Laurent Series and z-Transform
- Geometric Series
Applications B

20201223 Wed

Geometric Series - a unit start term

Laurent Series unshifted

Geometric Series - a unit start term

z-Transform unshifted

Geometric Series - a unit start term

Laurent Series vs. z-Transform unshifted

Geometric Series - a unit start term

Laurent Series

(1) $\boxed{+\dfrac{1}{1-\boxed{az}}}$ $\boxed{|z| < a^{-1}}$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

a^n u(n)	$(n \geq 0)$

(2) $\boxed{+\dfrac{1}{1-\boxed{a^{-1}z}}}$ $\boxed{|z| < a}$

$(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$((\frac{1}{a})^0 z^0 + (\frac{1}{a})^1 z^1 + (\frac{1}{a})^2 z^2 + \cdots)$

$(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(3) $\boxed{-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}}$ $\boxed{|z| > a^{-1}}$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-a^n$ u(-n)	$(n < 1)$

(4) $\boxed{-\dfrac{1}{1-\boxed{az^{-1}}}}$ $\boxed{|z| > a}$

$-(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-((\frac{1}{a})^0 z^0 + (\frac{1}{a})^{-1} z^{-1} + (\frac{1}{a})^{-2} z^{-2} + \cdots)$

$-(\frac{1}{a})^n$ u(-n)	$(n < 1)$

Geometric Series - a unit start term

z-Transform (n → -n)

(1) $+\dfrac{1}{1-\boxed{az}}$ $\boxed{|z|<a^{-1}}$

$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^1 + (\frac{1}{a})^{-2}z^2 + \cdots)$

a^{-n} u((-n))	$(-n \geq 0)$
$(\frac{1}{a})^n$ u(-n)	$(n < 1)$

(2) $+\dfrac{1}{1-\boxed{a^{-1}z}}$ $\boxed{|z|<a}$

$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$

$((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^1 + (\frac{1}{a})^2z^2 + \cdots)$

$(\frac{1}{a})^{-n}$ u((-n))	$(-n \geq 0)$
a^n u(-n)	$(n < 1)$

(3) $-\dfrac{1}{1-\boxed{a^{-1}z^{-1}}}$ $\boxed{|z|>a^{-1}}$

$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^1z^{-1} + (\frac{1}{a})^2z^{-2} + \cdots)$

$-a^{-n}$ u(-(-n))	$(-n < 1)$
$-(\frac{1}{a})^n$ u(n)	$(n \geq 0)$

(4) $-\dfrac{1}{1-\boxed{az^{-1}}}$ $\boxed{|z|>a}$

$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$

$-((\frac{1}{a})^0z^0 + (\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + \cdots)$

$-(\frac{1}{a})^{-n}$ u(-(-n))	$(-n < 1)$
$-a^n$ u(n)	$(n \geq 0)$

Geometric Series

Laurent Series vs. z-Transform (n → -n)

(1)

$$+\frac{1}{1-\boxed{az}} \qquad \boxed{|z|<a^{-1}}$$

$$(a^0z^0 + a^1z^1 + a^2z^2 + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^1 + \left(\tfrac{1}{a}\right)^{-2} z^2 + \cdots\right)$$

Laurent	a^n	u(n)	$(n \geq 0)$
z-Trans	$\left(\frac{1}{a}\right)^n$	u(-n)	$(n < 1)$

(2)

$$+\frac{1}{1-\boxed{a^{-1}z}} \qquad \boxed{|z|<a}$$

$$(a^0z^0 + a^{-1}z^1 + a^{-2}z^2 + \cdots)$$

$$\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{1} z^1 + \left(\tfrac{1}{a}\right)^{2} z^2 + \cdots\right)$$

Laurent	$\left(\frac{1}{a}\right)^n$	u(n)	$(n \geq 0)$
z-Trans	a^n	u(-n)	$(n < 1)$

(3)

$$-\frac{1}{1-\boxed{a^{-1}z^{-1}}} \qquad \boxed{|z|>a^{-1}}$$

$$-(a^0z^0 + a^{-1}z^{-1} + a^{-2}z^{-2} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{1} z^{-1} + \left(\tfrac{1}{a}\right)^{2} z^{-2} + \cdots\right)$$

Laurent	$-a^n$	u(-n)	$(n < 1)$
z-Trans	$-\left(\frac{1}{a}\right)^n$	u(n)	$(n \geq 0)$

(4)

$$-\frac{1}{1-\boxed{az^{-1}}} \qquad \boxed{|z|>a}$$

$$-(a^0z^0 + a^1z^{-1} + a^2z^{-2} + \cdots)$$

$$-\left(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$$

Laurent	$-\left(\frac{1}{a}\right)^n$	u(-n)	$(n < 1)$
z-Trans	$-a^n$	u(n)	$(n \geq 0)$

Geometric Series - a non-unit start term

Laurent Series **unshifted, complementary**

Geometric Series - a non-unit start term

z-Transform **unshifted, complementary**

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform **unshifted, complementary**

Geometric Series - a non-unit start term

Laurent Series

(5) $-\dfrac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}$ $\quad |z| > a^{-1}$

$-(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots)$

$-a^{n}\ u(-n-1) \quad (n < 0)$

(6) $-\dfrac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}$ $\quad |z| > a$

$-(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots)$

$-((\frac{1}{a})^{-1}z^{-1} + (\frac{1}{a})^{-2}z^{-2} + (\frac{1}{a})^{-3}z^{-3} + \cdots)$

$-(\frac{1}{a})^{n}\ u(-n-1) \quad (n < 0)$

(7) $+\dfrac{\boxed{az}}{1-\boxed{az}}$ $\quad |z| < a^{-1}$

$(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots)$

$a^{n}\ u(n-1) \quad (n \geq 1)$

(8) $+\dfrac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}$ $\quad |z| < a$

$(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots)$

$((\frac{1}{a})^{1}z^{1} + (\frac{1}{a})^{2}z^{2} + (\frac{1}{a})^{3}z^{3} + \cdots)$

$(\frac{1}{a})^{n}\ u(n-1) \quad (n \geq 1)$

Geometric Series - a non-unit start term

z-Transform (n → -n)

(5) $$-\frac{a^{-1}z^{-1}}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$$

$-a^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-\left(\frac{1}{a}\right)^{n}$ u(n-1)	$(n \geq 1)$

(6) $$-\frac{az^{-1}}{1-az^{-1}} \qquad |z| > a$$

$$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$$

$-\left(\frac{1}{a}\right)^{-n}$ u(-(-n)-1)	$(-n < 0)$
$-a^{n}$ u(n-1)	$(n \geq 1)$

(7) $$+\frac{az}{1-az} \qquad |z| < a^{-1}$$

$$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$$

a^{-n} u((-n)-1)	$(-n \geq 1)$
$\left(\frac{1}{a}\right)^{n}$ u(-n-1)	$(n < 0)$

(8) $$+\frac{a^{-1}z}{1-a^{-1}z} \qquad |z| < a$$

$$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$$

$\left(\frac{1}{a}\right)^{-n}$ u((-n)-1)	$(-n \geq 1)$
a^{n} u(-n-1)	$(n < 0)$

Geometric Series - a non-unit start term

Laurent Series vs. z-Transform (n → -n)

(5)

$$-\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}} \qquad \boxed{|z|>a^{-1}}$$

$-(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{1}z^{-1}+(\frac{1}{a})^{2}z^{-2}+(\frac{1}{a})^{3}z^{-3}+\cdots)$

Laurent	$-a^{n}$	u(-n-1)	$(n<0)$
z-Trans	$-(\frac{1}{a})^{n}$	u(n-1)	$(n\geq 1)$

(6)

$$-\frac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}} \qquad \boxed{|z|>a}$$

$-(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots)$

$-((\frac{1}{a})^{-1}z^{-1}+(\frac{1}{a})^{-2}z^{-2}+(\frac{1}{a})^{-3}z^{-3}+\cdots)$

Laurent	$-(\frac{1}{a})^{n}$	u(-n-1)	$(n<0)$
z-Trans	$-a^{n}$	u(n-1)	$(n\geq 1)$

(7)

$$+\frac{\boxed{az}}{1-\boxed{az}} \qquad \boxed{|z|<a^{-1}}$$

$(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots)$

$((\frac{1}{a})^{-1}z^{1}+(\frac{1}{a})^{-2}z^{2}+(\frac{1}{a})^{-3}z^{3}+\cdots)$

Laurent	a^{n}	u(n-1)	$(n\geq 1)$
z-Trans	$(\frac{1}{a})^{n}$	u(-n-1)	$(n<0)$

(8)

$$+\frac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}} \qquad \boxed{|z|<a}$$

$(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots)$

$((\frac{1}{a})^{1}z^{1}+(\frac{1}{a})^{2}z^{2}+(\frac{1}{a})^{3}z^{3}+\cdots)$

Laurent	$(\frac{1}{a})^{n}$	u(n-1)	$(n\geq 1)$
z-Trans	a^{n}	u(-n-1)	$(n<0)$

2 formulas

Simple Pole Form

	in terms of p			in terms of $1/p$	
	simple pole p	simple pole $1/p$		simple pole $1/p$	simple pole p
	$\boxed{\dfrac{1}{z-p}}$	$\boxed{\dfrac{1}{z^{-1}-p}}$		$\boxed{\dfrac{1}{z-p^{-1}}}$	$\boxed{\dfrac{1}{z^{-1}-p^{-1}}}$
/p	$\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$	/p $\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$	*p	$\boxed{-\dfrac{p}{1-pz}}$	*p $\boxed{-\dfrac{p}{1-pz^{-1}}}$
/z	$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$	*z $\boxed{\dfrac{z}{1-pz}}$	/z	$\boxed{\dfrac{z^{-1}}{1-p^{-1}z^{-1}}}$	*z $\boxed{\dfrac{z}{1-p^{-1}z}}$

z^{-1} z^{-1}

p^{-1}

when the pole is expressed as p

2 formulas **Simple Pole Form**

$$\boxed{\frac{1}{z-p}} \qquad \boxed{\frac{1}{z^{-1}-p}}$$

2 representations each **Shifted Geometric Series Form**

causal Laurent — anti- causal z-transform

$$\boxed{\frac{1}{z-p}} \quad \begin{cases} \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \triangleq f(z) = X(z^{-1}) \\ \qquad\qquad\qquad\quad \| \qquad\quad \| \\ \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \triangleq Y(z) = g(z^{-1}) \end{cases}$$

causal z-transform — anti- causal Laurent

causal z-transform — anti- causal Laurent

$$\boxed{\frac{1}{z^{-1}-p}} \quad \begin{cases} \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \triangleq X(z) = f(z^{-1}) \\ \qquad\qquad\qquad\quad \| \qquad\quad \| \\ \boxed{\dfrac{z}{1-pz}} \triangleq g(z) = Y(z^{-1}) \end{cases}$$

causal Laurent — anti- causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

when the pole is expressed as 1/p

2 formulas

Simple Pole Form

$$\boxed{\frac{1}{z - p^{-1}}} \qquad \boxed{\frac{1}{z^{-1} - p^{-1}}}$$

2 representations each

Shifted Geometric Series Form

$$\boxed{\frac{1}{z - p^{-1}}}$$

causal Laurent / anti- causal z-transform

$$\boxed{-\frac{p}{1 - pz}} \triangleq f(z) = X(z^{-1})$$

$$f(z) \;\|\; Y(z), \qquad X(z^{-1}) \;\|\; g(z^{-1})$$

$$\boxed{\frac{z^{-1}}{1 - p^{-1}z^{-1}}} \triangleq Y(z) = g(z^{-1})$$

causal z-transform / anti- causal Laurent

$$\boxed{\frac{1}{z^{-1} - p^{-1}}}$$

causal z-transform / anti- causal Laurent

$$\boxed{-\frac{p}{1 - pz^{-1}}} \triangleq X(z) = f(z^{-1})$$

$$X(z) \;\|\; g(z), \qquad f(z^{-1}) \;\|\; Y(z^{-1})$$

$$\boxed{\frac{z}{1 - p^{-1}z}} \triangleq g(z) = Y(z^{-1})$$

causal Laurent / anti- causal z-transform

Simple Pole Form **Shifted Geometric Series Form**

Shifted Geometric Series (1a) p

from unshifted series

2 formulas **Simple Pole Form**

$\boxed{\dfrac{1}{z-p}}$ $\boxed{\dfrac{1}{z^{-1}-p}}$

2 representations each

Simple Pole Form	**shifted**		**unshifted**
$\boxed{\dfrac{1}{z-p}}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$ $p^{-n-1}u(n)$	$\xleftarrow{/p}$	$-\dfrac{1}{1-p^{-1}z}$ $p^{-n}u(n)$
	$\dfrac{z^{-1}}{1-pz^{-1}}$ $p^{-n-1}u(-n-1)$	$\xleftarrow{/z}$	$\dfrac{1}{1-pz^{-1}}$ $p^{-n}u(-n)$
$\boxed{\dfrac{1}{z^{-1}-p}}$	$-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}$ $p^{n-1}u(-n)$	$\xleftarrow{/p}$	$-\dfrac{1}{1-p^{-1}z^{-1}}$ $p^{n}u(-n)$
	$\dfrac{z}{1-pz}$ $p^{n-1}u(n-1)$	$\xleftarrow{*z}$	$\dfrac{1}{1-pz}$ $p^{n}u(n)$

Simple Pole Form **Unit nominator**

Shifted Geometric Series Form **Un-shifted Geometric Series Form**

u(n) u(-n)

Shifted Geometric Series (1b) 1/p

from unshifted series

2 formulas **Simple Pole Form**

$\boxed{\dfrac{1}{z - p^{-1}}}$ $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$

2 representations each

	shifted		**unshifted**
$\boxed{\dfrac{1}{z - p^{-1}}}$	$\boxed{-\dfrac{p}{1 - p z}}$ $p^{n+1}u(n)$	$\xleftarrow{*p}$	$\boxed{-\dfrac{1}{1 - p z}}$ $p^{n}u(n)$
	$\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}}$ $p^{-n-1}u(-n-1)$	$\xleftarrow{/z}$	$\boxed{\dfrac{1}{1 - p z^{-1}}}$ $p^{-n}u(-n)$
$\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$	$\boxed{-\dfrac{p}{1 - p z^{-1}}}$ $p^{-n+1}u(-n)$	$\xleftarrow{*p}$	$\boxed{-\dfrac{1}{1 - p z^{-1}}}$ $p^{-n}u(-n)$
	$\boxed{\dfrac{z}{1 - p^{-1} z}}$ $p^{-n-1}u(n-1)$	$\xleftarrow{*z}$	$\boxed{\dfrac{1}{1 - p^{-1} z}}$ $p^{-n}u(n)$

Simple Pole Form **Unit nominator**

Shifted Geometric Series Form **Un-shifted Geometric Series**

u(n) u(-n)

Shifted Geometric Series (2a) p

from complementary unshifted series

2 formulas

Simple Pole Form

$\boxed{\frac{1}{z-p}}$ $\boxed{\frac{1}{z^{-1}-p}}$

2 representations each

	shifted		unshifted, complementary
$\boxed{\frac{1}{z-p}}$	$\boxed{-\frac{p^{-1}}{1-p^{-1}z}}$ $p^{-n-1}\,u(n)$	$\xleftarrow{/z}$	$\boxed{-\frac{p^{-1}z}{1-p^{-1}z}}$ $p^{-n}\,u(n-1)$
	$\boxed{\frac{z^{-1}}{1-pz^{-1}}}$ $p^{-n-1}\,u(-n-1)$	$\xleftarrow{/p}$	$\boxed{\frac{pz^{-1}}{1-pz^{-1}}}$ $p^{-n}\,u(-n-1)$
$\boxed{\frac{1}{z^{-1}-p}}$	$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$ $p^{n-1}\,u(-n)$	$\xleftarrow{*z}$	$\boxed{-\frac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}}$ $p^{n}\,u(-n-1)$
	$\boxed{\frac{z}{1-pz}}$ $p^{n-1}\,u(n-1)$	$\xleftarrow{/p}$	$\boxed{\frac{pz}{1-pz}}$ $p^{n}\,u(n-1)$

Simple Pole Form

Common Ratio nominator

Shifted Geometric Series Form

Un-shifted Geometric Series Form

$u(n-1)$ $u(-n-1)$

Shifted Geometric Series (2b) 1/p

from complementary unshifted series

2 formulas **Simple Pole Form**

$\boxed{\dfrac{1}{z - p^{-1}}}$ $\boxed{\dfrac{1}{z^{-1} - p^{-1}}}$

2 representations each

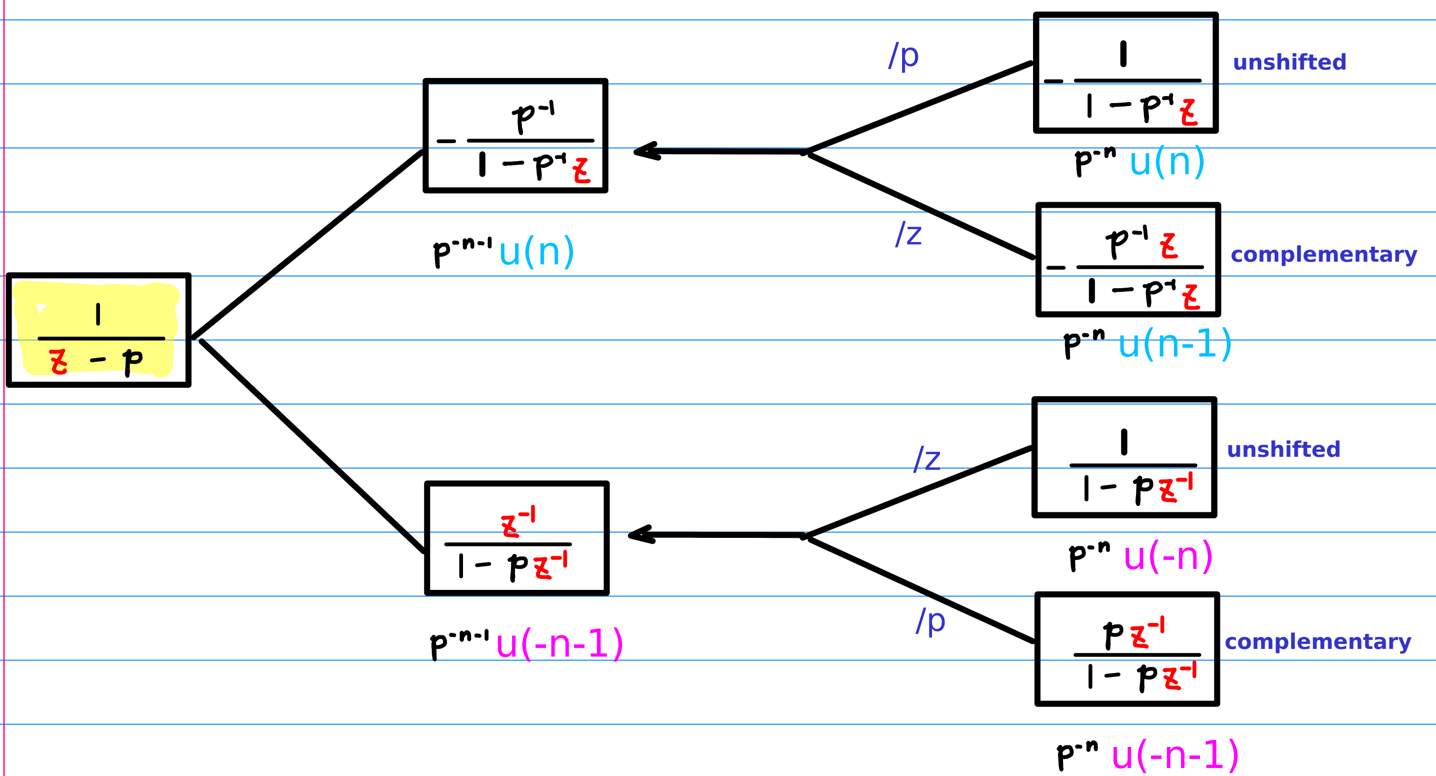
	shifted		**unshifted, complementary**
$\dfrac{1}{z - p^{-1}}$	$-\dfrac{p}{1-pz}$ p^{n+1} u(n)	/z ←	$-\dfrac{pz}{1-pz}$ p^{n} u(n-1)
	$\dfrac{z^{-1}}{1-p^{-1}z^{-1}}$ p^{n+1} u(-n-1)	*p ←	$\dfrac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}$ p^{n} u(-n-1)
$\dfrac{1}{z^{-1} - p^{-1}}$	$-\dfrac{p}{1-pz^{-1}}$ p^{-n+1} u(-n)	*z ←	$-\dfrac{pz^{-1}}{1-pz^{-1}}$ p^{-n} u(-n-1)
	$\dfrac{z}{1-p^{-1}z}$ p^{-n+1} u(n-1)	*p ←	$\dfrac{p^{-1}z}{1-p^{-1}z}$ p^{-n} u(n-1)

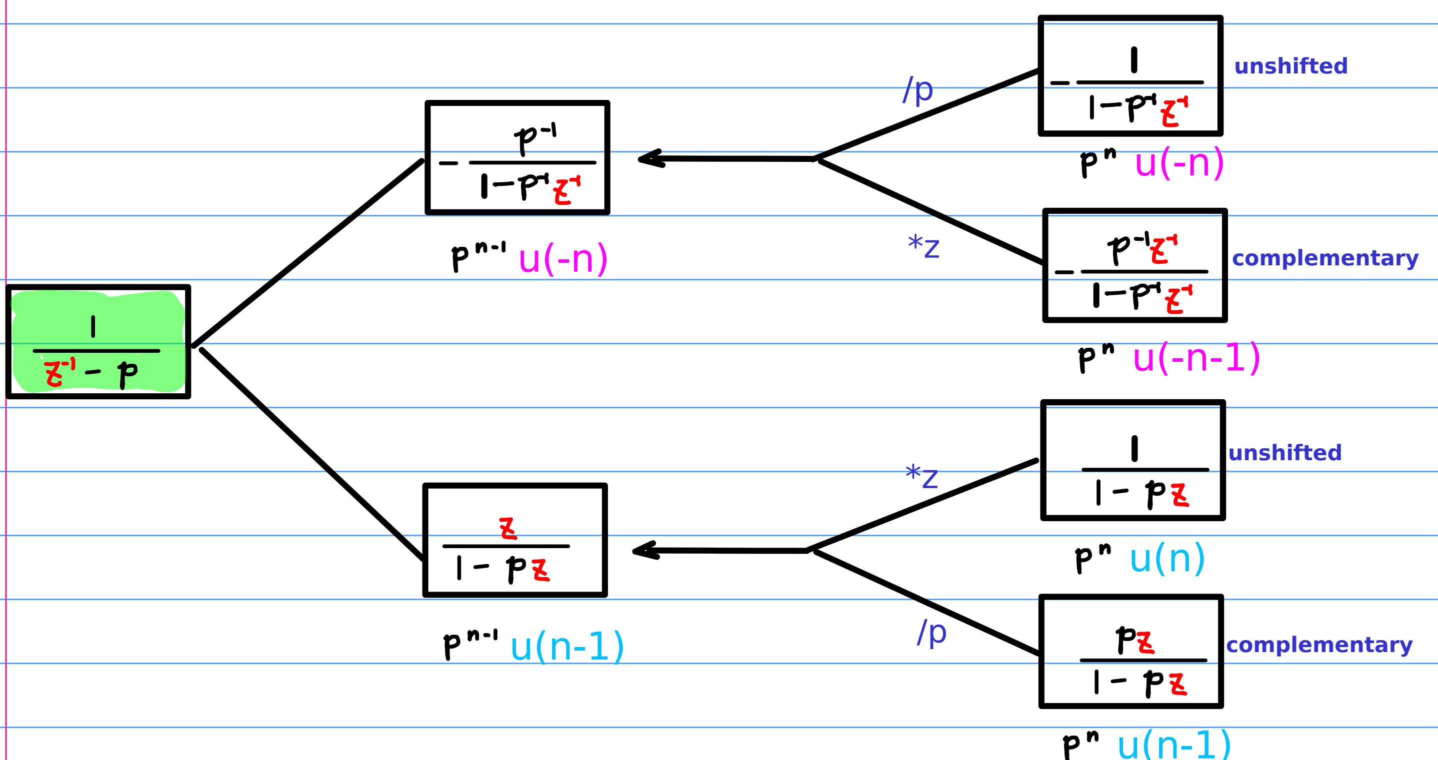
Simple Pole Form **Common Ratio nominator**

Shifted Geometric Series Form **Un-shifted Geometric Series For**

u(n-1) u(-n-1)

Shifted Geometric Series (3a) p





Shifted Geometric Series (3b) 1/p

$\boxed{\frac{1}{z - p^{-1}}}$

- $\boxed{-\frac{p}{1-pz}}$ $p^{n+1}u(n)$
  - $\xleftarrow{*p}$ $\boxed{-\frac{1}{1-pz}}$ $p^{n}u(n)$
  - $\xleftarrow{/z}$ $\boxed{-\frac{pz}{1-pz}}$ $p^{n}u(n-1)$
- $\boxed{\frac{z^{-1}}{1-p^{-1}z^{-1}}}$ $p^{n+1}u(-n-1)$
  - $\xleftarrow{/z}$ $\boxed{\frac{1}{1-p^{-1}z^{-1}}}$ $p^{n}u(-n)$
  - $\xleftarrow{*p}$ $\boxed{\frac{p^{-1}z^{-1}}{1-p^{-1}z^{-1}}}$ $p^{n}u(-n-1)$

$\boxed{\frac{1}{z^{-1} - p^{-1}}}$

- $\boxed{\frac{z}{1-p^{-1}z}}$ $p^{-n+1}u(n-1)$
  - $\xleftarrow{*z}$ $\boxed{\frac{1}{1-p^{-1}z}}$ $p^{-n}u(n)$
  - $\xleftarrow{*p}$ $\boxed{\frac{p^{-1}z}{1-p^{-1}z}}$ $p^{-n}u(n-1)$
- $\boxed{-\frac{p}{1-pz^{-1}}}$ $p^{-n+1}u(-n)$
  - $\xleftarrow{*p}$ $\boxed{-\frac{1}{1-pz^{-1}}}$ $p^{-n}u(-n)$
  - $\xleftarrow{*z}$ $\boxed{-\frac{pz^{-1}}{1-pz^{-1}}}$ $p^{-n}u(-n-1)$

2 formulas   Simple Pole Form

$\boxed{\dfrac{1}{z - p}}$ $\boxed{\dfrac{1}{z^{-1} - p}}$

2 representations each   Geometric Series Form

$\boxed{\dfrac{1}{z - p}}$

$\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}} \triangleq f(z) = X(z^{-1})$

causal   anti- causal

$\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}} \triangleq Y(z) = g(z^{-1})$

causal   anti- causal

$\boxed{\dfrac{1}{z^{-1} - p}}$

causal   anti- causal

$\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}} \triangleq X(z) = f(z^{-1})$

$\boxed{\dfrac{z}{1 - p z}} \triangleq g(z) = Y(z^{-1})$

causal   anti- causal

Simple Pole Form | Geometric Series Form

Simple Pole Form

①

$-\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$-a^n$	$(n \geq 0)$

②

$-\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$-\left(\frac{1}{a}\right)^n$	$(n < 1)$

③

$\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
a^n	$(n < 1)$

④

$\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑤

$-\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$-\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑥

$-\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$-a^n$	$(n < 1)$

⑦

$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$\left(\frac{1}{a}\right)^n$	$(n < 1)$

⑧

$\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
a^n	$(n \geq 0)$

Geometric Series : $f(z)$, $g(z)$, $\bar{f}(z)$, $\bar{g}(z)$

$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n+1}$	$(n \geq 0)$

$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^{n-1}$	$(n \geq 1)$

Geometric Series :

$f(z) = g(z^{-1})$ the same algebraic formula but the complement ROC's

$g(z) = f(z^{-1})$ the same algebraic formula but the complement ROC's

$f(z) = f_a(z)$ two variable function

$g(z) = g_a(z)$ two variable function

$\bar{f}(z) = f_{a^{-1}}(z)$ inverse a

$\bar{g}(z) = g_{a^{-1}}(z)$ inverse a

associated simple pole forms

$a^{-1} f(z),\ z^{-1} g(z),\ a\bar{f}(z),\ z^{-1}\bar{g}(z)$

①

$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = -a^n$	$(n \geq 0)$

②

$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n < 1)$

③

$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

④

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑤

$a\bar{f}(z) = -\dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -\left(\frac{1}{a}\right)^n$	$(n \geq 0)$

⑥

$a\bar{f}(z^{-1}) = -\dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

⑦

$z\,\bar{g}(z^{-1}) = \dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^n$	$(n < 1)$

⑧

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$
$\bar{f}(z)$	$\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$	$\bar{g}(z)$

①

$a_n = -a^{n+1}$ $(n \geq 0)$

$f(z) = -\dfrac{a}{1-az}$ $|z| < a^{-1}$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$ $(n < 1)$

$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$ $|z| > a$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

$a_n = a^{n+1}$ $(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-a^{-1}z}$ $|z| < a$

$a_n = \left(\frac{1}{a}\right)^{n-1}$ $(n \geq 1)$

⑤

$a_n = -\left(\frac{1}{a}\right)^{n+1}$ $(n \geq 0)$

$\bar{f}(z) = -\dfrac{a^{-1}}{1-a^{-1}z}$ $|z| < a$

neg(n) Sym(n)

inv(z) inv(z)

⑥

$a_n = -a^{n-1}$ $(n < 1)$

$\bar{f}(z^{-1}) = -\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$ $|z| > a^{-1}$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

⑦

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1-az^{-1}}$ $|z| > a$

$a_n = \left(\frac{1}{a}\right)^{n+1}$ $(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

⑧

$\bar{g}(z) = \dfrac{z}{1-az}$ $|z| < a^{-1}$

$a_n = a^{n-1}$ $(n \geq 1)$

(1) (3) (5) (7)

(2) (4) (6) (8)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$f(z)$	$f(z^{-1})$
$g(z^{-1})$	$g(z)$
$\bar{f}(z)$	$\bar{f}(z^{-1})$
$\bar{g}(z^{-1})$	$\bar{g}(z)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$\bar{f}(z) = -\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -(\frac{1}{a})^{n+1}$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$\bar{f}(z^{-1}) = -\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^{n-1}$	$(n < 1)$

③

$a_n = a^{n+1}$	$(n < 0)$
$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$
$g(z) = \frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$\bar{g}(z^{-1}) = \frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = (\frac{1}{a})^{n+1}$	$(n < 0)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$\bar{g}(z) = \frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$ $\quad a^{-1}f(z^{-1})$

$zg(z^{-1})$ $\quad z^{-1}g(z)$

$a\bar{f}(z)$ $\quad a\bar{f}(z^{-1})$

$z\bar{g}(z^{-1})$ $\quad z^{-1}\bar{g}(z)$

a unit nominator

1

$a_n = -a^n$	$(n \geq 0)$
$a^{-1}f(z) = -\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

2

$a_n = -(\frac{1}{a})^n$	$(n < 1)$
$a^{-1}f(z^{-1}) = -\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

3

$zg(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

4

$z^{-1}g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = (\frac{1}{a})^n$	$(n \geq 0)$

5

$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$
$a\bar{f}(z) = -\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

neg(n) sym(n)

inv(z) inv(z)

6

$a_n = -a^n$	$(n < 1)$
$a\bar{f}(z^{-1}) = -\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

7

$z\bar{g}(z^{-1}) = \frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = (\frac{1}{a})^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

8

$z^{-1}\bar{g}(z) = \frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

(1)	(2)
(3)	(4)
(5)	(6)
(7)	(8)

$a^{-1}f(z)$ $a^{-1}f(z^{-1})$

$zg(z^{-1})$ $z^{-1}g(z)$

$a\bar{f}(z)$ $a\bar{f}(z^{-1})$

$z\bar{g}(z^{-1})$ $z^{-1}\bar{g}(z)$

a unit nominator

①

$a_n = -a^n$	$(n \geq 0)$
$a^{-1}f(z) = -\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{a})^n$	$(n < 1)$
$a^{-1}f(z^{-1}) = -\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑤

$a\bar{f}(z) = -\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = -(\frac{1}{a})^n$	$(n \geq 0)$

inv(z) inv(z)

neg(n) sym(n)

⑥

$a\bar{f}(z^{-1}) = -\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = -a^n$	$(n < 1)$

③

$a_n = a^n$	$(n < 1)$
$zg(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

neg(n) sym(n)

inv(z) inv(z)

④

$a_n = (\frac{1}{a})^n$	$(n \geq 0)$
$z^{-1}g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

inv(a) inv(a)
inv(a)

inv(a) inv(a)
inv(a)

⑦

$z\bar{g}(z^{-1}) = \frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = (\frac{1}{a})^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

⑧

$z^{-1}\bar{g}(z) = \frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^n$	$(n \geq 0)$

	Simple Pole Forms				Geometric Series Forms	
1	$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot a^{-1}$	id	$a_n = -a^{n}$	$(n \geq 0)$
	$f(z) = -\frac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot a^{1}$	id	$a^{-1}f(z) = -\frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
2	$a_n = -(\frac{1}{a})^{n-1}$	$(n < 1)$	$\cdot a^{-1}$	id	$a_n = -(\frac{1}{a})^{n}$	$(n < 1)$
	$f(z^{-1}) = -\frac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot a^{-1}$	id	$a^{-1}f(z^{-1}) = -\frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
3	$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot z$	id	$z\,g(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
	$a_n = a^{n+1}$	$(n < 0)$	$\cdot a^{-1}$	S(c(n))	$a_n = a^{n}$	$(n < 1)$
4	$g(z) = \frac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot z^{-1}$	id	$z^{-1}g(z) = \frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
	$a_n = (\frac{1}{a})^{n-1}$	$(n \geq 1)$	$\cdot a^{-1}$	S(c(n))	$a_n = (\frac{1}{a})^{n}$	$(n \geq 0)$
5	$a_n = -(\frac{1}{a})^{n+1}$	$(n \geq 0)$	$\cdot a$	id	$a_n = -(\frac{1}{a})^{n}$	$(n \geq 0)$
	$\bar{f}(z) = -\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot a$	id	$a\bar{f}(z) = -\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
6	$a_n = -a^{n-1}$	$(n < 1)$	$\cdot a$	id	$a_n = -a^{n}$	$(n < 1)$
	$\bar{f}(z^{-1}) = -\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot a$	id	$a\bar{f}(z^{-1}) = -\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
7	$\bar{g}(z^{-1}) = \frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot z$	id	$z\bar{g}(z^{-1}) = \frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
	$a_n = (\frac{1}{a})^{n+1}$	$(n < 0)$	$\cdot a$	s(c(n))	$a_n = (\frac{1}{a})^{n}$	$(n < 1)$
8	$\bar{g}(z) = \frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot z^{-1}$	id	$z^{-1}\bar{g}(z) = \frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
	$a_n = a^{n-1}$	$(n \geq 1)$	$\cdot a$	s(c(n))	$a_n = a^{n}$	$(n \geq 0)$

Simple Pole Forms			Geometric Series Forms	

1.

Simple Pole Form			Geometric Series Form	
$a_n = -a^{n+1}$	$(n \geq 0)$	$\cdot a$ id $\longleftarrow$	$a_n = -a^n$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot a$ id	$a^{-1}f(z) = -\dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$

$-a\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$-\,(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

2.

Simple Pole Form			Geometric Series Form	
$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$	$\cdot a$ id $\longleftarrow$	$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot a$ id	$a^{-1}f(z^{-1}) = -\dfrac{1}{1-az^{-1}}$	$\lvert z\rvert > a$

$-a\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-a\,(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots)$

$-\,(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$-\,(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^{-1} z^{-1} + \left(\tfrac{1}{a}\right)^{-2} z^{-2} + \cdots)$

3.

Simple Pole Form			Geometric Series Form	
$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot z^{-1}$ id	$z\,g(z^{-1}) = \dfrac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$	$\longleftarrow$ $\cdot a$ S(c(n))	$a_n = a^n$	$(n < 1)$

$-\,z^{-1}(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-\,(a^0 z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + \cdots)$

$-\,(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

4.

Simple Pole Form			Geometric Series Form	
$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot z$ id	$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$	$\longleftarrow$ $\cdot a$ S(c(n))	$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$-\,z\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,(\left(\tfrac{1}{a}\right)^0 z^1 + \left(\tfrac{1}{a}\right)^1 z^2 + \left(\tfrac{1}{a}\right)^2 z^3 + \cdots)$

$-\,(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots)$

$-\,(\left(\tfrac{1}{a}\right)^0 z^0 + \left(\tfrac{1}{a}\right)^1 z^1 + \left(\tfrac{1}{a}\right)^2 z^2 + \cdots)$

Simple Pole Forms | Geometric Series Forms

⑤

Simple Pole Forms			Geometric Series Forms	
$a_n = -\left(\frac{1}{a}\right)^{n+1}$	$(n \geq 0)$	$\cdot a^{-1}$ id	$a_n = -\left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$
$\bar{f}(z) = -\frac{a^{-1}}{1-a^{-1}z}$	$\lvert z\rvert < a$	$\cdot a^{-1}$ id	$a\bar{f}(z) = -\frac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$

$-\left(\frac{1}{a}\right)\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^1 z^0 + \left(\frac{1}{a}\right)^2 z^1 + \left(\frac{1}{a}\right)^3 z^2 + \cdots\right)$

$-\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^1 z^1 + \left(\frac{1}{a}\right)^2 z^2 + \cdots\right)$

⑥

Simple Pole Forms			Geometric Series Forms	
$a_n = -a^{n-1}$	$(n < 1)$	$\cdot a^{-1}$ id	$a_n = -a^{n}$	$(n < 1)$
$\bar{f}(z^{-1}) = -\frac{a^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot a^{-1}$ id	$a\bar{f}(z^{-1}) = -\frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$

$-a^{-1}(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

$-(a^{-1} z^0 + a^{-2} z^{-1} + a^{-3} z^{-2} + \cdots)$

$-(a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$

⑦

Simple Pole Forms			Geometric Series Forms	
$\bar{g}(z^{-1}) = \frac{z^{-1}}{1-az^{-1}}$	$\lvert z\rvert > a$	$\cdot z^{-1}$ id	$z\bar{g}(z^{-1}) = \frac{1}{1-az^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$	$\cdot a^{-1}$s(c(n))	$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 1)$

$z^{-1}\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

$\left(\left(\frac{1}{a}\right)^0 z^{-1} + \left(\frac{1}{a}\right)^{-1} z^{-2} + \left(\frac{1}{a}\right)^{-2} z^{-3} + \cdots\right)$

$(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots)$

$\left(\left(\frac{1}{a}\right)^0 z^0 + \left(\frac{1}{a}\right)^{-1} z^{-1} + \left(\frac{1}{a}\right)^{-2} z^{-2} + \cdots\right)$

⑧

Simple Pole Forms			Geometric Series Forms	
$\bar{g}(z) = \frac{z}{1-az}$	$\lvert z\rvert < a^{-1}$	$\cdot z$ id	$z^{-1}\bar{g}(z) = \frac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$	$\cdot a^{-1}$s(c(n))	$a_n = a^{n}$	$(n \geq 0)$

$z(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

$(a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots)$

$(a^0 z^0 + a^1 z^1 + a^2 z^2 + \cdots)$

①

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

②

$a_n = -\left(\frac{1}{a}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\dfrac{a}{1-az^{-1}}$	$\lvert z\rvert > a$

neg(n) sym(n)

inv(z) inv(z)

$a_n = -a^{n+1}$	$(n \geq 0)$
$f(z) = -\dfrac{a}{1-az}$	$\lvert z\rvert < a^{-1}$

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n < 0)$

neg(n) sym(n)

inv(z) inv(z)

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\cdot\, z$ id

$\cdot\, a$ s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

④

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\cdot\, z$ id

$\cdot\, a$ s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$g(z^{-1}) = \frac{z^{-1}}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$	$\cdot z^{-1}$ id		$z\,g(z^{-1}) = \frac{1}{1-a^{-1}z^{-1}}$	$\lvert z\rvert > a^{-1}$
$a_n = a^{n+1}$	$(n<0)$	$\cdot a$ s(c(n))		$a_n = a^{n}$	$(n<1)$

$$X(z) = \frac{1}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$a_n = a^{n} \qquad (n<1)$$

$\bullet z^{-1}$

$+\ (a^0 z^0 + a^{-1} z^{-1} + a^{-2} z^{-2} + \cdots)$ $\qquad n = 0, -1, -2, \cdots$

$a^0 \quad a^{-1} \quad a^{-2} \quad \cdots$ $\qquad \bullet a$

$+\ (a^0 z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + \cdots)$ $\qquad n = -1, -2, -3, \cdots$

$$z^{-1}X(z) = \frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad |z| > a^{-1}$$

$$a_{n-1} = a^{n+1} \qquad (n<0)$$

$g(z) = \dfrac{z}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n-1}$	$(n \geq 1)$

$\cdot z$ id $\qquad \cdot a^{-1}$ s(c(n))

$z^{-1}g(z) = \dfrac{1}{1-a^{-1}z}$	$\lvert z\rvert < a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n \geq 0)$

$$X(z) = \frac{1}{1-a^{-1}z} \qquad \lvert z\rvert < a$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n \geq 0)$$

$\bullet z$

$$-\left(a^0 z^0 + a^{-1} z^1 + a^{-2} z^2 + \cdots\right) \qquad n = 0, 1, 2, \cdots$$

$$a^0 \qquad a^{-1} \qquad a^{-2} \qquad \cdots \qquad \bullet a^{-1}$$

$$-\left(a^0 z^1 + a^{-1} z^2 + a^{-2} z^3 + \cdots\right) \qquad n = 1, 2, 3, \cdots$$

$$z X(z) = \frac{z}{1-a^{-1}z} \qquad \lvert z\rvert < a$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n-1} \qquad (n \geq 1)$$

$\bar{g}(z^{-1}) = \dfrac{z^{-1}}{1 - a z^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n+1}$	$(n < 0)$

$\cdot\, z$ id

$\cdot\, a^{-1}$ s(c(n))

$z\,\bar{g}(z^{-1}) = \dfrac{1}{1 - a z^{-1}}$	$\lvert z\rvert > a$
$a_n = \left(\frac{1}{a}\right)^{n}$	$(n < 1)$

$$X(z) = \frac{1}{1 - a z^{-1}} \qquad \lvert z\rvert > a$$

$$a_n = \left(\frac{1}{a}\right)^{n} \qquad (n < 1)$$

$$-\left(a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} + \cdots\right) \qquad n = 0, -1, -2, \cdots$$

$\bullet\, z$ $\quad a^0 \quad a^1 \quad a^2 \quad \cdots$ $\quad \bullet\, a^{-1}$

$$-\left(a^0 z^{-1} + a^1 z^{-2} + a^2 z^{-3} + \cdots\right) \qquad n = -1, -2, -3, \cdots$$

$$z\,X(z) = \frac{z^{-1}}{1 - a z^{-1}} \qquad \lvert z\rvert > a$$

$$a_{n-1} = \left(\frac{1}{a}\right)^{n+1} \qquad (n < 0)$$

$\bar{g}(z) = \dfrac{z}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n-1}$	$(n \geq 1)$

$\cdot\, z^{-1}$ id

$\cdot\, a^{-1}$ s(c(n))

$z^{-1}\bar{g}(z) = \dfrac{1}{1-az}$	$\lvert z\rvert < a^{-1}$
$a_n = a^{n}$	$(n \geq 0)$

$$X(z) = \frac{1}{1-az} \qquad |z| < a^{-1}$$

$$a_n = a^n \qquad (n \geq 0)$$

$\bullet\, z^{-1}$

$+\ (a^0 z^0 + a^1 z^{-1} + a^2 z^{-2} \quad)$ $\quad n = 0, 1, 2, \cdots$

$a^0 \quad a^1 \quad a^2 \quad \cdots$ $\quad \bullet\, a^{-1}$

$+\ (a^0 z^1 + a^1 z^2 + a^2 z^3 + \cdots)$ $\quad n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z}{1-az} \qquad |z| < a^{-1}$$

$$a_{n-1} = a^{n-1} \qquad (n \geq 1)$$

①

$a_n = -2^n$	$(n \geq 0)$
$0.5 f(z) = -\frac{1}{1-2z}$	$\lvert z\rvert < 0.5$

neg(n) sym(n)

inv(z) inv(z)

②

$a_n = -(\frac{1}{2})^n$	$(n < 1)$
$0.5 f(z^{-1}) = -\frac{1}{1-2z^{-1}}$	$\lvert z\rvert > a$

sign, inv(a,z) comp(z)

sign comp(n)

sign, inv(a,z) comp(z)

sign comp(n)

③

$z\, g(z^{-1}) = \frac{1}{1-0.5z^{-1}}$	$\lvert z\rvert > 0.5$
$a_n = 2^n$	$(n < 1)$

inv(z) inv(z)

neg(n) sym(n)

④

$z^{-1} g(z) = \frac{1}{1-0.5z}$	$\lvert z\rvert < 2$
$a_n = (\frac{1}{2})^n$	$(n \geq 0)$

①

$a_n = -2^{n+1}$ $\quad (n \geq 0)$

$f(z) = -\dfrac{2}{1-2z}$ $\quad \lvert z \rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$ $\quad (n < 1)$

$f(z^{-1}) = -\dfrac{2}{1-2z^{-1}}$ $\quad \lvert z \rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \dfrac{z^{-1}}{1-0.5z^{-1}}$ $\quad \lvert z \rvert > 0.5$

$a_n = 2^{n+1}$ $\quad (n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \dfrac{z}{1-0.5z}$ $\quad \lvert z \rvert < 2$

$a_n = \left(\frac{1}{2}\right)^{n-1}$ $\quad (n \geq 1)$

①

$a_n = -2^{n+1}$	$(n \geq 0)$
$f(z) = -\frac{2}{1-2z}$	$\lvert z \rvert < 0.5$

neg(n) Sym(n)

inv(z) inv(z)

②

$a_n = -\left(\frac{1}{2}\right)^{n-1}$	$(n < 1)$
$f(z^{-1}) = -\frac{2}{1-2z^{-1}}$	$\lvert z \rvert > 2$

sign comp(z)

sign comp(n)

sign comp(z)

sign comp(n)

③

$g(z^{-1}) = \frac{z^{-1}}{1-0.5z^{-1}}$	$\lvert z \rvert > 0.5$
$a_n = 2^{n+1}$	$(n < 0)$

neg(n) Sym(n)

inv(z) inv(z)

④

$g(z) = \frac{z}{1-0.5z}$	$\lvert z \rvert < 2$
$a_n = \left(\frac{1}{2}\right)^{n-1}$	$(n \geq 1)$

①

$$a_n = -2^{n+1} \quad (n \geq 0)$$

$$f(z) = -\frac{2}{1-2z} \quad |z| < 0.5$$

②

$$a_n = -\left(\tfrac{1}{2}\right)^{n-1} \quad (n < 1)$$

$$f(z^{-1}) = -\frac{2}{1-2z^{-1}} \quad |z| > 2$$

neg(n) sym(n)

inv(z) inv(z)

$$a_n = -2^{n+1} \quad (n \geq 0)$$

$$f(z) = -\frac{2}{1-2z} \quad |z| < 0.5$$

③

$$g(z^{-1}) = \frac{z^{-1}}{1-0.5z^{-1}} \quad |z| > 0.5$$

$$a_n = 2^{n+1} \quad (n < 0)$$

neg(n) sym(n)

inv(z) inv(z)

$$g(z) = \frac{z}{1-0.5z} \quad |z| < 2$$

$$a_n = \left(\tfrac{1}{2}\right)^{n-1} \quad (n \geq 1)$$

- z id
- a s(c(n))

$$z^{-1}g(z) = \frac{1}{1-a^{-1}z} \quad |z| < a$$

$$a_n = \left(\tfrac{1}{a}\right)^{n} \quad (n \geq 0)$$

④

$$g(z) = \frac{z}{1-2z} \quad |z| < 2$$

$$a_n = \left(\tfrac{1}{2}\right)^{n-1} \quad (n \geq 1)$$

- z id
- a s(c(n))

$$z^{-1}g(z) = \frac{1}{1-0.5z} \quad |z| < 2$$

$$a_n = \left(\tfrac{1}{2}\right)^{n} \quad (n \geq 0)$$

$$\frac{3}{2}\ \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\frac{1}{2}\right)^{-1} z^0 + \left(\frac{1}{2}\right)^{-2} z^1 + \left(\frac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$$

$-n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\quad a^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\quad a^{n+1}$ $\quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

z^{-1} $\quad z^{-1}$ $\quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n}$ $\quad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\quad \left(\frac{1}{a}\right)^{-n+1}$ $\quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$

① $-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad |z|<0.5$

② $-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $\quad |z|>2$

③ $+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad |z|>2$

④ $+\frac{z}{1-0.5z} - \frac{z}{1-2z}$ $\quad |z|<0.5$

$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $\quad (n \geq 0)$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$|z|<0.5$

$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $\quad (n<1)$

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$|z|>2$

z^{-1} $\quad -n$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$|z|<0.5$

$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $\quad (n<0)$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$|z|>2$

z^{-1} $\quad -n$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z}$$

$|z|<0.5$

$\cdot z$ $\quad n-1$

$$+\frac{1}{1-0.5z} - \frac{1}{1-2z}$$

$|z|<0.5$

$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $\quad (n \geq 1)$

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z}$$

$|z|<0.5$

$\cdot z$ $\quad n-1$

$$+\frac{1}{1-z} - \frac{1}{1-2z}$$

$|z|<0.5$

$z^{-1}X(z)$ Shifted Sequence

$$X(z) = \underset{|z|>1}{\frac{1}{1-z^{-1}}} - \underset{|z|>2}{\frac{1}{1-2z^{-1}}} \qquad (|z|>2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

• z^{-1}

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$z\,f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z|<0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

$\cdot z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z\,f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\cdot z \quad (1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$z \quad (1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \quad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$z^{-1} \quad (1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \quad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $X(z)$ $a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n <$

$|z| > 2$ $X(z)$ $b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a$ $X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ a^{n+1} $n \geq 0$ $n \geq 1$ $n < 0$ $n <$

$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $-n$

$|z| > a$ $X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ a^{-n+1} $n < 0$ $n < 1$ $n \geq 0$ n

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az}$ $\sum_{n=0}^{\infty} a^{n+1} z^n$ $\frac{z}{1-az}$ $\sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $-\sum_{n=0}^{-\infty} a^{n-1} z^n$

$z\ X(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$\bullet z^{-1}$

$(1^0z^0 + 1^1z^{-1} + 1^2z^{-2} + \cdots) + (2^0z^0 + 2^1z^{-1} + 2^2z^{-2} \quad)$ n $n = 0, 1, 2, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots) + (2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots)$ $n-1$ $n = 1, 2, 3, \cdots$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots) + (2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots)$ n $n = 1, 2, 3, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1}

$(1^0z^{1} + 1^1z^{2} + 1^2z^{3} + \cdots) + (2^0z^{1} + 2^1z^{2} + 2^2z^{3} + \cdots)$ $-n$ $n = -1, -2, -3, \cdots$

$$z\,X(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

$a_{-(n+1)}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\quad = \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$\qquad = \left(\frac{1}{a}\right)^{n-1}$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n}$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \qquad z^{-1} \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$ $\qquad \left(\frac{1}{a}\right)^{-n+1} = a^{n-1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$\boxed{2z}$ $\boxed{2z^{-1}}$ $\boxed{2^{-1}z^{-1}}$ $\boxed{2^{-1}z}$

$\boxed{|z|<0.5}$ $\boxed{|z|>2}$ $\boxed{|z|>0.5}$ $\boxed{|z|<2}$

$$-\frac{2}{1-2z} \xleftrightarrow{z^{-1}} -\frac{2}{1-2z^{-1}}$$

$$\cdot\frac{(2z)^{-1}}{(2z)^{-1}} \Big\downarrow \Big\uparrow \cdot\frac{(2z)}{(2z)} \qquad \cdot\frac{(2z^{-1})^{-1}}{(2z^{-1})^{-1}} \Big\downarrow \Big\uparrow \cdot\frac{(2z^{-1})}{(2z^{-1})}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} \xleftrightarrow{z^{-1}} +\frac{z}{1-0.5z}$$

z-Transform $(n \rightarrow -n)$

(5) $\boxed{-\dfrac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}}}$ $\boxed{|z|>a^{-1}}$

$-\left(a^{-1}z^{-1}+a^{-2}z^{-2}+a^{-3}z^{-3}+\cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1}+\left(\tfrac{1}{a}\right)^{2}z^{-2}+\left(\tfrac{1}{a}\right)^{3}z^{-3}+\cdots\right)$

$-a^{-n}$ u(-(-n)-1)	$(-n<0)$
$-\left(\frac{1}{a}\right)^{n}$ u(n-1)	$(n\geq 1)$

(6) $\boxed{-\dfrac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}}}$ $\boxed{|z|>a}$

$-\left(a^{1}z^{-1}+a^{2}z^{-2}+a^{3}z^{-3}+\cdots\right)$

$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1}+\left(\tfrac{1}{a}\right)^{-2}z^{-2}+\left(\tfrac{1}{a}\right)^{-3}z^{-3}+\cdots\right)$

$-\left(\frac{1}{a}\right)^{-n}$ u(-(-n)-1)	$(-n<0)$
$-a^{n}$ u(n-1)	$(n\geq 1)$

(7) $\boxed{+\dfrac{\boxed{az}}{1-\boxed{az}}}$ $\boxed{|z|<a^{-1}}$

$\left(a^{1}z^{1}+a^{2}z^{2}+a^{3}z^{3}+\cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1}+\left(\tfrac{1}{a}\right)^{-2}z^{2}+\left(\tfrac{1}{a}\right)^{-3}z^{3}+\cdots\right)$

a^{-n} u((-n)-1)	$(-n\geq 1)$
$\left(\frac{1}{a}\right)^{n}$ u(-n-1)	$(n<0)$

(8) $\boxed{+\dfrac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}}}$ $\boxed{|z|<a}$

$\left(a^{-1}z^{1}+a^{-2}z^{2}+a^{-3}z^{3}+\cdots\right)$

$\left(\left(\tfrac{1}{a}\right)^{1}z^{1}+\left(\tfrac{1}{a}\right)^{2}z^{2}+\left(\tfrac{1}{a}\right)^{3}z^{3}+\cdots\right)$

$\left(\frac{1}{a}\right)^{-n}$ u((-n)-1)	$(-n\geq 1)$
a^{n} u(-n-1)	$(n<0)$

Laurent Series vs. z-Transform (n → -n)

(5)

$$-\frac{\boxed{a^{-1}z^{-1}}}{1-\boxed{a^{-1}z^{-1}}} \qquad |z| > a^{-1}$$

$$-\left(a^{-1}z^{-1} + a^{-2}z^{-2} + a^{-3}z^{-3} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{1}z^{-1} + \left(\tfrac{1}{a}\right)^{2}z^{-2} + \left(\tfrac{1}{a}\right)^{3}z^{-3} + \cdots\right)$$

Laurent	$-a^n$	u(-n-1)	$(n < 0)$
z-Trans	$-\left(\frac{1}{a}\right)^n$	u(n-1)	$(n \geq 1)$

(6)

$$-\frac{\boxed{az^{-1}}}{1-\boxed{az^{-1}}} \qquad |z| > a$$

$$-\left(a^{1}z^{-1} + a^{2}z^{-2} + a^{3}z^{-3} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{a}\right)^{-1}z^{-1} + \left(\tfrac{1}{a}\right)^{-2}z^{-2} + \left(\tfrac{1}{a}\right)^{-3}z^{-3} + \cdots\right)$$

Laurent	$-\left(\frac{1}{a}\right)^n$	u(-n-1)	$(n < 0)$
z-Trans	$-a^n$	u(n-1)	$(n \geq 1)$

(7)

$$+\frac{\boxed{az}}{1-\boxed{az}} \qquad |z| < a^{-1}$$

$$\left(a^{1}z^{1} + a^{2}z^{2} + a^{3}z^{3} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{-1}z^{1} + \left(\tfrac{1}{a}\right)^{-2}z^{2} + \left(\tfrac{1}{a}\right)^{-3}z^{3} + \cdots\right)$$

Laurent	a^n	u(n-1)	$(n \geq 1)$
z-Trans	$\left(\frac{1}{a}\right)^n$	u(-n-1)	$(n < 0)$

(8)

$$+\frac{\boxed{a^{-1}z}}{1-\boxed{a^{-1}z}} \qquad |z| < a$$

$$\left(a^{-1}z^{1} + a^{-2}z^{2} + a^{-3}z^{3} + \cdots\right)$$

$$\left(\left(\tfrac{1}{a}\right)^{1}z^{1} + \left(\tfrac{1}{a}\right)^{2}z^{2} + \left(\tfrac{1}{a}\right)^{3}z^{3} + \cdots\right)$$

Laurent	$\left(\frac{1}{a}\right)^n$	u(n-1)	$(n \geq 1)$
z-Trans	a^n	u(-n-1)	$(n < 0)$