5$

z^{-1} $\quad -n$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$|z|>2$

$\cdot z^{-1}$ $\quad n-1$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$|z|>2$

$f_c(z) \leftrightarrow a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$
$Y_a(z^{-1}) \leftrightarrow -y_n$	$X_c(z) \leftrightarrow x_n$

$f_a(z^{-1}) \leftrightarrow -a_n$	$g_c(z) \leftrightarrow b_n$
$Y_c(z) \leftrightarrow y_n$	$X_a(z^{-1}) \leftrightarrow -x_n$

A　1　$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$　　2　$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$

A-1　$f(z) = \boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}$　$\boxed{|z|<0.5}$　$\boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}$　$(n \geq 0)$

A-2　$g(z) = \boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}$　$\boxed{|z|>2}$　$\boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}$　$(n<1)$

A-1　$Y(z) = \boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}$　$\boxed{|z|<0.5}$　$\boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}$　$(n<1)$

A-2　$X(z) = \boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}$　$\boxed{|z|>2}$　$\boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}$　$(n \geq 0)$

B　1　$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$　　2　$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$

B-1　$f(z) = \boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}$　$\boxed{|z|>2}$　$\boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}$　$(n<0)$

B-2　$g(z) = \boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}$　$\boxed{|z|<0.5}$　$\boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}$　$(n \geq 1)$

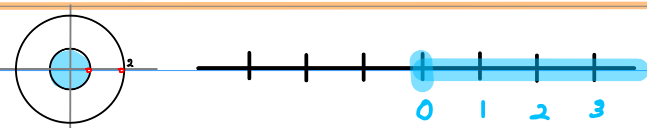
B-1　$Y(z) = \boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}$　$\boxed{|z|>2}$　$\boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}$　$(n \geq 1)$

B-2　$X(z) = \boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}$　$\boxed{|z|<0.5}$　$\boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}$　$(n<0)$

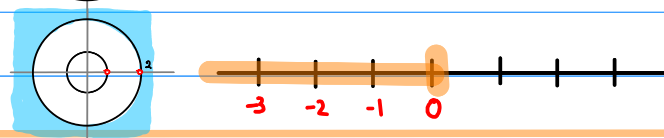
$f_c(z) \leftrightarrow a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$	$f_a(z^{-1}) \leftrightarrow -a_n$	$g_c(z) \leftrightarrow b_n$
$Y_a(z^{-1}) \leftrightarrow -y_n$	$X_c(z) \leftrightarrow x_n$	$Y_c(z) \leftrightarrow y_n$	$X_a(z^{-1}) \leftrightarrow -x_n$

A

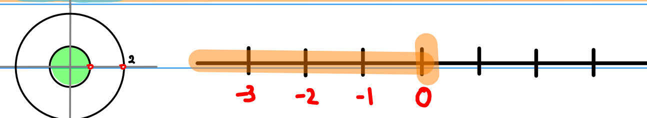
① $-\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ ② $-\dfrac{2}{1-2z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$

A−① $\boxed{-\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z| < 0.5}$ 

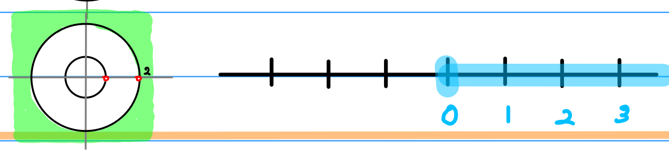
$(n \geq 0)$

A−② $\boxed{-\dfrac{2}{1-2z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z| > 2}$ 

$(n < 1)$

A−① $\boxed{-\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}}$ $\boxed{|z| < 0.5}$ 

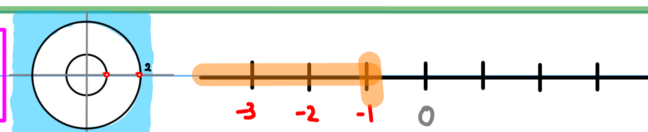
$(n < 1)$

A−② $\boxed{-\dfrac{2}{1-2z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z| > 2}$ 

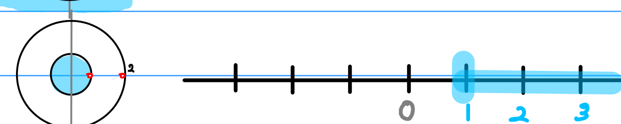
$(n \geq 0)$

B

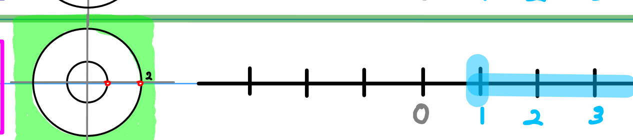
① $+\dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ ② $+\dfrac{z}{1-0.5z} - \dfrac{z}{1-2z}$

B−① $\boxed{+\dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z| > 2}$ 

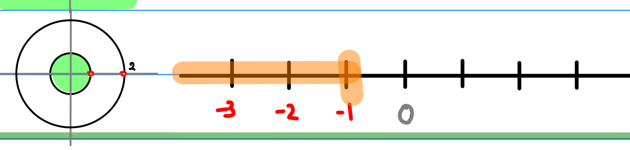
$(n < 0)$

B−② $\boxed{+\dfrac{z}{1-0.5z} - \dfrac{z}{1-2z}}$ $\boxed{|z| < 0.5}$ 

$(n \geq 1)$

B−① $\boxed{+\dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z| > 2}$ 

$(n \geq 1)$

B−② $\boxed{+\dfrac{z}{1-0.5z} - \dfrac{z}{1-2z}}$ $\boxed{|z| < 0.5}$ 

$(n < 0)$

$f_c(z) \leftrightarrow a_n$ $f_a(z^{-1}) \leftrightarrow -a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$ $g_c(z) \leftrightarrow b_n$	$Y_a(z^{-1}) \leftrightarrow -y_n$ $Y_c(z) \leftrightarrow y_n$	$X_c(z) \leftrightarrow x_n$ $X_a(z^{-1}) \leftrightarrow -x_n$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(-\frac{1}{z-2} + \frac{1}{z-0.5}\right)$

Ⓐ-① $f(z) = \boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<0.5}$ $\boxed{-2^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$

Ⓑ-① $f(z) = \boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+2^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$

Ⓐ-① $Y(z) = \boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<0.5}$ $\boxed{-(\frac{1}{2})^{n-1} + 2^{n-1}}$ $(n<1)$

Ⓑ-① $Y(z) = \boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{+(\frac{1}{2})^{n-1} - 2^{n-1}}$ $(n \geq 1)$

② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}\right)$

Ⓐ-② $g(z) = \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{-(\frac{1}{2})^{n-1} + 2^{n-1}}$ $(n<1)$

Ⓑ-② $g(z) = \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+(\frac{1}{2})^{n-1} - 2^{n-1}}$ $(n \geq 1)$

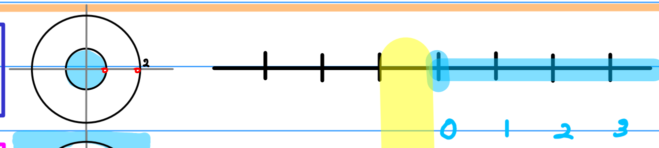
Ⓐ-② $X(z) = \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$ $\boxed{-2^{n+1} + (\frac{1}{2})^{n+1}}$ $(n \geq 0)$

Ⓑ-② $X(z) = \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$ $\boxed{+2^{n+1} - (\frac{1}{2})^{n+1}}$ $(n<0)$

$f_c(z) \leftrightarrow a_n$ $f_a(z^{-1}) \leftrightarrow -a_n$	$g_a(z^{-1}) \leftrightarrow -b_n$ $g_c(z) \leftrightarrow b_n$	$Y_a(z^{-1}) \leftrightarrow -y_n$ $Y_c(z) \leftrightarrow y_n$	$X_c(z) \leftrightarrow x_n$ $X_a(z^{-1}) \leftrightarrow -x_n$

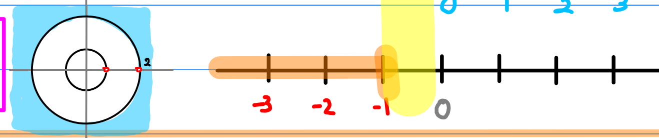
① $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(-\frac{1}{z-2} + \frac{1}{z-0.5}\right)$

A-① $\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<0.5}$



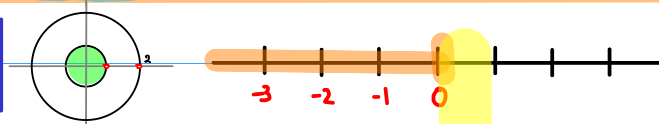
$(n \geq 0)$

B-① $\boxed{-\frac{z^{-1}}{1-2z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$



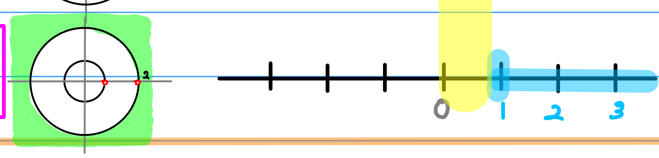
$(n<0)$

A-① $\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z|<0.5}$



$(n<1)$

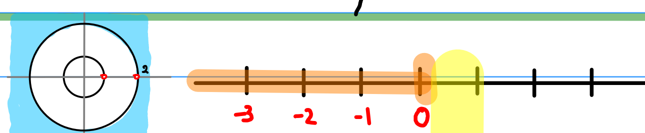
B-① $\boxed{-\frac{z^{-1}}{1-2z^{-1}} + \frac{z^{-1}}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$



$(n \geq 1)$

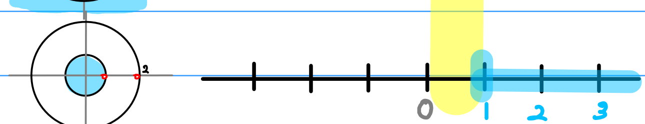
② $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}\right)$

A-② $\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$



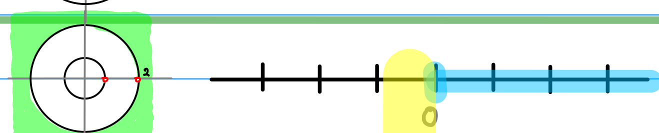
$(n<1)$

B-② $\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$



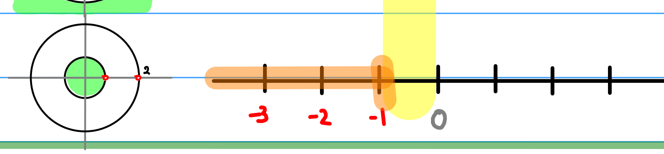
$(n \geq 1)$

A-② $\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z|>2}$



$(n \geq 0)$

B-② $\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z|<0.5}$



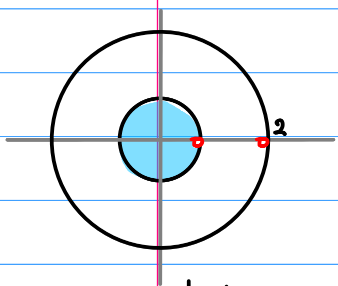
$(n<0)$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$$

$|z| < 0.5$ causal　　$|z| > 2$ anticausal

①-Ⓐ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$



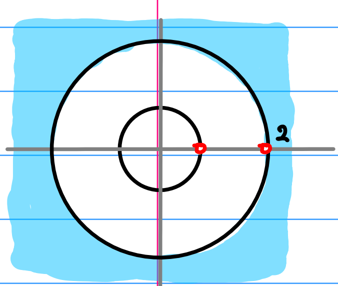
$|z| < 0.5$

$$\begin{aligned}
f(z) &= \frac{(-2)}{1-(2z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)} \\
&= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} (z)^n \\
&= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n
\end{aligned}$$

$(n \geq 0)$　　$a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

①-Ⓑ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

$$\begin{aligned}
f(z) &= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)} \qquad \neq \\
&= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\frac{1}{z}\right)^{n+1} \\
&= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \\
&= \sum_{n=-1}^{-\infty} (2)^{n+1} z^n - \sum_{n=-1}^{-\infty} \left(\frac{1}{2}\right)^{n+1} z^n
\end{aligned}$$

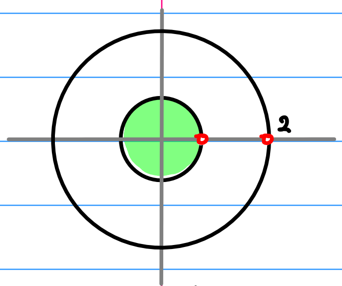
$(n < 0)$　　$a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$$

$|z| < 0.5$ anticausal $\qquad |z| > 2$ causal

①-Ⓐ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



$|z| < 0.5$

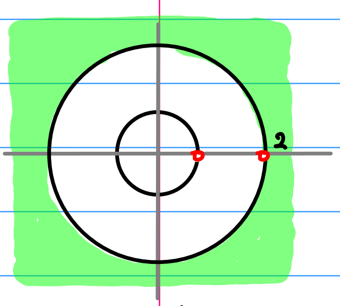
$$\boxed{\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1}(z)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty} \left(\tfrac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty} (2)^{n-1} z^{-n}$$

$(n \leq 0) \qquad a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}$

①-Ⓑ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)} \\ &= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \end{aligned}} \qquad \neq$$

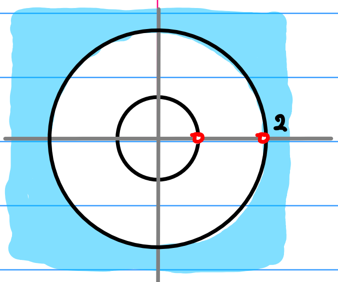
$(n > 0) \qquad a_n = \left(\tfrac{1}{2}\right)^{n-1} - (2)^{n-1}$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$$

$|z| > 2$ anticausal $\quad$ $|z| < 0.5$ causal

②-Ⓐ

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$



$|z| > 2$

$$f(z) = -\frac{(2)}{1-(\frac{2}{z})} + \frac{(\frac{1}{2})}{1-(\frac{1}{2z})}$$

$$= -\sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n$$

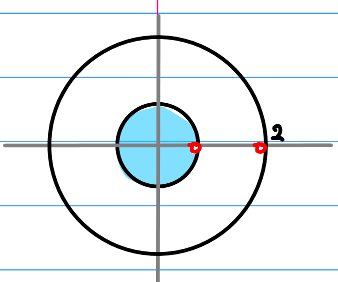
$$= -\sum_{n=0}^{\infty}(2)^{n+1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$$= -\sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1} z^{n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{n}$$

$(n \leq 0)$ $\quad a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$

②-Ⓑ

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = +\frac{z}{1-0.5z} - \frac{z}{1-2z}$$



$|z| < 0.5$

$$f(z) = +\frac{(z)}{1-(\frac{z}{2})} - \frac{(z)}{1-(2z)} \qquad \neq$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n}(z)^{n+1} - \sum_{n=0}^{\infty}(2)^{n}(z)^{n+1}$$

$$= +\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{n}$$

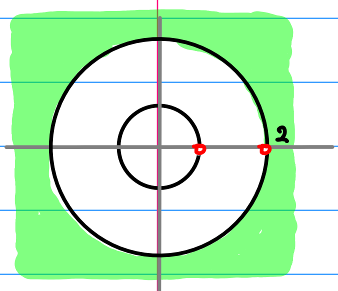
$(n > 0)$ $\quad a_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$$

$|z| > 2$ causal $\qquad$ $|z| < 0.5$ anticausal

②–Ⓐ

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$



$|z| > 2$

$$X(z) = \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)}$$

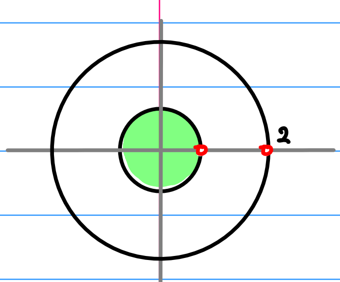
$$= -\sum_{n=0}^{\infty} (2)^{n+1}\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n$$

$$= -\sum_{n=0}^{\infty} (2)^{n+1} z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$(n \geq 0)$ $\qquad a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

②–Ⓑ

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = +\frac{z}{1-0.5z} - \frac{z}{1-2z}$$



$|z| < 0.5$

$$X(z) = -\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n (z)^{n+1} - \sum_{n=0}^{\infty} (2)^n (z)^{n+1}$$

$$= +\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^n - \sum_{n=1}^{\infty} (2)^{n-1} z^n$$

$\neq$

$$= +\sum_{n=-1}^{-\infty} (2)^{n+1} z^{-n} - \sum_{n=-1}^{-\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

$(n < 0)$ $\qquad a_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$