CORDIC Idea Lookahead

20160307

2-Step Lookahead (Quad Tree)

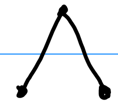
- 2 steps / a clock period
- Number of Adders
- Any benefit in determining rotation directions of angles in ATR (Arc Tangent Radix)?

Binary v.s. Quad Angle Tree

Original CORDIC

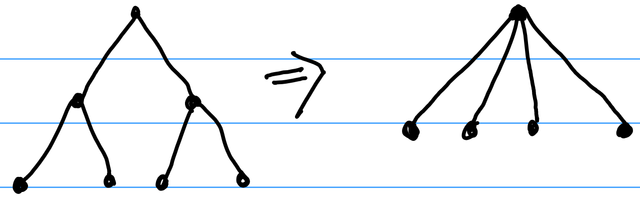
in each iteration
decide whether a CORDIC angle is added or subtracted

① Binary Angle Tree



- 2 choices
- ADD / SUB.

② Quad Angle Tree



- 4 choices
- Can reduce the latency by reducing the number of iteration.
- low power?
- good effect on the precision
- with the minimum hardware increase (adder)

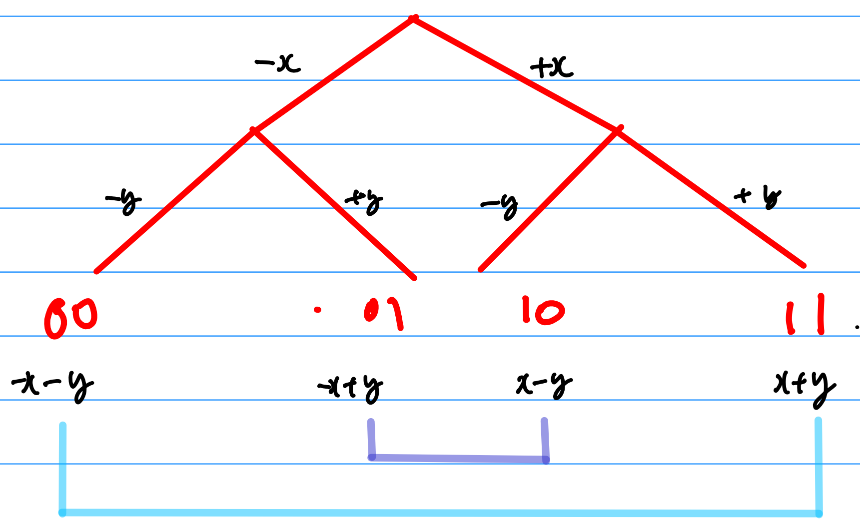
2-Step Lookahead (Quad Tree)

$m_0 = x+y = x+y$

$m_1 = x-y = x+\bar{y}+1$

$m_2 = -x+y = -(x-y) = \bar{x}+y+1$

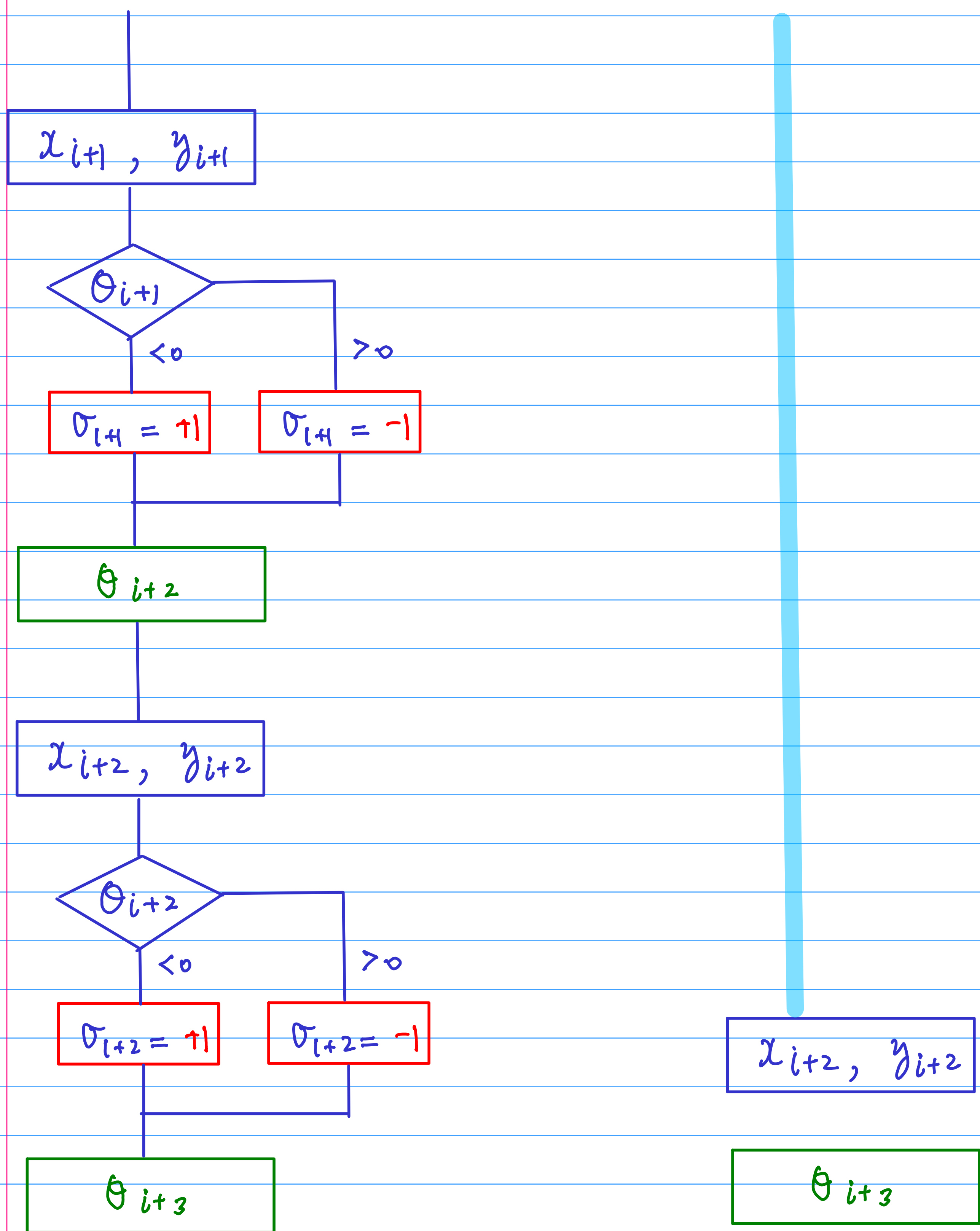
$m_3 = -x-y = -(x+y) = \bar{x}+\bar{y}+2$



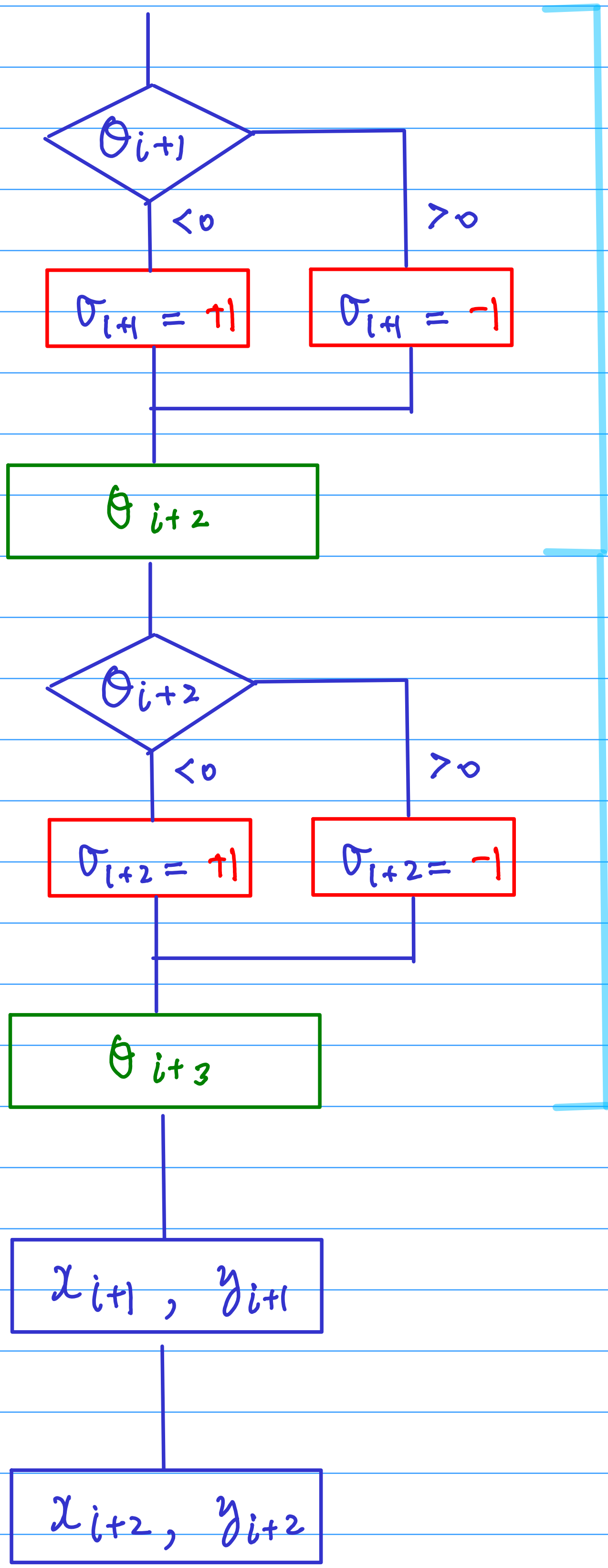
- 2's complementer – efficient incrementer/decrementer

- Comparator

$$\min_i \{|\theta-m_0|, |\theta-m_1|, |\theta-m_2|, |\theta-m_3|\}$$



x_{i+1}, y_{i+1}
θ_{i+1}
<0
>0
$\sigma_{i+1} = +1$
$\sigma_{i+1} = -1$
θ_{i+2}
x_{i+2}, y_{i+2}
θ_{i+2}
<0
>0
$\sigma_{i+2} = +1$
$\sigma_{i+2} = -1$
θ_{i+3}
x_{i+2}, y_{i+2}
θ_{i+3}

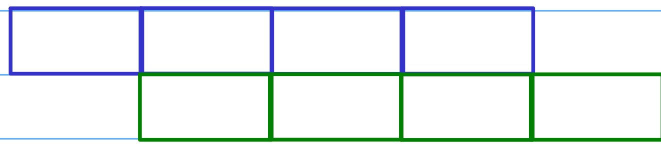


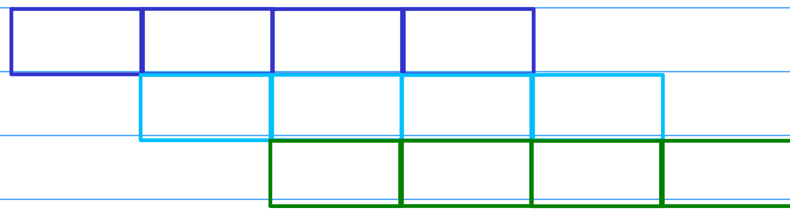
rotation direction
angles are
can be precalculated

finding rotation direction angles

applying angle rotation

no data dependence





$$x'_{i+1} = \left(x'_i - y'_i\,\sigma_i 2^{-i}\right)$$

$$y'_{i+1} = \left(x'_i\,\sigma_i 2^{-i} + y'_i\right)$$

$$\boxed{\theta_{i+1} = \theta_i - \tan^{-1}\left(\sigma_i 2^{-i}\right)}$$

no x_i, y_i affects
the determination of θ_{i+1}

Number of Adders

4 adders

$$
\begin{aligned}
m_0 &= x+y & &= x+y \\
m_1 &= x-y & &= x+\bar{y}+1 \\
m_2 &= -x+y &= -(x-y) &= \bar{x}+y+1 \\
m_3 &= -x-y &= -(x+y) &= \bar{x}+\bar{y}+2
\end{aligned}
$$

$$
\begin{aligned}
x+y &= x+y+00 \\
x-y &= x+\bar{y}+01 \\
-x+y &= \bar{x}+y+01 \\
-x-y &= \bar{x}+\bar{y}+10
\end{aligned}
$$

$$
\text{min}\left\{
\begin{aligned}
\theta_i-(x+y) &= \theta_i+\bar{x}+\bar{y}+10 \\
\theta_i-(x-y) &= \theta_i+\bar{x}+y+01 \\
\theta_i-(-x+y) &= \theta_i+x+\bar{y}+01 \\
\theta_i-(-x-y) &= \theta_i+x+y+00
\end{aligned}
\right.
$$

$\left(\begin{array}{l}\text{absolute operation} \\ \text{complement operation}\end{array}\right)$ any efficient way

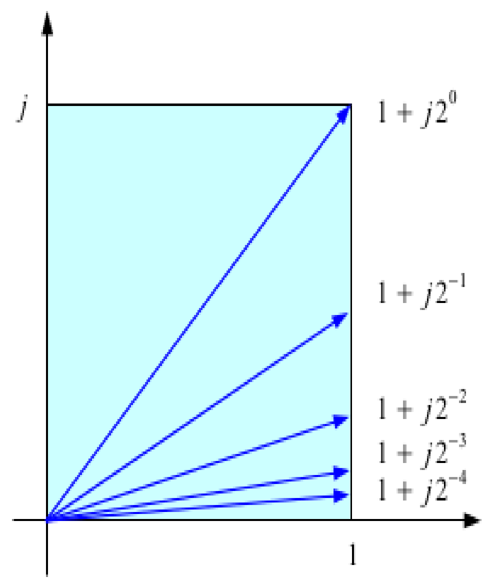
$$
\begin{aligned}
x &\longrightarrow |x| \\
x &\longrightarrow -x
\end{aligned}
$$

$$\theta_i - (x+y) = \theta_i + \bar{x} + \bar{y} + 10 \qquad 01$$
$$\theta_i - (x-y) = \theta_i + \bar{x} + y + 01 \qquad 10$$
$$\theta_i - (-x+y) = \theta_i + x + \bar{y} + 01 \qquad 10$$
$$\theta_i - (-x-y) = \theta_i + x + y + 00 \qquad 11$$

Choose θ_i such that $\tan\theta_i = \begin{cases} +2^{-i} \\ -2^{-i} \end{cases}$

$$\tan\theta_i = \sigma_i 2^{-i} \qquad \sigma_i \in \{+1, -1\}$$

$$x'_{i+1} = (x'_i - y'_i \sigma_i 2^{-i})$$
$$y'_{i+1} = (x'_i \sigma_i 2^{-i} + y'_i)$$
$$\theta_{i+1} = \theta_i - \tan^{-1}(\sigma_i 2^{-i})$$



$$\tan^{-1}\left(\frac{1}{2^i}\right) \approx \frac{1}{2^i}$$

@ $\boxed{\text{after initial iterations}}$

(%i3) `taylor(cos(x), x, 0, 8);`

(%o3)/T/ $1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}+...$

(%i4) `taylor(sin(x), x, 0, 8);`

(%o4)/T/ $x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{x^7}{5040}+...$

(%i11) `taylor(tan(x), x, 0, 8);`

(%o11)/T/ $x+\frac{x^3}{3}+\frac{2\,x^5}{15}+\frac{17\,x^7}{315}+...$

(%i21) `taylor(atan(x), x, 0, 16);`

(%o21)/T/ $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{15}}{15}+...$

(%i23) `f(x) := x-x^3/3+x^5/5-x^7/7+x^9/9-x^11/11+x^13/13-x^15/15;`

(%o23) $f(x):=x-\frac{x^3}{3}+\frac{x^5}{5}+\frac{-x^7}{7}+\frac{x^9}{9}+\frac{-x^{11}}{11}+\frac{x^{13}}{13}+\frac{-x^{15}}{15}$

(%i25) `float(f(1/2));`

(%o25) 0.46364724210793

(%i26) `float(f(1/2^2));`

(%o26) 0.24497866312362

(%i27) `float(f(1/2^3));`

(%o27) 0.12435499454676

(%i28) `float(f(1/2^4));`

(%o28) 0.062418809995957

```
octave:5> atan(1 ./ 2.^n)'
ans =

   0.7853982
   0.4636476
   0.2449787
   0.1243550
   0.0624188
   0.0312398
   0.0156237
   0.0078123
   0.0039062
```

```
i = 0 : 15;
A = atan(1./(2.^i));

B = A ./ A(length(A));
```

```
octave:16> B'
ans =

   2.5736e+04
   1.5193e+04
   8.0275e+03
   4.0749e+03
   2.0453e+03
   1.0237e+03
   5.1196e+02
   2.5599e+02
   1.2800e+02
   6.4000e+01
   3.2000e+01
   1.6000e+01
   8.0000e+00
   4.0000e+00
   2.0000e+00
   1.0000e+00
```

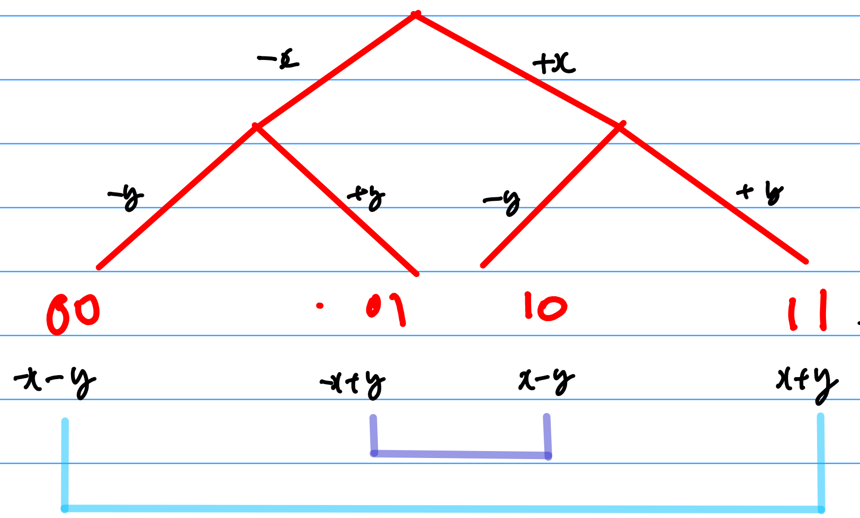
$$\boxed{\operatorname{atan}(2^{-i}) \approx 2^{-i}}$$

after initial iterations

$$\theta_i - (x+y) = \theta_i + \bar{x} + \bar{y} + 10 \qquad 01$$
$$\theta_i - (x-y) = \theta_i + \bar{x} + y + 01 \qquad 10$$
$$\theta_i - (-x+y) = \theta_i + x + \bar{y} + 01 \qquad 10$$
$$\theta_i - (-x-y) = \theta_i + x + y + 00 \qquad 11$$

Level Angles

$\theta_i = \frac{1}{2^i}$ all zero bits except one bit after initial iterations



$\theta_i = x$ | 0 ...... 0 | 1 | 0 | 0 ..... 0 |

$\theta_{i+1} = y$ | 0 ...... 0 | 0 | 1 | 0 ..... 0 |

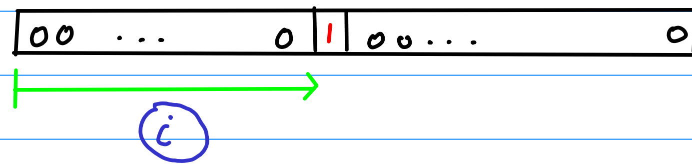
consecutive angles x, y $\quad x = 2y$

$$\begin{cases} x+y = 3y \\ x-y = y \end{cases}$$

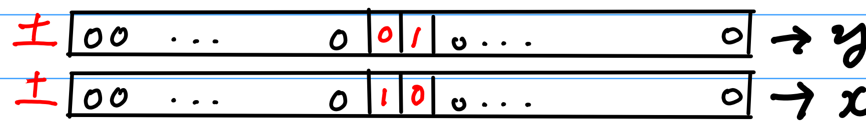
$$\begin{cases} -(x+y) = -3y \\ -(x-y) = -y \end{cases}$$

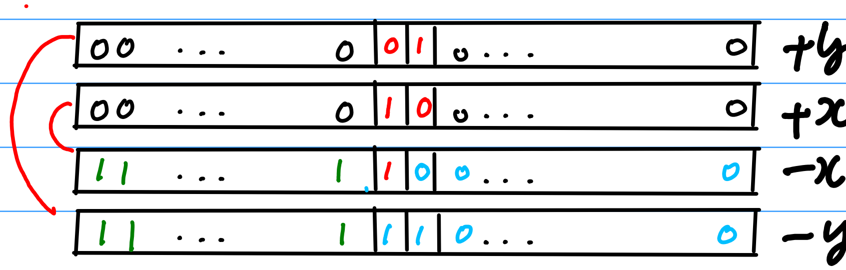
after a few iterations,

$$\theta_i = \tan^{-1}\left(\frac{1}{2^i}\right) \approx \left(\frac{1}{2^i}\right)$$



therefore, for Quad tree angles

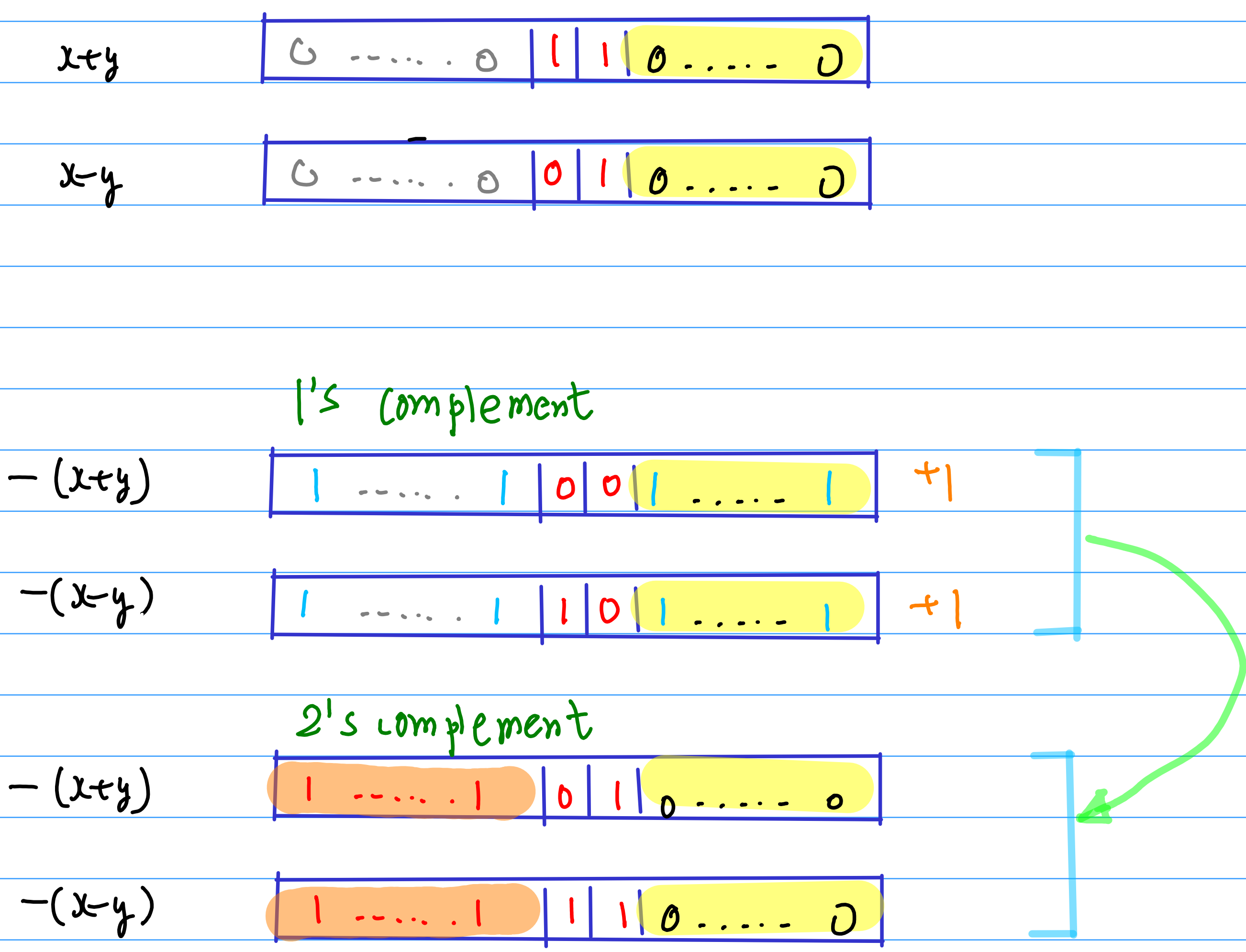




consecutive

$\underline{x = 2y}$ $\boxed{x-y = y}$

$\theta_i = x$	0 0	1	0	0 0	
$\theta_{i+1} = y$	0 0	0	1	0 0	
$x+y$	0 0	1	1	0 0	3
$x-y$	0 0	0	1	0 0	1
$-(x+y)$	1 1	0	1	0 0	-3
$-(x-y)$	1 1	1	1	0 0	-1

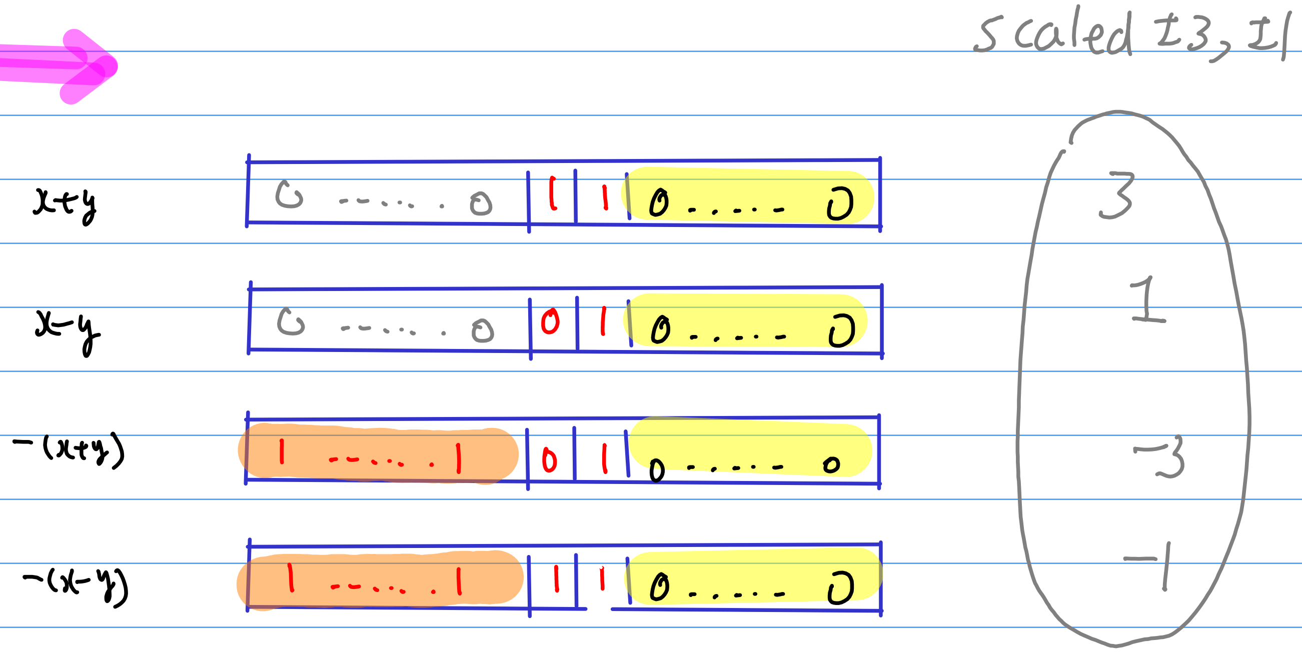

x+y
0 ...... 0 | 1 | 1 | 0 0
x−y
0 ...... 0 | 0 | 1 | 0 0
1's complement
−(x+y)
1 ...... 1 | 0 | 0 | 1 1
+1
−(x−y)
1 ...... 1 | 1 | 0 | 1 1
+1
2's complement
−(x+y)
1 ...... 1 | 0 | 1 | 0 0
−(x−y)
1 ...... 1 | 1 | 1 | 0 0

Angle Update

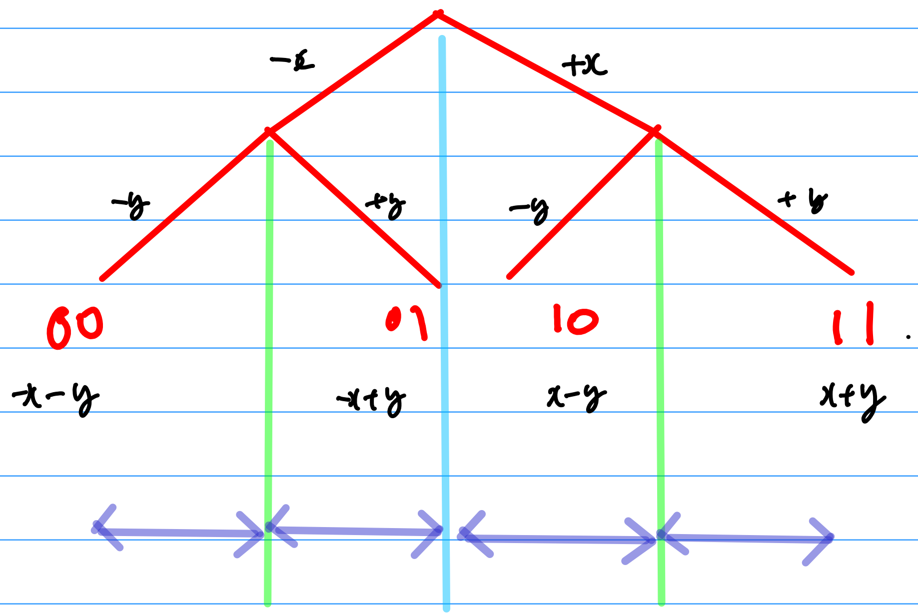
$$\text{min}\left\{\begin{aligned}
\theta_i - (x+y) &= \theta_i + \bar{x} + \bar{y} + 10 \\
\theta_i - (x-y) &= \theta_i + \bar{x} + y + 01 \\
\theta_i - (-x+y) &= \theta_i + x + \bar{y} + 01 \\
\theta_i - (-x-y) &= \theta_i + x + y + 00
\end{aligned}\right.$$

after a few iterations,

$$\theta_i = \tan^{-1}\left(\frac{1}{2^i}\right) \approx \left(\frac{1}{2^i}\right)$$



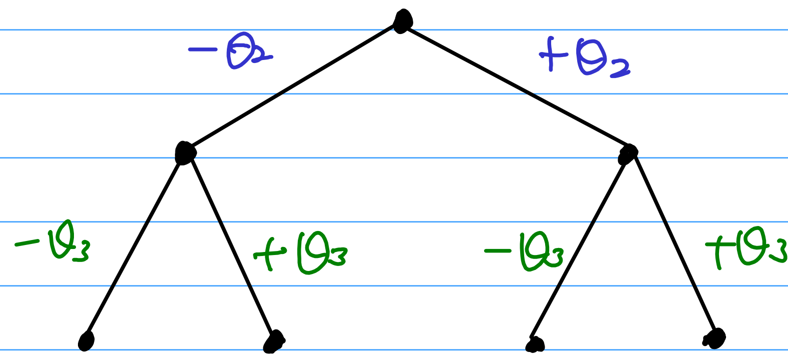
$$\begin{aligned}
\theta_i - (x+y) &= \theta_i + (-x-y) && -3 \\
\theta_i - (x-y) &= \theta_i + (-x+y) && -1 \\
\theta_i - (-x+y) &= \theta_i + (x-y) && +1 \\
\theta_i - (-x-y) &= \theta_i + (x+y) && +3
\end{aligned}$$



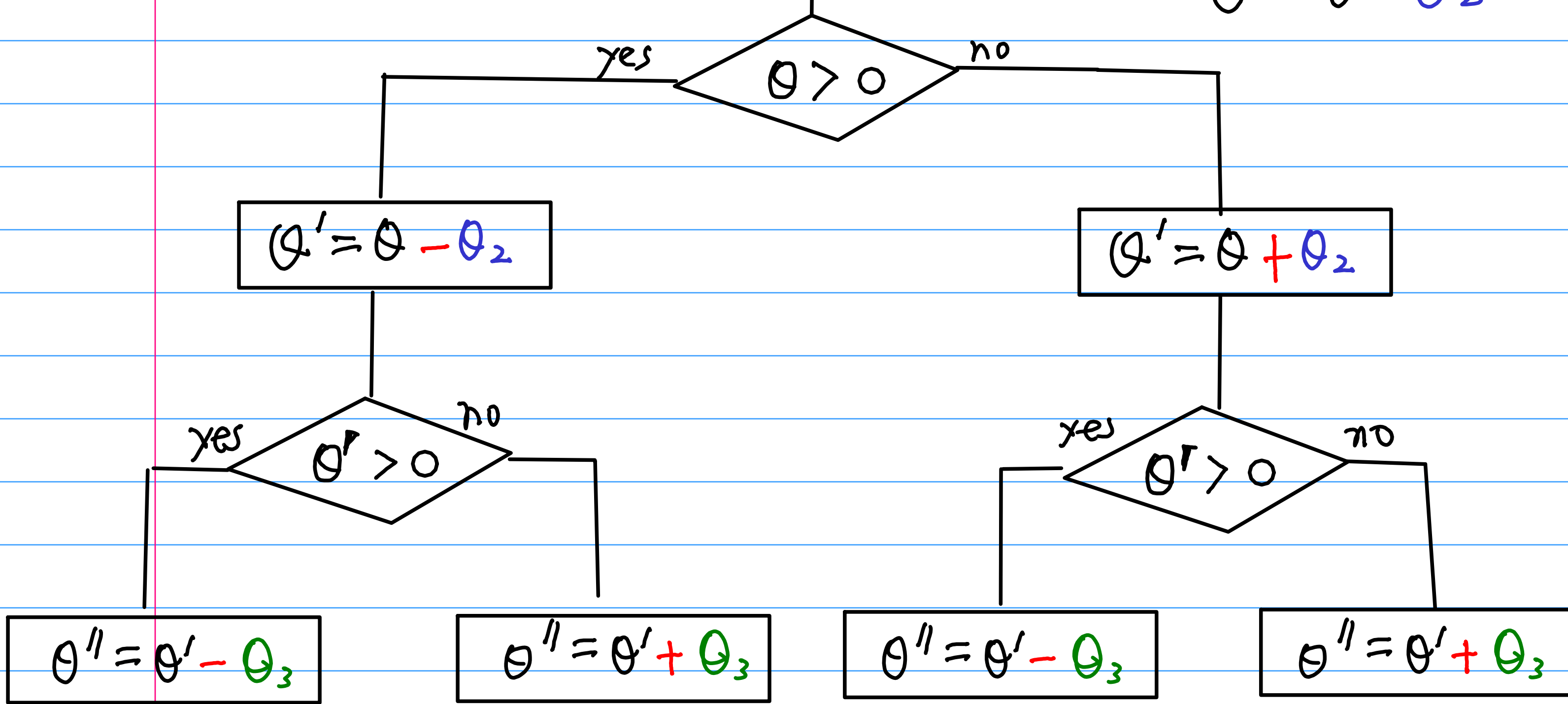
$\theta_i > 0$ $-x$ $\begin{cases} -(x+y) & -3 \\ -(x-y) & -1 \end{cases}$

$\theta_i < 0$ $+x$ $\begin{cases} (x+y) & +3 \\ (x-y) & +1 \end{cases}$

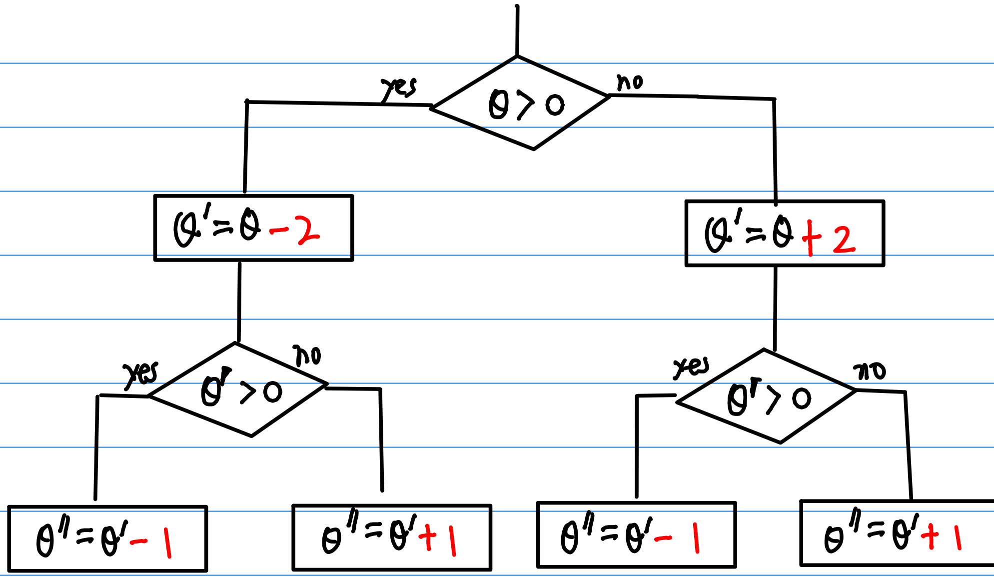
$$\theta_i - (x+y) = \theta_i + (-x-y)$$
$$\theta_i - (x-y) = \theta_i + (-x+y)$$
$$\theta_i - (-x+y) = \theta_i + (x-y)$$
$$\theta_i - (-x-y) = \theta_i + (x+y)$$

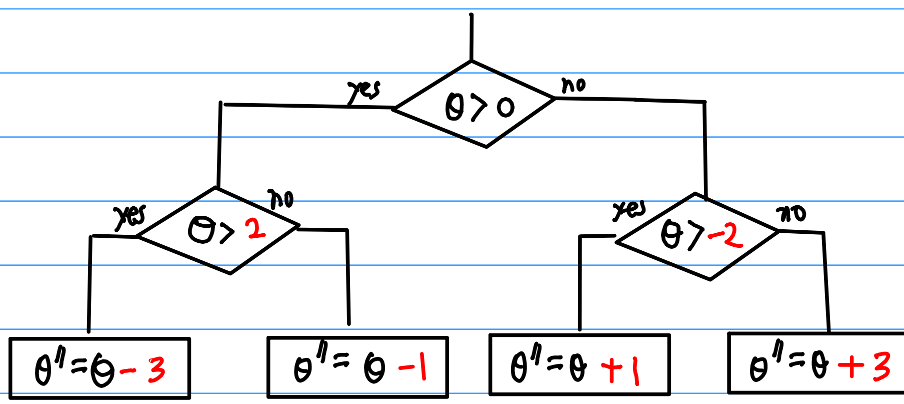


$\theta' = \theta - \theta_2$



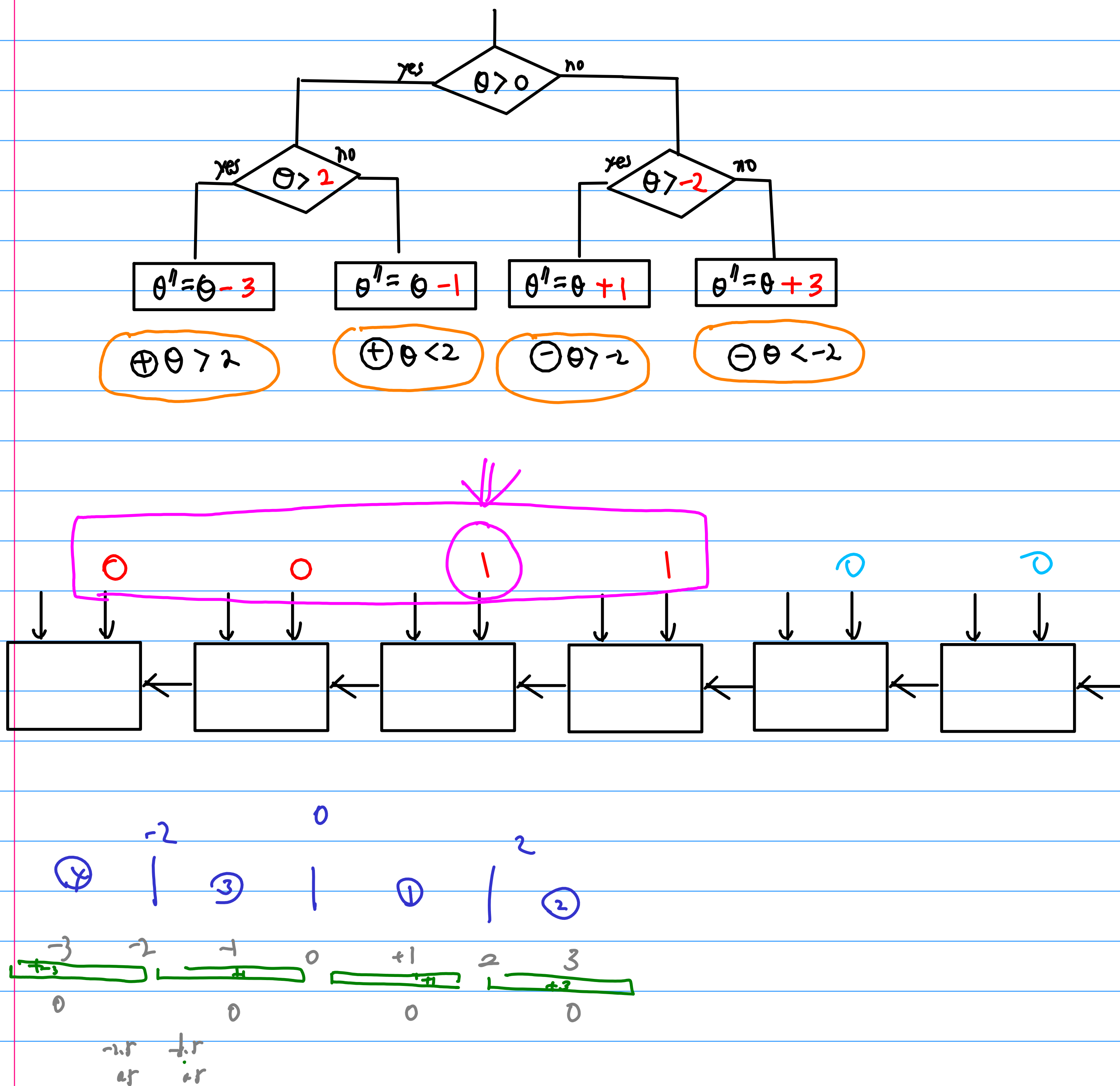
Decision adding ± 3, ± 1



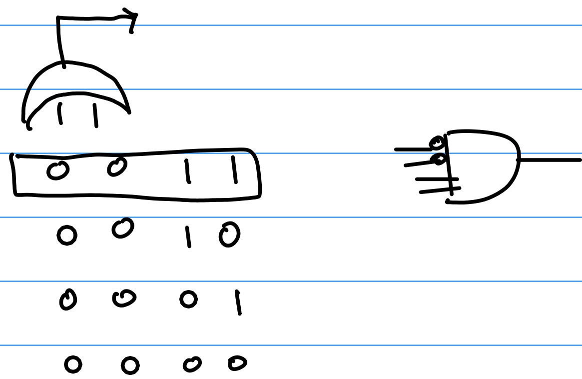


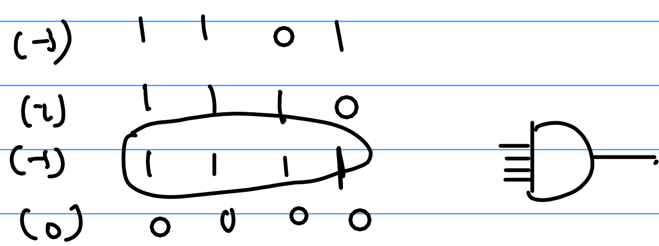
⊕ $\theta > 2$ ⊕ $\theta < 2$ ⊖ $\theta > -2$ ⊖ $\theta < -2$

In fact, adding scaled $\pm 3, \pm 1$.



A Comparator





angle update conditions

$\oplus\ \theta > 2$ $\quad$ $\oplus\ \theta < 2$ $\quad$ $\ominus\ \theta > -2$ $\quad$ $\ominus\ \theta < -2$

(13) $\boxed{\text{Decision}}$ Choose min

$$\min \left\{ \begin{aligned} \theta_i - (x+y) &= \theta_i + \bar{x} + \bar{y} + 10 \\ \theta_i - (x-y) &= \theta_i + \bar{x} + y + 01 \\ \theta_i - (-x+y) &= \theta_i + x + \bar{y} + 01 \\ \theta_i - (-x-y) &= \theta_i + x + y + 00 \end{aligned} \right.$$

* What's the difference?

* lookahead may be critical for the initial stage of cordic iteration

* Critically damped CORDIC
  adaptive parallel angle recording } $\boxed{\text{Swartzlander}}$

Angle Rotation

Choose θ_i such that $\tan\theta_i = \begin{cases} +2^{-i} \\ -2^{-i} \end{cases}$

$$\tan\theta_i = \sigma_i 2^{-i} \qquad \sigma_i \in \{+1, -1\}$$

$$\begin{aligned} x'_{i+1} &= (x'_i - y'_i \sigma_i 2^{-i}) \\ y'_{i+1} &= (x'_i \sigma_i 2^{-i} + y'_i) \\ \theta_{i+1} &= \theta_i - \tan^{-1}(\sigma_i 2^{-i}) \end{aligned}$$

$$\begin{pmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{pmatrix}$$

$$= \frac{1}{\sqrt{1+2^{-2\cdot n}}}\begin{pmatrix} +1 & \mp 2^{-n} \\ \pm 2^{-n} & +1 \end{pmatrix} \cdots \frac{1}{\sqrt{1+2^{-2\cdot 1}}}\begin{pmatrix} +1 & \mp 2^{-1} \\ \pm 2^{-1} & +1 \end{pmatrix} \cdot \frac{1}{\sqrt{1+2^{0}}}\begin{pmatrix} +1 & \mp 2^{0} \\ \pm 2^{0} & +1 \end{pmatrix}$$

$$= \underline{\frac{1}{\sqrt{1+2^{-2\cdot n}}} \cdots \frac{1}{\sqrt{1+2^{-2\cdot 1}}} \cdot \frac{1}{\sqrt{1+2^{-2\cdot 0}}}} \; \underline{\begin{pmatrix} +1 & \mp 2^{-n} \\ \pm 2^{-n} & +1 \end{pmatrix} \cdots \begin{pmatrix} +1 & \mp 2^{-1} \\ \pm 2^{-1} & +1 \end{pmatrix}\begin{pmatrix} +1 & \mp 2^{-0} \\ \pm 2^{-0} & +1 \end{pmatrix}}$$

$\Rightarrow$ $K = \prod 1/\sqrt{1+\tan^2\theta_i} = 0.607$

$\Rightarrow$ $\begin{pmatrix} +\cos(\Sigma\theta_i) & -\sin(\Sigma\theta_i) \\ +\sin(\Sigma\theta_i) & +\cos(\Sigma\theta_i) \end{pmatrix}$

$$\begin{pmatrix} +\cos(\Sigma\theta_i) & -\sin(\Sigma\theta_i) \\ +\sin(\Sigma\theta_i) & +\cos(\Sigma\theta_i) \end{pmatrix} = \frac{1}{K} \cdot \begin{pmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & +\cos\theta \end{pmatrix}$$

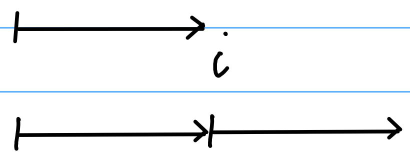
$$\begin{aligned} 1/K &= \prod \sqrt{1+\tan^2\theta_i} = 1.647 \\ &= A = \textit{CORDIC Gain} \end{aligned}$$

$$\begin{bmatrix} 1 & \mp 2^{-i} \\ \pm 2^{-i} & 1 \end{bmatrix} \begin{bmatrix} 1 & \mp 2^{-i-1} \\ \pm 2^{-i-1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2^{-2i-1} & \mp(2^{-i} + 2^{-i-1}) \\ \pm(2^{-i} + 2^{-i-1}) & 1 - 2^{-2i-1} \end{bmatrix}$$

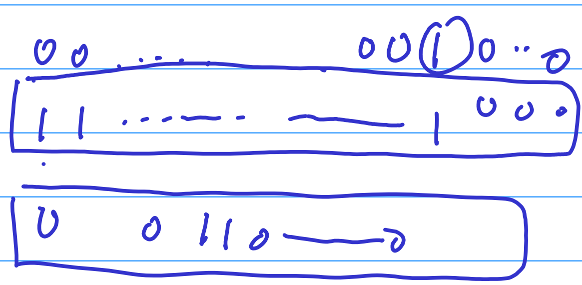
$$\begin{bmatrix} 1 - 2^{-2i-1} & \mp(2^{-i} + 2^{-i-1}) \\ \pm(2^{-i} + 2^{-i-1}) & 1 - 2^{-2i-1} \end{bmatrix}$$

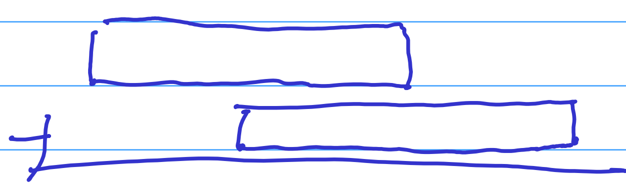
	1.0000 0000	1.
$i=1$	0.1000 0000	0.5
$i=2$	0.0100 0000	0.25
$i=3$	0.0010 0000	0.125
$i=4$	0.0001 0000	0.0625
$i=5$	0.0000 1000	



2^{-2i-1}

$2^{i} + 2^{i-1}$

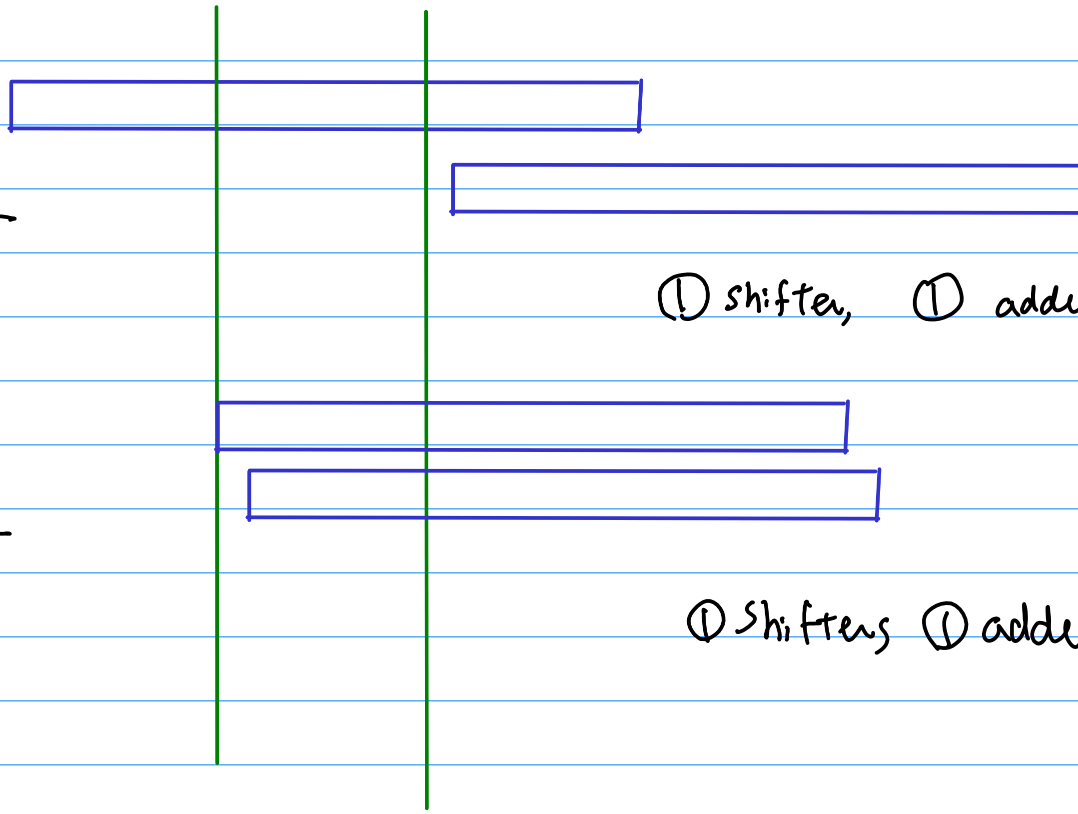




$$\begin{bmatrix} (1-2^{-2i-1})X & \mp(2^{-i}+2^{-i-1})Y \\ \pm(2^{-i}+2^{-i-1})X & (1-2^{-2i-1})Y \end{bmatrix}$$

X

$-)\ 2^{-2i-1}X$



① shifter, ① adder

$2^{-i}Y$

$+)\ 2^{-i-1}Y$

① shifters ① adders

total ② shifters ② adders

1. T shifter

2. T adder

Simple loop Unrolling

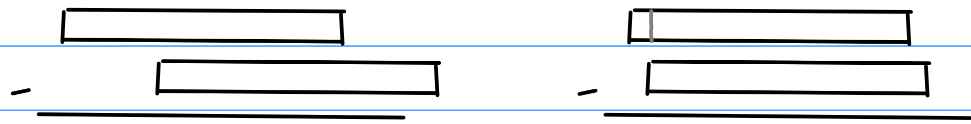
$$\begin{bmatrix} 1 & \mp 2^{-i} \\ \pm 2^{-i} & 1 \end{bmatrix} \begin{bmatrix} 1 & \mp 2^{-i-1} \\ \pm 2^{-i-1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2^{-2i-1} & \mp(2^{-i} + 2^{-i-1}) \\ \pm(2^{-i} + 2^{-i-1}) & 1 - 2^{-2i-1} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 - 2^{-2i-1} & \mp(2^{-i} + 2^{-i-1}) \\ \pm(2^{-i} + 2^{-i-1}) & 1 - 2^{-2i-1} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$(1-2^{-2i-1})x \mp (2^{-i}+2^{-i-1})y$

2 shifts



$$\begin{array}{r} 10000000 \\ 00000010 \\ \hline 01111111 \end{array}$$

3 shifts

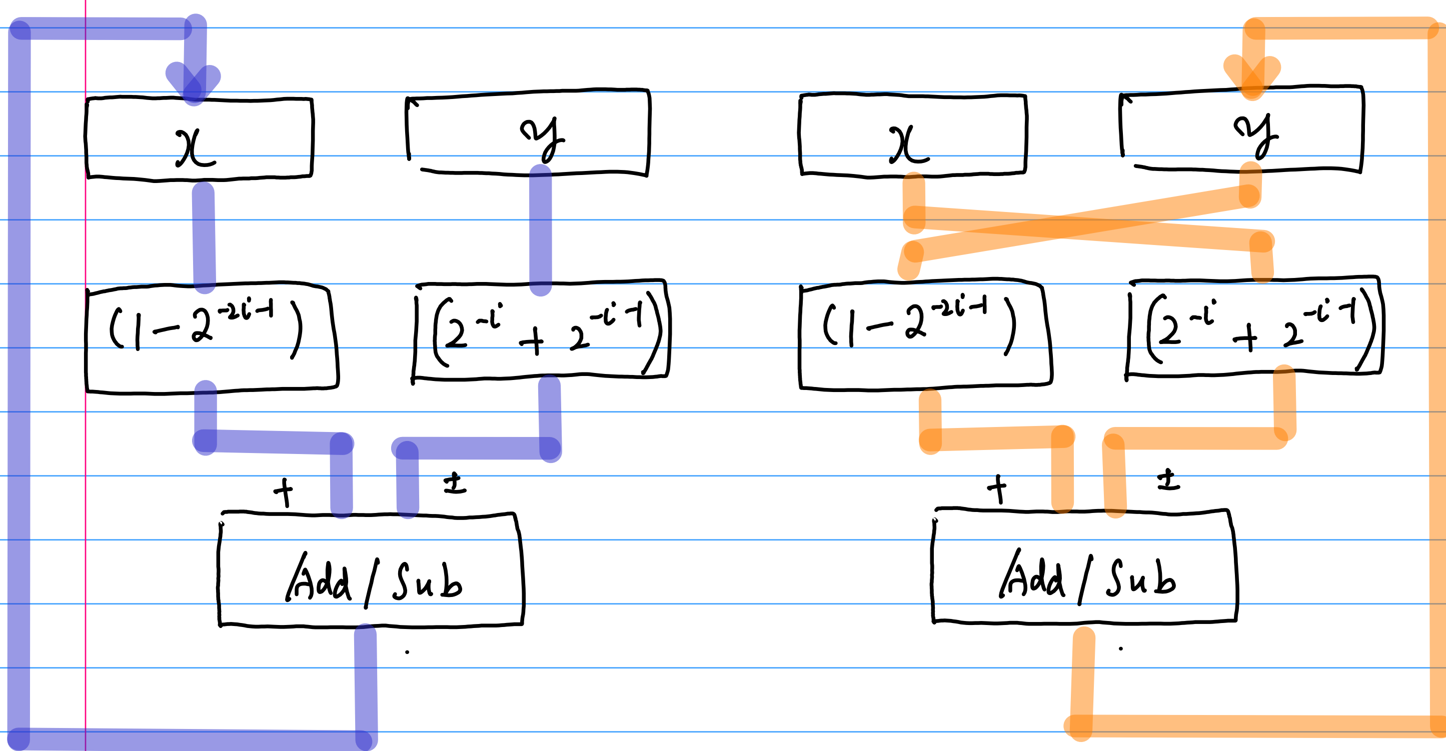
2 adders

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 - 2^{-2i-1} & \mp(2^{-i} + 2^{-i-1}) \\ \pm(2^{-i} + 2^{-i-1}) & 1 - 2^{-2i-1} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$= \begin{bmatrix} +\left(x - 2^{-2i-1}x\right) \mp \left(2^{-i}y + 2^{-i-1}y\right) \\ +\left(y - 2^{-2i-1}y\right) \pm \left(2^{-i}x + 2^{-i-1}x\right) \end{bmatrix}$$

Serialization

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} +\left(x - 2^{-2i-1}x\right) \mp \left(2^{-i}y + 2^{-i-1}y\right) \\ +\left(y - 2^{-2i-1}y\right) \pm \left(2^{-i}x + 2^{-i-1}x\right) \end{bmatrix}$$



2 shifters
3 adders

2 shifters
3 adders.

4 shifters
6 adders

may be significant

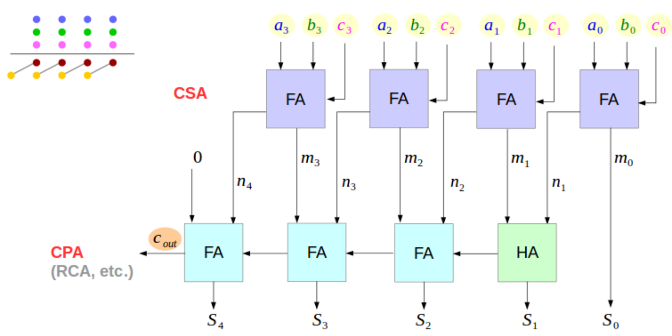
$\boxed{\text{1 T shifter}}$
2 T adder

4 32-bit binary number addition

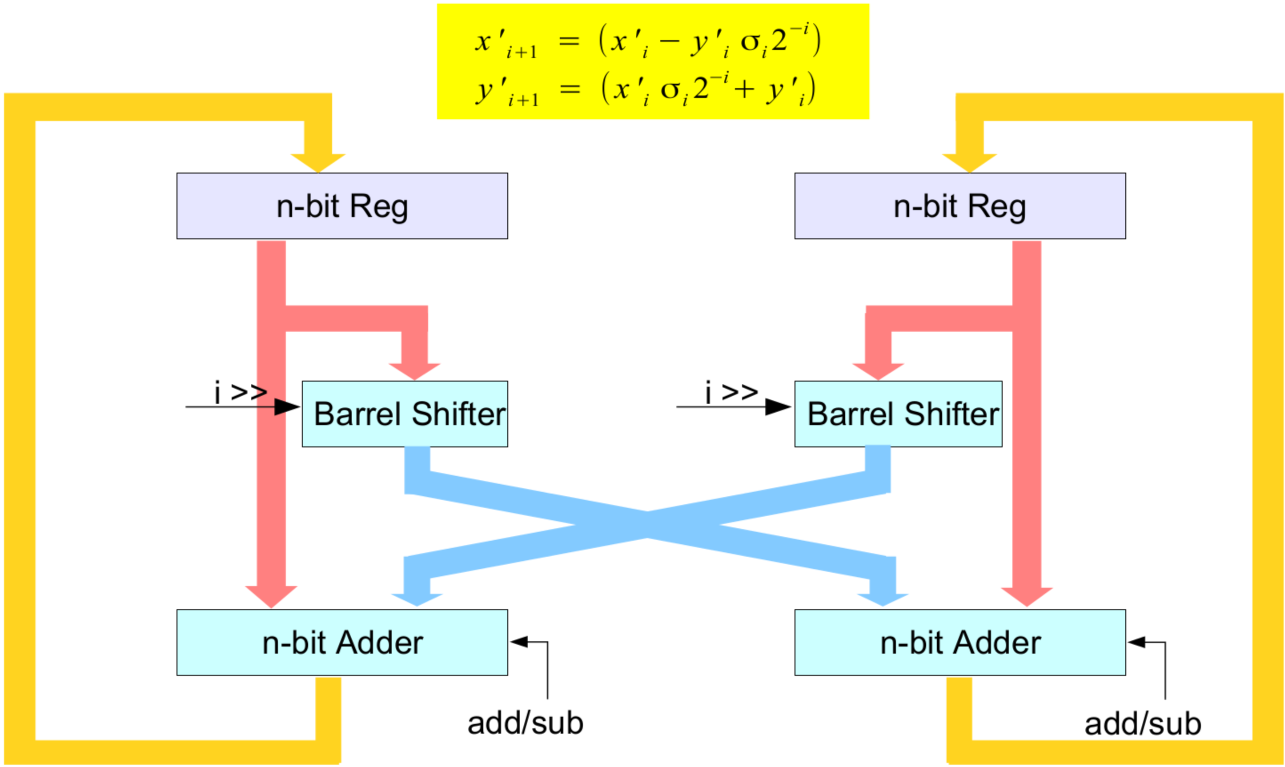
CSA (Carry Save Adder) → faster with the same number of FA's
CLA (Carry Lookahead Adder)

32-bit shifter
→ 5 level
→ more delay than FA adder

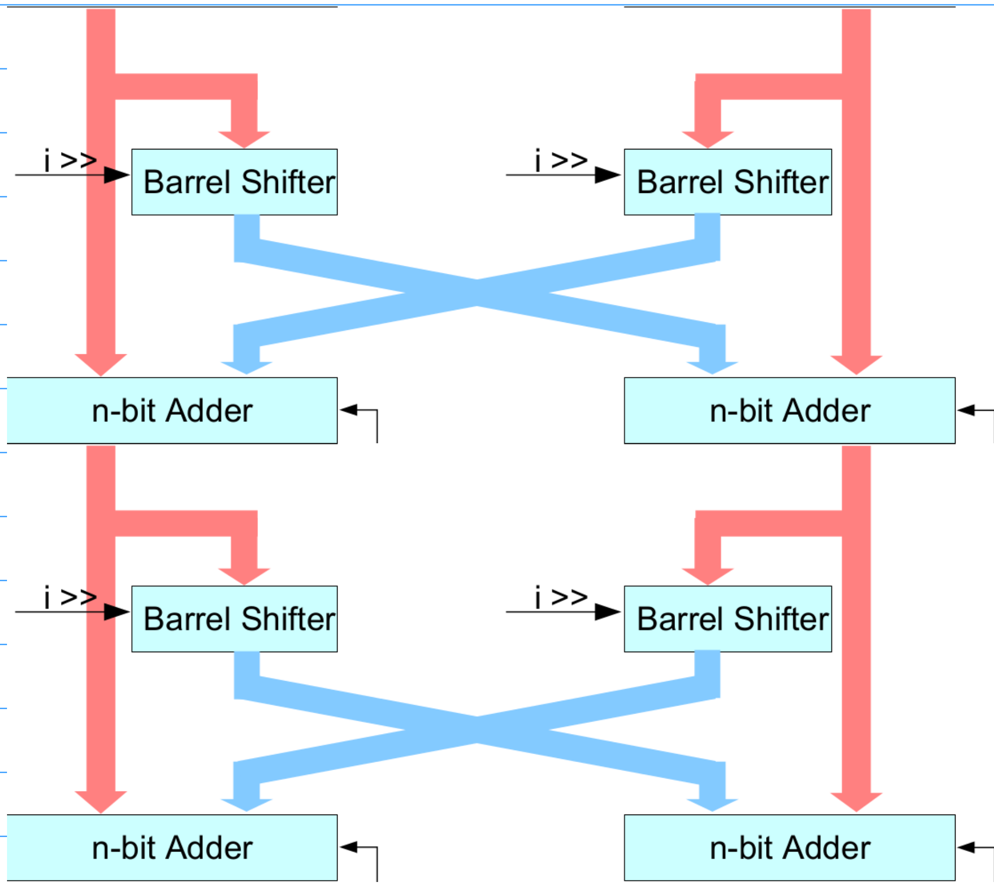
CSA + RCA



regular cordic



Loop Unrolled CORDIC

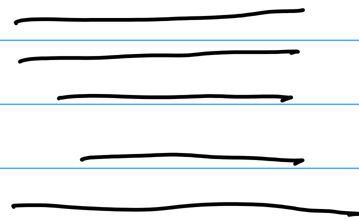


4 shifters   2. $T_{shifter}$

4 adders   2. T_{adder}

Maybe cos, sin values are also quantized along y axis.

shift-and-add operation



(%i5) A1;

(%o5) $\begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}$

(%i6) A2;

(%o6) $\begin{bmatrix} 1 & -m^2 \\ m^2 & 1 \end{bmatrix}$

(%i7) A3;

(%o7) $\begin{bmatrix} 1 & -m^3 \\ m^3 & 1 \end{bmatrix}$

(%i8) A4;

(%o8) $\begin{bmatrix} 1 & -m^4 \\ m^4 & 1 \end{bmatrix}$

2-steps

(%i9) A1.A2;

(%o9) $\begin{bmatrix} 1-m^3 & -m^2-m \\ m^2+m & 1-m^3 \end{bmatrix}$

3-steps

(%i11) ratsimp(A1.A2.A3);

(%o11) $\begin{bmatrix} -m^5-m^4-m^3+1 & m^6-m^3-m^2-m \\ -m^6+m^3+m^2+m & -m^5-m^4-m^3+1 \end{bmatrix}$

4-steps

(%i12) ratsimp(A1.A2.A3.A4);

(%o12) $\begin{bmatrix} m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 & m^9+m^8+m^7+m^6-m^4-m^3-m^2-m \\ -m^9-m^8-m^7-m^6+m^4+m^3+m^2+m & m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 \end{bmatrix}$

2-steps

(%i10) A2.A1;

(%o10) $\begin{bmatrix} 1-m^3 & -m^2-m \\ m^2+m & 1-m^3 \end{bmatrix}$

3-steps

(%i12) ratsimp(A3.A2.A1);

(%o12) $\begin{bmatrix} -m^5-m^4-m^3+1 & m^6-m^3-m^2-m \\ -m^6+m^3+m^2+m & -m^5-m^4-m^3+1 \end{bmatrix}$

4-steps

(%i13) ratsimp(A4.A3.A2.A1);

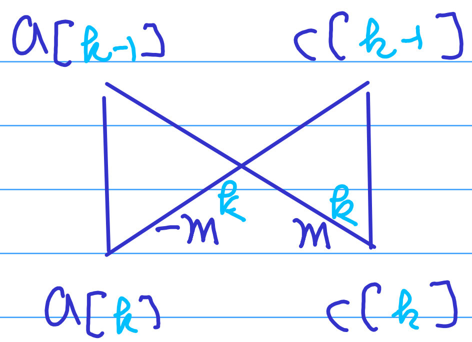
(%o13) $\begin{bmatrix} m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 & m^9+m^8+m^7+m^6-m^4-m^3-m^2-m \\ -m^9-m^8-m^7-m^6+m^4+m^3+m^2+m & m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 \end{bmatrix}$

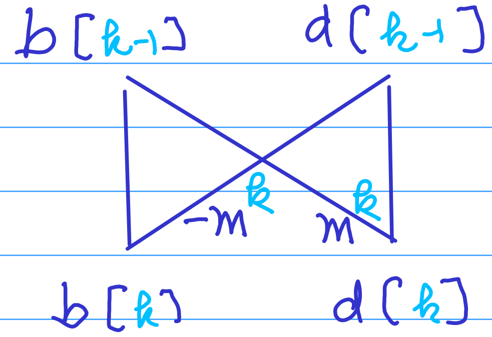
$$A[k] = \begin{bmatrix} 1 & -m^k \\ m^k & 1 \end{bmatrix} A[k-1]$$

$$= \begin{bmatrix} 1 & -m^k \\ m^k & 1 \end{bmatrix} \begin{bmatrix} 1 & -m^{k-1} \\ m^{k-1} & 1 \end{bmatrix} \cdots \begin{bmatrix} 1 & -m^1 \\ m^1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a[k] & b[k] \\ c[k] & d[k] \end{bmatrix} = \begin{bmatrix} 1 & -m^k \\ m^k & 1 \end{bmatrix} \begin{bmatrix} a[k-1] & b[k-1] \\ c[k-1] & d[k-1] \end{bmatrix}$$

$$= \begin{bmatrix} a[k-1] - m^k c[k-1] & b[k-1] - m^k d[k-1] \\ m^k a[k-1] + c[k-1] & m^k b[k-1] + d[k-1] \end{bmatrix}$$



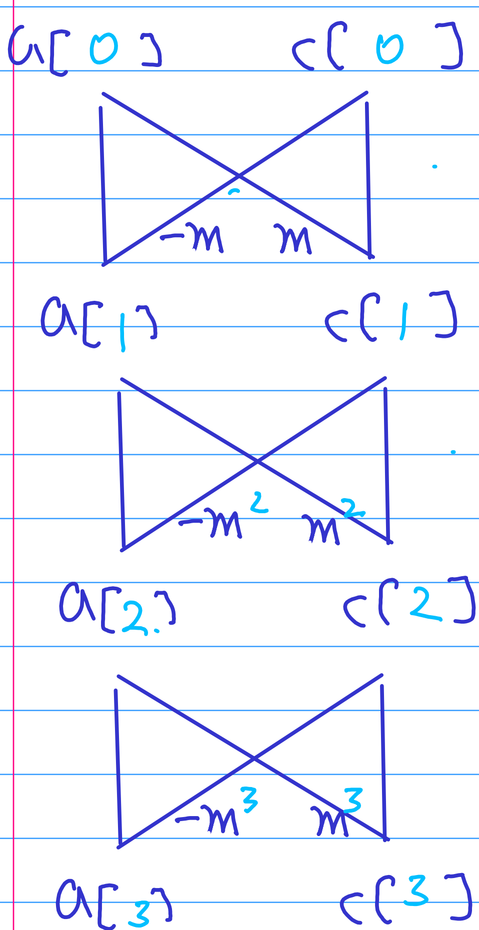


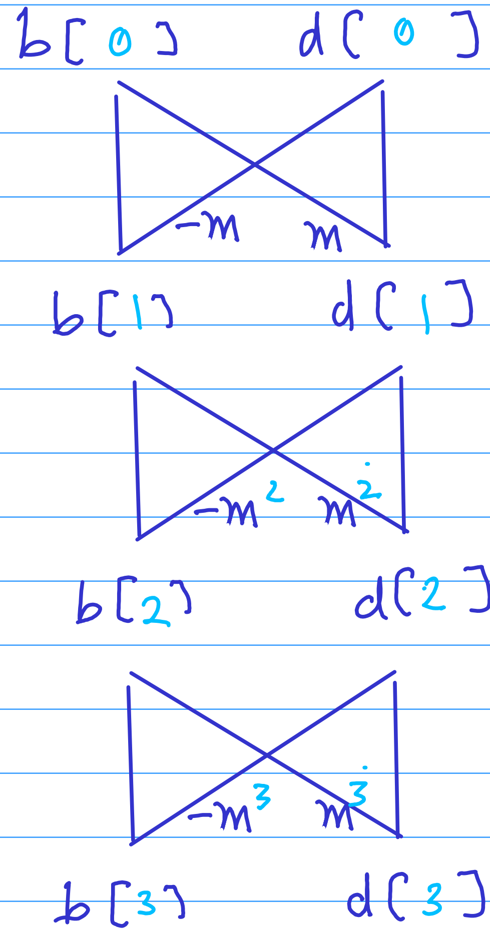
$a[k] = a[k-1] - m^k c[k-1]$

$c[k] = m^k a[k-1] + c[k-1]$

$b[k] = b[k-1] - m^k d[k-1]$

$d[k] = m^k b[k-1] + d[k-1]$





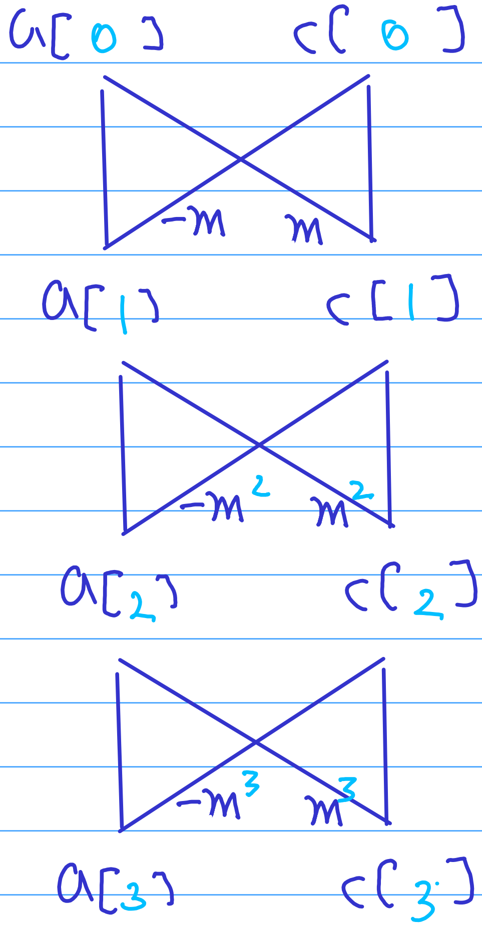
$a[k] = a[k-1] - m^k c[k-1]$

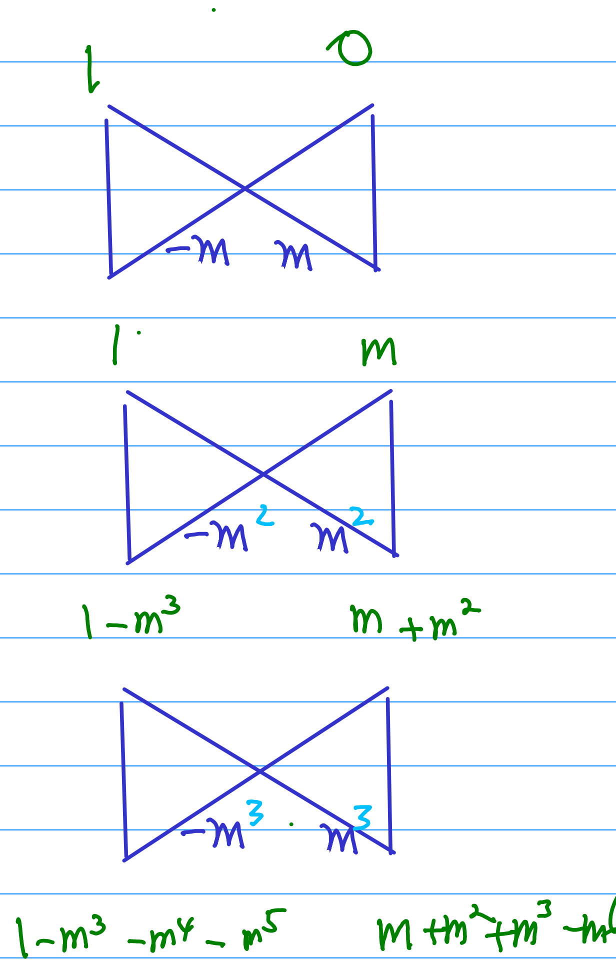
$c[k] = m^k a[k-1] + c[k-1]$

$b[k] = b[k-1] - m^k d[k-1]$

$d[k] = m^k b[k-1] + d[k-1]$

$$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \qquad \begin{bmatrix} & \end{bmatrix}$$





$$= \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix} \begin{bmatrix} a[0] & b[0] \\ c[0] & d[0] \end{bmatrix}$$

$$\begin{bmatrix} -m^5-m^4-m^3+1 & m^6-m^3-m^2-m \\ -m^6+m^3+m^2+m & -m^5-m^4-m^3+1 \end{bmatrix}$$

```
(%i51) for i : 1 thru 10 do
       (A[i] : matrix([1, -m^i], [m^i, 1]),
       print("A[", i, "]= ", A[i]) )$
```

$A[\,1\,]=\begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}$

$A[\,2\,]=\begin{bmatrix} 1 & -m^2 \\ m^2 & 1 \end{bmatrix}$

$A[\,3\,]=\begin{bmatrix} 1 & -m^3 \\ m^3 & 1 \end{bmatrix}$

$A[\,4\,]=\begin{bmatrix} 1 & -m^4 \\ m^4 & 1 \end{bmatrix}$

$A[\,5\,]=\begin{bmatrix} 1 & -m^5 \\ m^5 & 1 \end{bmatrix}$

$A[\,6\,]=\begin{bmatrix} 1 & -m^6 \\ m^6 & 1 \end{bmatrix}$

$A[\,7\,]=\begin{bmatrix} 1 & -m^7 \\ m^7 & 1 \end{bmatrix}$

$A[\,8\,]=\begin{bmatrix} 1 & -m^8 \\ m^8 & 1 \end{bmatrix}$

$A[\,9\,]=\begin{bmatrix} 1 & -m^9 \\ m^9 & 1 \end{bmatrix}$

$A[\,10\,]=\begin{bmatrix} 1 & -m^{10} \\ m^{10} & 1 \end{bmatrix}$

$\boxed{\text{without } \sigma_i}$

always $\underline{\sigma_i = 1}$

```
(%i48) for i : 1 thru 5 do (
       T : I,

       (for j : i thru 1 step -1 do
           T : (A[j].T) ),
       print(expand(T) ) );
```

Step

1

$$\begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}$$

2 1

$$\begin{bmatrix} 1-m^3 & -m^2-m \\ m^2+m & 1-m^3 \end{bmatrix}$$

3 2 1

$$\begin{bmatrix} -m^5-m^4-m^3+1 & m^6-m^3-m^2-m \\ -m^6+m^3+m^2+m & -m^5-m^4-m^3+1 \end{bmatrix}$$

4 3 2 1

$$\begin{bmatrix} m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 & m^9+m^8+m^7+m^6-m^4-m^3-m^2-m \\ -m^9-m^8-m^7-m^6+m^4+m^3+m^2+m & m^{10}-m^7-m^6-2\,m^5-m^4-m^3+1 \end{bmatrix}$$

5 4 3 2 1

$$\begin{bmatrix} m^{14}+m^{13}+m^{12}+m^{11}+m^{10}-m^9-m^8-2\,m^7-2\,m^6-2\,m^5-m^4-m^3+1 & -m^{15}+m^{12}+m^{11}+2\,m^{10}+2\,m^9+2\,m^8+m^7+m^6-m^5-m^4-m^3-m^2- \\ m^{15}-m^{12}-m^{11}-2\,m^{10}-2\,m^9-2\,m^8-m^7-m^6+m^5+m^4+m^3+m^2+m & m^{14}+m^{13}+m^{12}+m^{11}+m^{10}-m^9-m^8-2\,m^7-2\,m^6-2\,m^5-m^4-m^3+ \end{bmatrix}$$

(%o48) *done*

Octave cordic

```
for j = 1 : n
  if ( theta < 0.0 )
    sigma = -1.0;
  else
    sigma = 1.0;
  end

  factor = sigma * poweroftwo;

  R = [ 1.0,    -factor;...
        factor, 1.0       ];

  v = R * v;

  xn = [xn; v(1)];
  yn = [yn; v(2)];
  zn = [zn; theta];

%
% Update the remaining angle.
%
  theta = theta - sigma * angle;

  poweroftwo = poweroftwo / 2.0;
```

s_i

m^i

$\boxed{\sigma_i = \pm 1}$

(%i52)
```
for i : 1 thru 10 do
(A[i] : matrix([1, -s[i]*m^i], [s[i]*m^i, 1]),
print("A[", i, "]= ", A[i]) )$
```

Step

① $A[\,1\,] = \begin{bmatrix} 1 & -s_1\,m \\ s_1\,m & 1 \end{bmatrix}$

② $A[\,2\,] = \begin{bmatrix} 1 & -s_2\,m^2 \\ s_2\,m^2 & 1 \end{bmatrix}$

③ $A[\,3\,] = \begin{bmatrix} 1 & -s_3\,m^3 \\ s_3\,m^3 & 1 \end{bmatrix}$

④ $A[\,4\,] = \begin{bmatrix} 1 & -s_4\,m^4 \\ s_4\,m^4 & 1 \end{bmatrix}$

⑤ $A[\,5\,] = \begin{bmatrix} 1 & -s_5\,m^5 \\ s_5\,m^5 & 1 \end{bmatrix}$

⑥ $A[\,6\,] = \begin{bmatrix} 1 & -s_6\,m^6 \\ s_6\,m^6 & 1 \end{bmatrix}$

⑦ $A[\,7\,] = \begin{bmatrix} 1 & -s_7\,m^7 \\ s_7\,m^7 & 1 \end{bmatrix}$

⑧ $A[\,8\,] = \begin{bmatrix} 1 & -s_8\,m^8 \\ s_8\,m^8 & 1 \end{bmatrix}$

⑨ $A[\,9\,] = \begin{bmatrix} 1 & -s_9\,m^9 \\ s_9\,m^9 & 1 \end{bmatrix}$

⑩ $A[\,10\,] = \begin{bmatrix} 1 & -s_{10}\,m^{10} \\ s_{10}\,m^{10} & 1 \end{bmatrix}$

```
(%i57) for i : 1 thru 4 do (
         T : I,

         (for j : i thru 1 step -1 do
             T : (A[j].T) ),
         print(expand(T) ) );
```

1 $$\begin{bmatrix} 1 & -s_1 m \\ s_1 m & 1 \end{bmatrix}$$

2 1 $$\begin{bmatrix} 1-s_1 s_2 m^3 & -s_2 m^2-s_1 m \\ s_2 m^2+s_1 m & 1-s_1 s_2 m^3 \end{bmatrix}$$

3 2 1 $$\begin{bmatrix} -s_2 s_3 m^5-s_1 s_3 m^4-s_1 s_2 m^3+1 & s_1 s_2 s_3 m^6-s_3 m^3-s_2 m^2-s_1 m \\ -s_1 s_2 s_3 m^6+s_3 m^3+s_2 m^2+s_1 m & -s_2 s_3 m^5-s_1 s_3 m^4-s_1 s_2 m^3+1 \end{bmatrix}$$

4 3 2 1 $$\begin{bmatrix} s_1 s_2 s_3 s_4 m^{10}-s_3 s_4 m^7-s_2 s_4 m^6-s_1 s_4 m^5-s_2 s_3 m^5-s_1 s_3 m^4-s_1 s_2 m^3+1 & s_2 s_3 s_4 m^9+s_1 s_3 s_4 m^8+s_1 s_2 s_4 m^7+s_1 s_2 s_3 m^6-s_4 m^4-s_3 m^3-s_2 m^2-s_1 \\ -s_2 s_3 s_4 m^9-s_1 s_3 s_4 m^8-s_1 s_2 s_4 m^7-s_1 s_2 s_3 m^6+s_4 m^4+s_3 m^3+s_2 m^2+s_1 m & s_1 s_2 s_3 s_4 m^{10}-s_3 s_4 m^7-s_2 s_4 m^6-s_1 s_4 m^5-s_2 s_3 m^5-s_1 s_3 m^4-s_1 s_2 m^3+ \end{bmatrix}$$

(%o57) *done*

CC. Kao's $\begin{bmatrix} P_{xi}, & -P_{yi} \\ P_{yi} & P_{xi} \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

16-bit

16 15 14 ...... 2 1 → precompute → LUT.

Barrel shifter → actually <u>multiplier</u>

→ double steps — Quad Tree

tripple steps — Octa Tree

for a given iteration number ($nIter = 10$)

$\boxed{\text{precomputation}}$

$m^i \rightarrow$ binary digit

$\underline{a_1 a_2 a_3}$

$\Downarrow$

+1 or -1

* similar

Booth Recoding

no iteration on the scaling process

array adder, multiplier

- Table Lookup approach
- Coarse - Fine

Let's see the quantized effect first.

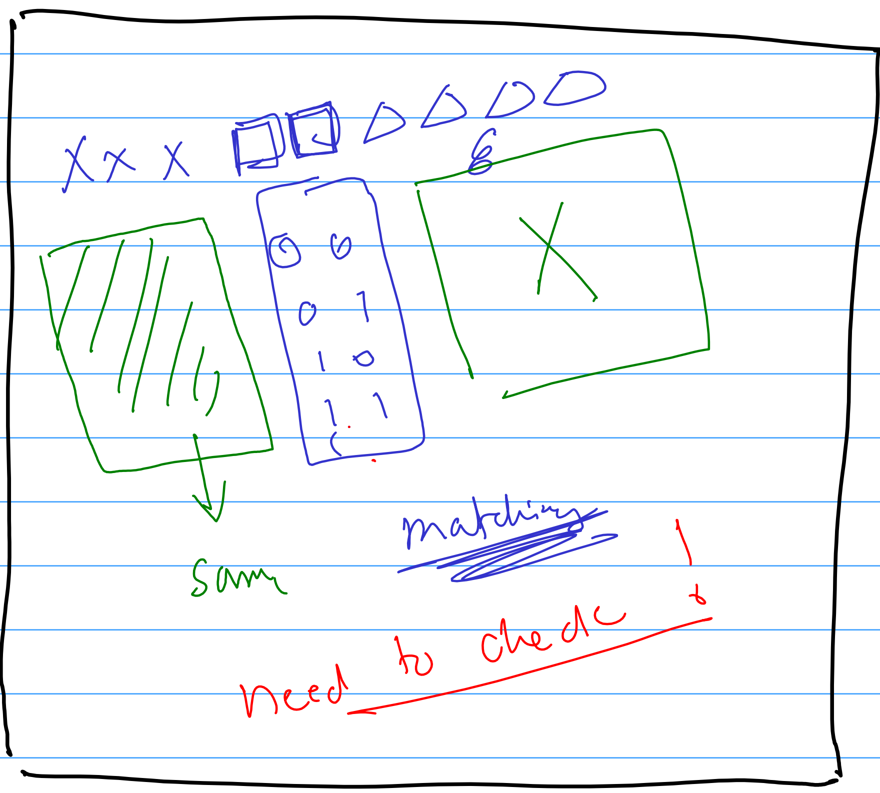
at a certain iteration,

we can find out which bits are added or subtracted from the current accumulated value.

↳ cos or sin values

add / subtract operation is determined by a coefficients multiplication

σ_i's



```
(%i32) batch("/home/young/MyWork/7.cordic_accuracy/2.wxMaxima/TestK.wxm");
read and interpret file: #p/home/young/MyWork/7.cordic_accuracy/2.wxMaxima/TestK.wxm
(%i33) for i from 0 thru 20 do A[i]:matrix([1,-s[i]*m^i],[s[i]*m^i,1])
(%i34) I:matrix([1,0],[0,1])
```

(%o34) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

```
(%i35) for i from 10 thru 20 do
        (T:I,for j from i step -1 thru i-2 do T:A[j] . T,B[i]:expand(T),
         print(B[i]))
```

10 9 8 $\begin{bmatrix} -s_9 s_{10} m^{19}-s_8 s_{10} m^{18}-s_8 s_9 m^{17}+1 & s_8 s_9 s_{10} m^{27}-s_{10} m^{10}-s_9 m^9-s_8 m^8 \\ -s_8 s_9 s_{10} m^{27}+s_{10} m^{10}+s_9 m^9+s_8 m^8 & -s_9 s_{10} m^{19}-s_8 s_{10} m^{18}-s_8 s_9 m^{17}+1 \end{bmatrix}$

11 10 9 $\begin{bmatrix} -s_{10} s_{11} m^{21}-s_9 s_{11} m^{20}-s_9 s_{10} m^{19}+1 & s_9 s_{10} s_{11} m^{30}-s_{11} m^{11}-s_{10} m^{10}-s_9 m^9 \\ -s_9 s_{10} s_{11} m^{30}+s_{11} m^{11}+s_{10} m^{10}+s_9 m^9 & -s_{10} s_{11} m^{21}-s_9 s_{11} m^{20}-s_9 s_{10} m^{19}+1 \end{bmatrix}$

12 11 10 $\begin{bmatrix} -s_{11} s_{12} m^{23}-s_{10} s_{12} m^{22}-s_{10} s_{11} m^{21}+1 & s_{10} s_{11} s_{12} m^{33}-s_{12} m^{12}-s_{11} m^{11}-s_{10} m^{10} \\ -s_{10} s_{11} s_{12} m^{33}+s_{12} m^{12}+s_{11} m^{11}+s_{10} m^{10} & -s_{11} s_{12} m^{23}-s_{10} s_{12} m^{22}-s_{10} s_{11} m^{21}+1 \end{bmatrix}$

13 12 11 $\begin{bmatrix} -s_{12} s_{13} m^{25}-s_{11} s_{13} m^{24}-s_{11} s_{12} m^{23}+1 & s_{11} s_{12} s_{13} m^{36}-s_{13} m^{13}-s_{12} m^{12}-s_{11} m^{11} \\ -s_{11} s_{12} s_{13} m^{36}+s_{13} m^{13}+s_{12} m^{12}+s_{11} m^{11} & -s_{12} s_{13} m^{25}-s_{11} s_{13} m^{24}-s_{11} s_{12} m^{23}+1 \end{bmatrix}$

14 13 12 $\begin{bmatrix} -s_{13} s_{14} m^{27}-s_{12} s_{14} m^{26}-s_{12} s_{13} m^{25}+1 & s_{12} s_{13} s_{14} m^{39}-s_{14} m^{14}-s_{13} m^{13}-s_{12} m^{12} \\ -s_{12} s_{13} s_{14} m^{39}+s_{14} m^{14}+s_{13} m^{13}+s_{12} m^{12} & -s_{13} s_{14} m^{27}-s_{12} s_{14} m^{26}-s_{12} s_{13} m^{25}+1 \end{bmatrix}$

15 14 13 $\begin{bmatrix} -s_{14} s_{15} m^{29}-s_{13} s_{15} m^{28}-s_{13} s_{14} m^{27}+1 & s_{13} s_{14} s_{15} m^{42}-s_{15} m^{15}-s_{14} m^{14}-s_{13} m^{13} \\ -s_{13} s_{14} s_{15} m^{42}+s_{15} m^{15}+s_{14} m^{14}+s_{13} m^{13} & -s_{14} s_{15} m^{29}-s_{13} s_{15} m^{28}-s_{13} s_{14} m^{27}+1 \end{bmatrix}$

16 15 14 $\begin{bmatrix} -s_{15} s_{16} m^{31}-s_{14} s_{16} m^{30}-s_{14} s_{15} m^{29}+1 & s_{14} s_{15} s_{16} m^{45}-s_{16} m^{16}-s_{15} m^{15}-s_{14} m^{14} \\ -s_{14} s_{15} s_{16} m^{45}+s_{16} m^{16}+s_{15} m^{15}+s_{14} m^{14} & -s_{15} s_{16} m^{31}-s_{14} s_{16} m^{30}-s_{14} s_{15} m^{29}+1 \end{bmatrix}$

17 16 15 $\begin{bmatrix} -s_{16} s_{17} m^{33}-s_{15} s_{17} m^{32}-s_{15} s_{16} m^{31}+1 & s_{15} s_{16} s_{17} m^{48}-s_{17} m^{17}-s_{16} m^{16}-s_{15} m^{15} \\ -s_{15} s_{16} s_{17} m^{48}+s_{17} m^{17}+s_{16} m^{16}+s_{15} m^{15} & -s_{16} s_{17} m^{33}-s_{15} s_{17} m^{32}-s_{15} s_{16} m^{31}+1 \end{bmatrix}$

18 17 16 $\begin{bmatrix} -s_{17} s_{18} m^{35}-s_{16} s_{18} m^{34}-s_{16} s_{17} m^{33}+1 & s_{16} s_{17} s_{18} m^{51}-s_{18} m^{18}-s_{17} m^{17}-s_{16} m^{16} \\ -s_{16} s_{17} s_{18} m^{51}+s_{18} m^{18}+s_{17} m^{17}+s_{16} m^{16} & -s_{17} s_{18} m^{35}-s_{16} s_{18} m^{34}-s_{16} s_{17} m^{33}+1 \end{bmatrix}$

19 18 17 $\begin{bmatrix} -s_{18} s_{19} m^{37}-s_{17} s_{19} m^{36}-s_{17} s_{18} m^{35}+1 & s_{17} s_{18} s_{19} m^{54}-s_{19} m^{19}-s_{18} m^{18}-s_{17} m^{17} \\ -s_{17} s_{18} s_{19} m^{54}+s_{19} m^{19}+s_{18} m^{18}+s_{17} m^{17} & -s_{18} s_{19} m^{37}-s_{17} s_{19} m^{36}-s_{17} s_{18} m^{35}+1 \end{bmatrix}$

20 19 18 $\begin{bmatrix} -s_{19} s_{20} m^{39}-s_{18} s_{20} m^{38}-s_{18} s_{19} m^{37}+1 & s_{18} s_{19} s_{20} m^{57}-s_{20} m^{20}-s_{19} m^{19}-s_{18} m^{18} \\ -s_{18} s_{19} s_{20} m^{57}+s_{20} m^{20}+s_{19} m^{19}+s_{18} m^{18} & -s_{19} s_{20} m^{39}-s_{18} s_{20} m^{38}-s_{18} s_{19} m^{37}+1 \end{bmatrix}$

```
(%o35) /home/young/MyWork/7.cordic_accuracy/2.wxMaxima/TestK.wxm
```

$s_8 s_9 s_{10} m^{27} - s_{10} m^{10} - s_9 m^9 - s_8 m^8$

$-s_9 s_{10} m^{19} - s_8 s_{10} m^{18} - s_8 s_9 m^{17} + 1$

n=10	$s_8 s_9 s_{10} m^{27} - s_{10} m^{10} - s_9 m^9 - s_8 m^8$				$-s_9 s_{10} m^{19} - s_8 s_{10} m^{18} - s_8 s_9 m^{17} + 1$		
11	30	11	10	9	21	20	19
12	33	12	11	10	23	22	21
13	36	13	12	11	25	24	23
14	39	14	13	12	27	26	25
15	42	15	14	13	29	28	27
16	45	16	15	14	31	30	29
17	48	17	16	15	33	32	31
18	51	18	17	16	35	34	33
19	54		18	17	37	36	35
20	57			18	39	38	37
	$3(n-1)$	n	$n-1$	$n-2$	$2n-1$	$2n-2$	$2n-3$

2 1 0

$$\begin{bmatrix} -s_1 s_2 m^3 - s_0 s_2 m^2 - s_0 s_1 m + 1 & s_0 s_1 s_2 m^3 - s_2 m^2 - s_1 m - s_0 \\ -s_0 s_1 s_2 m^3 + s_2 m^2 + s_1 m + s_0 & -s_1 s_2 m^3 - s_0 s_2 m^2 - s_0 s_1 m + 1 \end{bmatrix}$$

3 2 1

$$\begin{bmatrix} -s_2 s_3 m^5 - s_1 s_3 m^4 - s_1 s_2 m^3 + 1 & s_1 s_2 s_3 m^6 - s_3 m^3 - s_2 m^2 - s_1 m \\ -s_1 s_2 s_3 m^6 + s_3 m^3 + s_2 m^2 + s_1 m & -s_2 s_3 m^5 - s_1 s_3 m^4 - s_1 s_2 m^3 + 1 \end{bmatrix}$$

4 3 2

$$\begin{bmatrix} -s_3 s_4 m^7 - s_2 s_4 m^6 - s_2 s_3 m^5 + 1 & s_2 s_3 s_4 m^9 - s_4 m^4 - s_3 m^3 - s_2 m^2 \\ -s_2 s_3 s_4 m^9 + s_4 m^4 + s_3 m^3 + s_2 m^2 & -s_3 s_4 m^7 - s_2 s_4 m^6 - s_2 s_3 m^5 + 1 \end{bmatrix}$$

5 4 3

$$\begin{bmatrix} -s_4 s_5 m^9 - s_3 s_5 m^8 - s_3 s_4 m^7 + 1 & s_3 s_4 s_5 m^{12} - s_5 m^5 - s_4 m^4 - s_3 m^3 \\ -s_3 s_4 s_5 m^{12} + s_5 m^5 + s_4 m^4 + s_3 m^3 & -s_4 s_5 m^9 - s_3 s_5 m^8 - s_3 s_4 m^7 + 1 \end{bmatrix}$$

6 5 4

$$\begin{bmatrix} -s_5 s_6 m^{11} - s_4 s_6 m^{10} - s_4 s_5 m^9 + 1 & s_4 s_5 s_6 m^{15} - s_6 m^6 - s_5 m^5 - s_4 m^4 \\ -s_4 s_5 s_6 m^{15} + s_6 m^6 + s_5 m^5 + s_4 m^4 & -s_5 s_6 m^{11} - s_4 s_6 m^{10} - s_4 s_5 m^9 + 1 \end{bmatrix}$$

7 6 5

$$\begin{bmatrix} -s_6 s_7 m^{13} - s_5 s_7 m^{12} - s_5 s_6 m^{11} + 1 & s_5 s_6 s_7 m^{18} - s_7 m^7 - s_6 m^6 - s_5 m^5 \\ -s_5 s_6 s_7 m^{18} + s_7 m^7 + s_6 m^6 + s_5 m^5 & -s_6 s_7 m^{13} - s_5 s_7 m^{12} - s_5 s_6 m^{11} + 1 \end{bmatrix}$$

8 7 6

$$\begin{bmatrix} -s_7 s_8 m^{15} - s_6 s_8 m^{14} - s_6 s_7 m^{13} + 1 & s_6 s_7 s_8 m^{21} - s_8 m^8 - s_7 m^7 - s_6 m^6 \\ -s_6 s_7 s_8 m^{21} + s_8 m^8 + s_7 m^7 + s_6 m^6 & -s_7 s_8 m^{15} - s_6 s_8 m^{14} - s_6 s_7 m^{13} + 1 \end{bmatrix}$$

9 8 7

$$\begin{bmatrix} -s_8 s_9 m^{17} - s_7 s_9 m^{16} - s_7 s_8 m^{15} + 1 & s_7 s_8 s_9 m^{24} - s_9 m^9 - s_8 m^8 - s_7 m^7 \\ -s_7 s_8 s_9 m^{24} + s_9 m^9 + s_8 m^8 + s_7 m^7 & -s_8 s_9 m^{17} - s_7 s_9 m^{16} - s_7 s_8 m^{15} + 1 \end{bmatrix}$$

8 7 $$\begin{bmatrix} 1-s_7 s_8 m^{15} & -s_8 m^8 - s_7 m^7 \\ s_8 m^8 + s_7 m^7 & 1-s_7 s_8 m^{15} \end{bmatrix}$$

9 8 $$\begin{bmatrix} 1-s_8 s_9 m^{17} & -s_9 m^9 - s_8 m^8 \\ s_9 m^9 + s_8 m^8 & 1-s_8 s_9 m^{17} \end{bmatrix}$$

10 9 $$\begin{bmatrix} 1-s_9 s_{10} m^{19} & -s_{10} m^{10} - s_9 m^9 \\ s_{10} m^{10} + s_9 m^9 & 1-s_9 s_{10} m^{19} \end{bmatrix}$$

11 10 $$\begin{bmatrix} 1-s_{10} s_{11} m^{21} & -s_{11} m^{11} - s_{10} m^{10} \\ s_{11} m^{11} + s_{10} m^{10} & 1-s_{10} s_{11} m^{21} \end{bmatrix}$$

12 11 $$\begin{bmatrix} 1-s_{11} s_{12} m^{23} & -s_{12} m^{12} - s_{11} m^{11} \\ s_{12} m^{12} + s_{11} m^{11} & 1-s_{11} s_{12} m^{23} \end{bmatrix}$$

13 12 $$\begin{bmatrix} 1-s_{12} s_{13} m^{25} & -s_{13} m^{13} - s_{12} m^{12} \\ s_{13} m^{13} + s_{12} m^{12} & 1-s_{12} s_{13} m^{25} \end{bmatrix}$$

14 13 $$\begin{bmatrix} 1-s_{13} s_{14} m^{27} & -s_{14} m^{14} - s_{13} m^{13} \\ s_{14} m^{14} + s_{13} m^{13} & 1-s_{13} s_{14} m^{27} \end{bmatrix}$$

15 14 $$\begin{bmatrix} 1-s_{14} s_{15} m^{29} & -s_{15} m^{15} - s_{14} m^{14} \\ s_{15} m^{15} + s_{14} m^{14} & 1-s_{14} s_{15} m^{29} \end{bmatrix}$$

16 15 $$\begin{bmatrix} 1-s_{15} s_{16} m^{31} & -s_{16} m^{16} - s_{15} m^{15} \\ s_{16} m^{16} + s_{15} m^{15} & 1-s_{15} s_{16} m^{31} \end{bmatrix}$$

17 16 $$\begin{bmatrix} 1-s_{16} s_{17} m^{33} & -s_{17} m^{17} - s_{16} m^{16} \\ s_{17} m^{17} + s_{16} m^{16} & 1-s_{16} s_{17} m^{33} \end{bmatrix}$$

18 17 $$\begin{bmatrix} 1-s_{17} s_{18} m^{35} & -s_{18} m^{18} - s_{17} m^{17} \\ s_{18} m^{18} + s_{17} m^{17} & 1-s_{17} s_{18} m^{35} \end{bmatrix}$$

19 18 $$\begin{bmatrix} 1-s_{18} s_{19} m^{37} & -s_{19} m^{19} - s_{18} m^{18} \\ s_{19} m^{19} + s_{18} m^{18} & 1-s_{18} s_{19} m^{37} \end{bmatrix}$$

20 19 $$\begin{bmatrix} 1-s_{19} s_{20} m^{39} & -s_{20} m^{20} - s_{19} m^{19} \\ s_{20} m^{20} + s_{19} m^{19} & 1-s_{19} s_{20} m^{39} \end{bmatrix}$$

```
for i : 0 thru 20 do
(A[i] : matrix([1, -s[i]*m^i], [s[i]*m^i, 1]) /* ,
print("A[", i, "]= ", A[i]) */
)$

I : matrix([1, 0], [0, 1]);

/*
for i : 0 thru 10 do (
  T : I,
  (for j : i thru 0 step -1 do   T : (A[j].T) ),
  B[i] : expand(T) /*,
     print(B[i])  */
                                   i, i-1, i-2 ...., 0
)$
*/


/*
for i : 2 thru 20 do (
  T : I,
  (for j : i thru i-2 step -1 do   T : (A[j].T) ),
                                   i, i-1, i-2
  B[i] : expand(T)  ,
     print(B[i])
)$
*/

for i : 2 thru 20 do (
  T : I,
  (for j : i thru i-1 step -1 do   T : (A[j].T) ),
                                   i, i-1
  B[i] : expand(T)  ,
     print(B[i])
)$
```

$$\begin{pmatrix} 1 & -\left(\frac{1}{2}\right)^{i+1} d_{i+1} \\ \left(\frac{1}{2}\right)^{i+1} d_{i+1} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\left(\frac{1}{2}\right)^{i} d_{i} \\ \left(\frac{1}{2}\right)^{i} d_{i} & 1 \end{pmatrix}$$

```
(%i25) mm: m2.m1;
```

(%o25) $$\begin{pmatrix} 1-2^{-2\,i-1}\,d_i\,d_{i+1} & -2^{-i-1}\,d_{i+1}-\frac{d_i}{2^i} \\ 2^{-i-1}\,d_{i+1}+\frac{d_i}{2^i} & 1-2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$$

```
(%i26) m2;
```

(%o26) $$\begin{pmatrix} 1 & -2^{-i-1}\,d_{i+1} \\ 2^{-i-1}\,d_{i+1} & 1 \end{pmatrix}$$

```
(%i27) m1;
```

(%o27) $$\begin{pmatrix} 1 & -\frac{d_i}{2^i} \\ \frac{d_i}{2^i} & 1 \end{pmatrix}$$

```
(%i28) mm;
```

(%o28) $$\begin{pmatrix} 1-2^{-2\,i-1}\,d_i\,d_{i+1} & -2^{-i-1}\,d_{i+1}-\frac{d_i}{2^i} \\ 2^{-i-1}\,d_{i+1}+\frac{d_i}{2^i} & 1-2^{-2\,i-1}\,d_i\,d_{i+1} \end{pmatrix}$$

```
(%i29) ratsimp(matrix([1-2^(-2*i-1)*d[i]*d[i+1],-2^(-i-1)*d[i+1]-d[i]/2^i],[2^(-i-1)*d[i+1]+
```

(%o29) $$\begin{pmatrix} -2^{-2\,i-1}\,\left(d_i\,d_{i+1}-2^{2\,i+1}\right) & -2^{-i-1}\,\left(d_{i+1}+2\,d_i\right) \\ 2^{-i-1}\,\left(d_{i+1}+2\,d_i\right) & -2^{-2\,i-1}\,\left(d_i\,d_{i+1}-2^{2\,i+1}\right) \end{pmatrix}$$

```
(%i30) 2^(i+1) * mm;
```

(%o30) $$\begin{pmatrix} 2^{i+1}\,\left(1-2^{-2\,i-1}\,d_i\,d_{i+1}\right) & 2^{i+1}\,\left(-2^{-i-1}\,d_{i+1}-\frac{d_i}{2^i}\right) \\ 2^{i+1}\,\left(2^{-i-1}\,d_{i+1}+\frac{d_i}{2^i}\right) & 2^{i+1}\,\left(1-2^{-2\,i-1}\,d_i\,d_{i+1}\right) \end{pmatrix}$$

```
(%i31) expand(matrix([2^(i+1)*(1-2^(-2*i-1)*d[i]*d[i+1]),2^(i+1)*(-2^(-i-1)*d[i+1]-d[i]/2^i)
```

(%o31) $$\begin{pmatrix} 2^{i+1}-\frac{d_i\,d_{i+1}}{2^i} & -d_{i+1}-2\,d_i \\ d_{i+1}+2\,d_i & 2^{i+1}-\frac{d_i\,d_{i+1}}{2^i} \end{pmatrix}$$

2-step look ahead operation

$$\begin{pmatrix} 1 & -\left(\frac{1}{2}\right)^{i+1} d_{i+1} \\ \left(\frac{1}{2}\right)^{i+1} d_{i+1} & 1 \end{pmatrix} \begin{pmatrix} 1 & -\left(\frac{1}{2}\right)^{i} d_{i} \\ \left(\frac{1}{2}\right)^{i} d_{i} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - \left(\frac{1}{2}\right)^{i+1} \left(\frac{1}{2}\right)^{i} d_{i+1} d_i & -\left(\frac{1}{2}\right)^{i} d_i - \left(\frac{1}{2}\right)^{i+1} d_{i+1} \\ \left(\frac{1}{2}\right)^{i+1} d_{i+1} + \left(\frac{1}{2}\right)^{i} d_i & -\left(\frac{1}{2}\right)^{i+1} \left(\frac{1}{2}\right)^{i} d_{i+1} d_i + 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 - m^{i+1} m^{i} d_{i+1} d_i & -m^{i+1} d_{i+1} - m^{i} d_i \\ m^{i+1} d_{i+1} + m^{i} d_i & -m^{i+1} m^{i} d_{i+1} d_i + 1 \end{pmatrix}$$

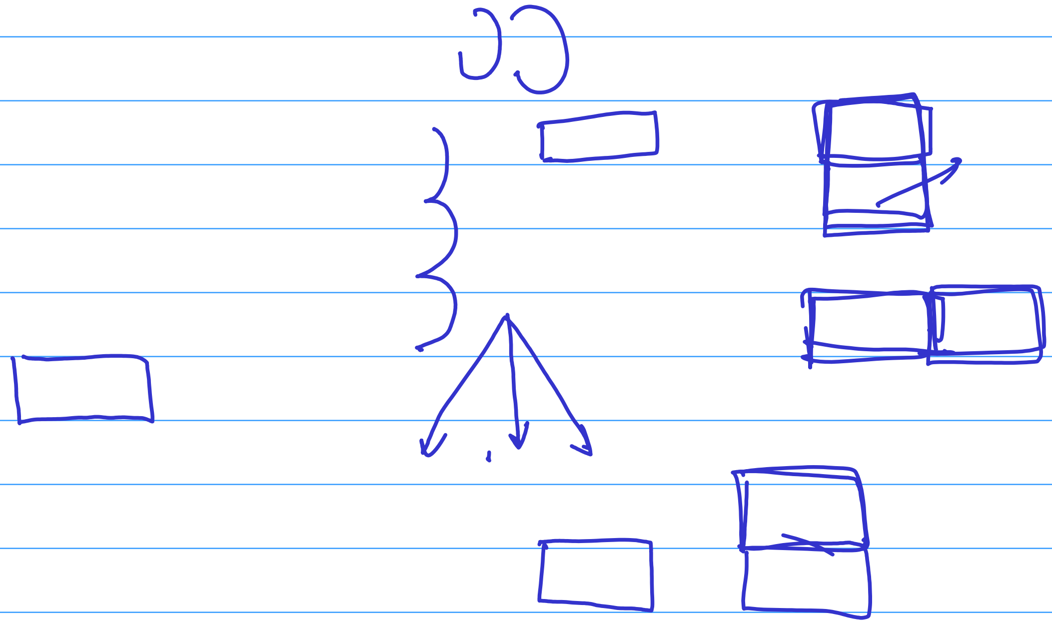
$$= \begin{pmatrix} 1 - m^{2i+1} d_{i+1} d_i & -m^{i+1} d_{i+1} - m^{i} d_i \\ m^{i+1} d_{i+1} + m^{i} d_i & -m^{2i+1} d_{i+1} d_i + 1 \end{pmatrix}$$

$$i=7 \quad \begin{bmatrix} 1 - s_7 s_8 m^{15} & \underline{-s_8 m^8 - s_7 m^7} \\ s_8 m^8 + s_7 m^7 & \underline{1 - s_7 s_8 m^{15}} \end{bmatrix}$$

$$i=8 \quad \begin{bmatrix} 1 - s_8 s_9 m^{17} & \underline{-s_9 m^9 - s_8 m^8} \\ s_9 m^9 + s_8 m^8 & \underline{1 - s_8 s_9 m^{17}} \end{bmatrix}$$

$$i=9 \quad \begin{bmatrix} 1 - s_9 s_{10} m^{19} & \underline{-s_{10} m^{10} - s_9 m^9} \\ s_{10} m^{10} + s_9 m^9 & \underline{1 - s_9 s_{10} m^{19}} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} +\left(x - 2^{-2i-1} x\right) \mp \left(2^{-i} y + 2^{-i-1} y\right) \\ +\left(y - 2^{-2i-1} y\right) \pm \left(2^{-i} x + 2^{-i-1} x\right) \end{bmatrix}$$



☆ CORDIC implementation

how to handle the K

scaling factor ?

multiplier?