Difference Equation Second Order (H.2)

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Based on
Complex Analysis for Mathematics and Engineering
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$$y[n+2] - 2a\,y[n+1] + b\,y[n] = 0$$

Let $y[n] = r^n \Rightarrow r^{n+2} - 2a\,r^{n+1} + b\,r^n = 0$

$$r^n(r^2 - 2ar + b) = 0$$

$(r^2 - 2ar + b) = 0$ Characteristic Equation

① $\boxed{a^2 > b}$ $r_1 = a + \sqrt{a^2 - b}$ **overdamped**

$r_2 = a - \sqrt{a^2 - b}$

$$y[n] = c_1 r_1^n + c_2 r_2^n$$

② $\boxed{a^2 = b}$ $r_1 = r_2 = a$ **Critically damped**

$$y[n] = c_1 r_1^n + C_2\, n\, r_1^n$$

③ $\boxed{a^2 < b}$ $r_1 = a + i\sqrt{b - a^2} = re^{i\phi}$ **underdamped**

$r_2 = a - i\sqrt{b - a^2} = re^{-i\phi}$

$r = \sqrt{b}$

$y[n] = k_1 r_1^n + k_2 r_2^n$

$\phi = \tan^{-1}\left(\frac{\sqrt{b - a^2}}{a}\right)$

$$= C_1 r^n \cos(n\phi) + C_2 r^n \sin(n\phi)$$

real coefficien function $f(z) = Y(z)\, z^{n-1}$

z_j pole
$\bar{z}_j$ pole

$$\text{Res}\left[Y(z)\, z^{n-1},\ \bar{z}_j \right] = \overline{\text{Res}\left[Y(z)\, z^{n-1},\ z_j \right]}$$

$$\text{Res}\left[f(z),\ \bar{z}_j \right] = \overline{\text{Res}\left[f(z),\ z_j \right]}$$

$$a\,y[n-2] - b\,y[n-1] + y[n] = u[n-1] \qquad y[-2] = y[-1] = 0$$

$$a z^{-2} Y(z) - b z^{-1} Y(z) + Y(z) = z^{-1} \frac{z}{z-1}$$

$$Y(z) = \frac{1}{(a z^{-2} - b z^{-1} + 1)} \frac{1}{z-1} = \frac{z}{(z^2 - bz + a)} \cdot \frac{z}{(z-1)}$$

$$z_1, z_2 = \frac{+b \pm \sqrt{b^2 - 4a}}{2}$$

$b^2 - 4a > 0$ overdamped

$b^2 - 4a = 0$ critically damped

$b^2 - 4a < 0$ underdamped

Overdamped

$$\boxed{b^2 > 4a}$$

$b^2 > 4a \qquad (0.7)^2 = 0.49 > 4 \cdot 0.1 \qquad b = 0.7$

$a = 0.1$

$$(z^2 - bz + a) = z^2 - 0.7z + 0.1 = (z - 0.2)(z - 0.5)$$

$$Y(z) = \frac{z}{(z^2 - bz + a)} \cdot \frac{z}{(z-1)} = \frac{z}{(z-0.2)(z-0.5)} \cdot \frac{z}{(z-1)}$$

$$z^{-1} Y(z) = \frac{A}{(z-1)} + \frac{B}{(z-0.2)} + \frac{C}{(z-0.5)}$$

$$A = (z-1)\frac{1}{(z-0.2)(z-0.5)} \cdot \frac{z}{(z-1)}\bigg|_{z=1} = \frac{1}{(0.8)(0.5)} = \frac{1}{0.4} = \frac{5}{2}$$

$$B = (z-0.2)\frac{1}{(z-0.2)(z-0.5)} \cdot \frac{z}{(z-1)}\bigg|_{z=0.2} = \frac{0.2}{(-0.3)(-0.7)} = \frac{2}{21}$$

$$C = (z-0.5)\frac{1}{(z-0.2)(z-0.5)} \cdot \frac{z}{(z-1)}\bigg|_{z=0.5} = \frac{0.5}{(0.3)(-0.5)} = -\frac{10}{3}$$

$$Y(z) = \frac{5}{2}\frac{z}{(z-1)} + \frac{2}{21}\frac{z}{(z-0.2)} - \frac{10}{3}\frac{z}{(z-0.5)}$$

$$y[n] = \frac{5}{2}u[n] + \frac{2}{21}\left(\frac{1}{5}\right)^n u[n] - \frac{10}{3}\left(\frac{1}{2}\right)^n u[n]$$

Critically Damped $\boxed{b^2 = 4a}$

$b^2 = 4a \qquad (0.8)^2 = 0.64 = 4 \cdot 0.16 \qquad b = 0.8$

$a = 0.16$

$z^2 - bz + \frac{b^2}{4} = \left(z - \frac{b}{2}\right)^2 \qquad \boxed{z = \frac{b}{2}} \quad 0.4$

$$Y(z) = \frac{z}{(z^2 - bz + a)} \cdot \frac{z}{(z-1)} = \frac{z}{(z-0.4)^2} \cdot \frac{z}{(z-1)}$$

$$z^{-1} Y(z) = \frac{A}{z-1} + \frac{B}{(z-0.4)^2} + \frac{C}{z-0.4}$$

$$A = (z-1)\frac{z}{(z-0.4)^2} \cdot \frac{1}{(z-1)}\Big|_{z=1} = \frac{1}{0.6^2} = \frac{1}{(0.6)^2}$$

$$B = (z-0.4)^2 \frac{z}{(z-0.4)^2} \cdot \frac{1}{(z-1)}\Big|_{z=0.4} = -\frac{0.4}{0.6}$$

$$C = \frac{d}{dz}\left[(z-0.4)^2 \frac{z}{(z-0.4)^2} \cdot \frac{1}{(z-1)}\right]\Big|_{z=0.4}$$

$$= \frac{d}{dz}\left[\frac{z}{z-1}\right]\Big|_{z=0.4} = \frac{z-1-z}{(z-1)^2}\Big|_{z=0.4} = \frac{-1}{(0.6)^2}$$

$$z^{-1} Y(z) = \frac{\frac{1}{(0.6)^2}}{z-1} + \frac{-\frac{0.4}{0.6}}{(z-0.4)^2} + \frac{\frac{-1}{(0.6)^2}}{z-0.4}$$

$$Y(z) = \frac{1}{(0.6)^2}\left\{\frac{z}{(z-1)} - 0.24\frac{z}{(z-0.4)^2} - \frac{z}{(z-0.4)}\right\}$$

$$y[n] = \frac{1}{(0.6)^2}\left\{u[n] - 0.24\, n\, (0.4)^{n-1} u[n] - (0.4)^n u[n]\right\}$$

$$y[n] = \frac{1}{(0.6)^2}\left\{1 - (0.6n + 1)(0.4)^n\right\} u[n]$$

Underdamped

$$\boxed{b^2 < 4a}$$

$$z^2 - bz + a = (z - ce^{j\theta})(z - ce^{-j\theta}) \qquad z = ce^{j\theta},\ ce^{-j\theta}$$

$$\begin{cases} a = c^2 \\ -b = -c(e^{j\theta} + e^{-j\theta}) = -2c\cos\theta \end{cases}$$

$$Y(z) = \frac{z}{(z^2 - bz + a)} \cdot \frac{z}{(z-1)} = \frac{z}{(z-ce^{j\theta})(z-ce^{-j\theta})} \cdot \frac{z}{(z-1)}$$

$$z^{-1}Y(z) = \frac{A}{(z-ce^{j\theta})} + \frac{B}{(z-ce^{-j\theta})} + \frac{C}{(z-1)}$$

$$A = (z-ce^{j\theta}) \frac{1}{(z-ce^{j\theta})(z-ce^{-j\theta})} \cdot \frac{z}{(z-1)} \Bigg|_{z=ce^{j\theta}} = \frac{1}{(ce^{j\theta} - ce^{-j\theta})} \cdot \frac{ce^{j\theta}}{(ce^{j\theta} - 1)}$$

$$B = (z-ce^{-j\theta}) \frac{1}{(z-ce^{j\theta})(z-ce^{-j\theta})} \cdot \frac{z}{(z-1)} \Bigg|_{z=ce^{-j\theta}} = \frac{1}{(ce^{-j\theta} - ce^{j\theta})} \cdot \frac{ce^{-j\theta}}{(ce^{-j\theta} - 1)}$$

$$C = (z-1) \frac{1}{(z-ce^{j\theta})(z-ce^{-j\theta})} \cdot \frac{z}{(z-1)} \Bigg|_{z=1} = \frac{1}{(1-ce^{j\theta})(1-ce^{-j\theta})}$$

$$Y(z) = + \frac{ce^{j\theta}}{2cj\sin\theta\,(ce^{j\theta}-1)} \frac{z}{(z-ce^{-j\theta})} - \frac{ce^{-j\theta}}{2cj\sin\theta\,(ce^{-j\theta}-1)} \frac{z}{(z-ce^{j\theta})} + \frac{1}{(1-2c\cos\theta+c^2)} \frac{z}{(z-1)}$$

$$y[n] = - \frac{ce^{j\theta}}{2cj\sin\theta\,(1-ce^{j\theta})} (ce^{j\theta})^n u[n] + \frac{ce^{-j\theta}}{2cj\sin\theta\,(1-ce^{-j\theta})} (ce^{-j\theta})^n u[n] + \frac{1}{(1-2c\cos\theta+c^2)} u[n]$$

$$\boxed{\begin{array}{ll} 1 - ce^{j\theta} = re^{-j\phi} & (1-ce^{j\theta})(1-ce^{-j\theta}) = r^2 \\ 1 - ce^{-j\theta} = re^{+j\phi} & \end{array}}$$

$y[n] =$

$$-\frac{1}{2cj\sin\theta\,(1-ce^{j\theta})}(ce^{j\theta})^{n+1}u[n]$$
$$+\frac{1}{2cj\sin\theta\,(1-ce^{-j\theta})}(e^{-j\theta})^{n+1}u[n]$$
$$+\frac{1}{(1-2c\cos\theta+c^2)}u[n]$$

$y[n] =$

$$-\frac{e^{+j\phi}}{2rcj\sin\theta}(ce^{j\theta})^{n+1}u[n]$$
$$+\frac{e^{-j\phi}}{2rcj\sin\theta}(e^{-j\theta})^{n+1}u[n]$$
$$+\frac{1}{r^2}u[n]$$

$$y[n] = \frac{e^{+j\phi}}{2rcj\sin\theta}(ce^{j\theta})^{n+1}u[n] - \frac{e^{-j\phi}}{2rcj\sin\theta}(e^{-j\theta})^{n+1}u[n] + \frac{1}{r^2}u[n]$$

$$y[n] = \frac{1}{r^2}\left[-\frac{e^{+j\phi}}{2j\sin\theta}rc^n e^{j(n+1)\theta} + \frac{e^{-j\phi}}{2j\sin\theta}rc^n e^{-j(n+1)\theta} + 1\right]u[n]$$

$$y[n] = \frac{1}{r^2}\left[-\frac{rc^n}{\sin\theta}\left(\frac{e^{j((n+1)\theta+\phi)} - e^{-j((n+1)\theta+\phi)}}{2j}\right) + 1\right]u[n]$$

$$y[n] = \frac{1}{r^2}\left[-\frac{rc^n}{\sin\theta}\sin((n+1)\theta+\phi) + 1\right]u[n]$$

Solving Nohomogeneous Equation

① homogeneous equation

$$y_h[n+2] - 2a\, y_h[n+1] + b\, y_h[n] = 0 \qquad y_h[n]$$

② transfer function

$$H(z) = \frac{1}{1 - 2a z^{-1} + b z^{-2}} \qquad h[n]$$

③ particular solution

$$y_p[n] = \mathcal{Z}^{-1}[H(z)X(z)]$$

$$y_p[n] = \sum_{i=0}^{n} h[i]\, x[n-i]$$

Zero State Response

Ambardar 583p

$$H(z) = \frac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}}$$

$x[n] = 4\,u[n]$

$y[-1] = 0, \quad y[-2] = 12$

$$Y_{zs}(z) = H(z)X(z) = \frac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}} \cdot \frac{4z}{z-1}$$

$$= \frac{4z^3}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}$$

$$z^{-1}Y_{zs}(z) = \frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}$$

$$= \frac{A}{(z-\frac{1}{2})} + \frac{B}{(z+\frac{1}{3})} + \frac{C}{(z-1)}$$

$$A = (z-\tfrac{1}{2})\frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}\Bigg|_{z=\frac{1}{2}} = \frac{4(\frac{1}{2})^2}{(\frac{1}{2}+\frac{1}{3})(\frac{1}{2}-1)} = \frac{1}{\frac{5}{6}(-\frac{1}{2})} = -\frac{12}{5}$$

$$B = (z+\tfrac{1}{3})\frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}\Bigg|_{z=-\frac{1}{3}} = \frac{4(\frac{1}{3})^2}{(-\frac{1}{3}-\frac{1}{2})(-\frac{1}{3}-1)} = \frac{\frac{4}{9}}{\frac{-5}{6}\cdot\frac{-4}{3}} = \frac{2}{5}$$

$$C = (z-1)\frac{4z^2}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}\Bigg|_{z=1} = \frac{4(1)^2}{(1-\frac{1}{2})(1+\frac{1}{3})} = \frac{4}{\frac{1}{2}\cdot\frac{4}{3}} = 6$$

$$Y_{zs}(z) = -\frac{12}{5}\cdot\frac{z}{(z-\frac{1}{2})} + \frac{2}{5}\frac{z}{(z+\frac{1}{3})} + 6\frac{z}{(z-1)}$$

$$y_{zs}[n] = -\frac{12}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n] + 6\,u[n]$$

Zero Input Response

Ambardar 583p

$$H(z) = \frac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}} \qquad x[n] = 4\,u[n]$$

$$y[-1] = 0, \quad y[-2] = 12$$

ZSR ⇒ $\dfrac{Y(z)}{X(z)} = \dfrac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}}$ $\qquad \left(z^2 - \frac{1}{6}z - \frac{1}{6}\right)Y(z) = z^2 X(z)$

$$y[n+2] - \frac{1}{6}y[n] - \frac{1}{6}y[n] = x[n+2]$$

Diff EQ ⇒ $y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n]$

I.C. $Y(z) - \frac{1}{6}\left(z^{-1}Y(z) + y[-1]\right) - \frac{1}{6}\left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]\right) = X(z)$

$$\left(1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right)Y(z) = \frac{1}{6}\left(y[-1] + y[-2] + z^{-1}y[-1]\right) + X(z) = 2 + X(z)$$

$$Y(z) = \underbrace{\boxed{\frac{2}{\left(1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right)}}}_{ZIR} + \underbrace{\boxed{\frac{X(z)}{\left(1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right)}}}_{ZSR}$$

$$Y_{ZI}(z) = \frac{2z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}} = \frac{2z^2}{(z - \frac{1}{2})(z + \frac{1}{3})}$$

$$z^{-1}Y_{ZI}(z) = \frac{2z}{(z - \frac{1}{2})(z + \frac{1}{3})} = \frac{A}{(z - \frac{1}{2})} + \frac{B}{(z + \frac{1}{3})}$$

$$A = (z - \tfrac{1}{2})\frac{2z}{(z - \frac{1}{2})(z + \frac{1}{3})}\Bigg|_{z=\frac{1}{2}} = \frac{1}{\frac{5}{6}} = \frac{6}{5}$$

$$B = (z + \tfrac{1}{3})\frac{2z}{(z - \frac{1}{2})(z + \frac{1}{3})}\Bigg|_{z=-\frac{1}{3}} = \frac{-\frac{2}{3}}{-\frac{5}{6}} = \frac{4}{5}$$

$$Y_{ZI}(z) = \frac{6}{5}\frac{z}{(z - \frac{1}{2})} + \frac{4}{5}\frac{z}{(z + \frac{1}{3})}$$

$$y_{ZI}[n] = \frac{6}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{5}\left(-\frac{1}{3}\right)^n u[n]$$

Total Response

$$\boxed{\begin{array}{l} y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = x[n] \qquad y[-1]=0,\ y[-2]=12 \\ x[n] = 4\,u[n] \end{array}}$$

$$y_{zs}[n] = -\frac{12}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{2}{5}\left(-\frac{1}{3}\right)^n u[n] + 6\,u[n]$$

$$y_{zi}[n] = \frac{6}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{4}{5}\left(-\frac{1}{3}\right)^n u[n]$$

$$\boxed{y[n] = y_{zs}[n] + y_{zi}[n]}$$

$$y[n] = -\frac{6}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{5}\left(-\frac{1}{3}\right)^n u[n] + 6\,u[n]$$

Forced Response

$$y_f[n] = 6\,u[n] \qquad \boxed{x[n] = 4\,u[n]}$$

Natural Response

$$y_n[n] = -\frac{6}{5}\left(\frac{1}{2}\right)^n u[n] + \frac{6}{5}\left(-\frac{1}{3}\right)^n u[n] \qquad \boxed{y[n] - \frac{1}{6}y[n-1] - \frac{1}{6}y[n-2] = 0}$$

$$\left(z - \frac{1}{2}\right)\left(z + \frac{1}{3}\right) = 0$$

Forced and Natural Responses

ZSR $\Rightarrow$ $\boxed{H(z) = \dfrac{Y(z)}{X(z)}} = \dfrac{z^2}{z^2 - \frac{1}{6}z - \frac{1}{6}}$

$$\left(z^2 - \tfrac{1}{6}z - \tfrac{1}{6}\right)Y(z) = z^2 X(z)$$

$$\boxed{y[n+2] - \tfrac{1}{6}y[n] - \tfrac{1}{6}y[n] = x[n+2]}$$

Diff EQ $\Rightarrow$ $\boxed{y[n] - \tfrac{1}{6}y[n-1] - \tfrac{1}{6}y[n-2] = x[n]}$ $\quad\boxed{y[-1] = 0,\ y[-2] = 12}$

I.C. $\boxed{Y(z) - \tfrac{1}{6}\left(z^{-1}Y(z) + y[-1]\right) - \tfrac{1}{6}\left(z^{-2}Y(z) + z^{-1}y[-1] + y[-2]\right) = X(z)}$

$$\left(1 - \tfrac{1}{6}z^{-1} - \tfrac{1}{6}z^{-2}\right)Y(z) = \tfrac{1}{6}\left(y[-1] + y[-2] + z^{-1}y[-1]\right) + X(z) = 2 + X(z)$$

$\boxed{x[n] = 4\,u[n]}$ $\qquad$ $\boxed{X(z) = \dfrac{4z}{z-1}}$

$$Y(z) = \underbrace{\boxed{\frac{2}{\left(1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right)}}}_{\text{ZIR}} + \underbrace{\boxed{\frac{X(z)}{\left(1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right)}}}_{\text{ZSR}}$$

$$Y_{zs}(z) = -\frac{12}{5}\cdot\frac{z}{(z - \frac{1}{2})} + \frac{2}{5}\frac{z}{(z + \frac{1}{3})} + 6\frac{z}{(z-1)}$$

$$Y_{ZI}(z) = \frac{6}{5}\frac{z}{(z - \frac{1}{2})} + \frac{4}{5}\frac{z}{(z + \frac{1}{3})}$$

$$Y(z) = \underbrace{\boxed{-\frac{6}{5}\frac{z}{(z - \frac{1}{2})} + \frac{6}{5}\frac{z}{(z + \frac{1}{3})}}}_{\text{natural response}} + \underbrace{\boxed{6\frac{z}{(z-1)}}}_{\text{forced response}}$$

$$\left(1 - \tfrac{1}{6}z^{-1} - \tfrac{1}{6}z^{-2}\right)$$
$$= z^{-2}\left(z^2 - \tfrac{1}{6}z - \tfrac{1}{6}\right)$$
$$= z^{-2}\left(z - \tfrac{1}{2}\right)\left(z + \tfrac{1}{3}\right)$$

$$Y(z) = \left(\underbrace{\boxed{\frac{2}{(1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2})}}}_{ZIR} + \underbrace{\boxed{\frac{X(z)}{(1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2})}}}_{ZSR} \right) \times \frac{z^2}{z^2}$$

$$= \frac{2z^2}{(z^2-\frac{1}{6}z-\frac{1}{6})} + \frac{4z^3}{(z^2-\frac{1}{6}z-\frac{1}{6})(z-1)}$$

$$= \frac{2z^2}{(z-\frac{1}{2})(z+\frac{1}{3})}\left(1+\frac{2z}{z-1}\right) = \frac{2z^2(3z-1)}{(z-\frac{1}{2})(z+\frac{1}{3})(z-1)}$$

$$(z-1)Y(z) = \frac{2z^2(3z-1)}{(z-\frac{1}{2})(z+\frac{1}{3})}$$

poles $\frac{1}{2}, -\frac{1}{3}$

inside the unit circle

→ final value theorem

$$y_n[n] = \lim_{z\to1}(z-1)Y(z) = \lim_{z\to1}\frac{2z^2(3z-1)}{(z-\frac{1}{2})(z+\frac{1}{3})}$$

$$= \frac{2\cdot2}{(-\frac{1}{2})(\frac{4}{3})} = 6$$

17)	initial value	x_0	$\longleftrightarrow$	$\lim_{z\to\infty} X(z)$
18	final value	$\lim_{n\to\infty} x_n$	$\longleftrightarrow$	$\lim_{z\to1}(z-1)X(z)$

Undetermined Coefficients

Sig & Sys
Poularikas

input $x[n]$:

1. n^k $\quad n \geq 0$ integer
2. a^n $\quad a \neq 0$ nonzero constants
3. $\cos an$ $\quad a \neq 0$ non-zero constants
4. $\sin an$ $\quad a \neq 0$ non-zero constants
5. product combinations

Sig & Sys
Poulorikas

$x[n]$	
n^m	$A_1 n^m + A_2 n^{m-1} + \cdots + A_m n + A_{m+1}$
a^n	$A a^n$
$\cos\theta n$ / $\sin\theta n$	$A_1 \cos\theta n + B_1 \sin\theta n$
$n^m a^n$	$a^n (A_1 n^m + A_2 n^{m-1} + \cdots + A_m n + A_{m+1})$
$a^n \cos\theta n$ / $a^n \sin\theta n$	$a^n (A_1 \cos\theta n + B_1 \sin\theta n)$

Sig & Sys
Poularikas

$$y[n] - 5y[n-1] + 6y[n-2] = 3^n \qquad n = 0, 1, 2, \cdots$$

$$x[n] = 3^n u[n], \qquad y[-1] = 0 \qquad y[-2] = 1$$

$$\lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3) = 0 \qquad \lambda_1 = 2, \quad \lambda_2 = 3$$

$$y_1[n] = 2^n \qquad y_2[n] = 3^n$$

$$y_h[n] = C_1 2^n + C_2 3^n$$

$$y_p[n] = A\, n\, 3^n \qquad \begin{cases} y_2[n] = 3^n \\ x[n] = 3^n u[n] \end{cases}$$

$$y_p[n] - 5y_p[n-1] + 6y_p[n-2] = 3^n$$

$$An3^n - 5A(n-1)3^{n-1} + 6A(n-2)3^{n-2} = 3^n$$

$$An \cdot 9 - 15A(n-1) + 6A(n-2) = 9$$

$$(9A - 15A + 6A)n + 15A - 12A = 9 \qquad 3A = 9 \qquad A = 3 \qquad y_p = n3^{n+1}$$

$$y[n] = y_h[n] + y_p[n] = C_1 2^n + C_2 3^n + n3^{n+1}$$

$$y[-1] = C_1 2^{-1} + C_2 3^{-1} - 1 = 0 \qquad \tfrac{1}{2}C_1 + \tfrac{1}{3}C_2 = 1$$

$$y[-2] = C_1 2^{-2} + C_2 3^{-2} - 2 \cdot 3^{-1} = 1 \qquad \tfrac{1}{4}C_1 + \tfrac{1}{9}C_2 = \tfrac{5}{3}$$

$$3C_1 + 2C_2 = 6 \qquad 6C_1 + 4C_2 = 12 \qquad C_1 = 16$$

$$9C_1 + 4C_2 = 60 \qquad -\ 9C_1 + 4C_2 = 60 \qquad C_2 = (6 - 48)/2$$

$$3C_1 = 48 \qquad = -21$$

$$y[n] = 16\,2^n - 21 \cdot 3^n + n3^{n+1}$$

$y[n] - y[n-2] = 5n^2$

$(\lambda^2 - 1) = 0 \qquad \lambda_1 = 1 \quad \lambda_2 = -1$

$y_h[n] = C_1(1)^n + C_2(-1)^n$

$$\begin{cases} x[n] = 5n^2 \\ y_h[n] = C_1(1)^n + C_2(-1)^n \end{cases}$$

$An^2 + Bn + C$

$y_p[n] = n(An^2 + Bn + c)$

$= An^3 + Bn^2 + Cn$

$An^3 + Bn^2 + Cn - A(n-2)^3 - B(n-2)^2 - C(n-1) = 0$

$A(n^3 - (n-2)^3) = A(n-(n-2))(n^2 + n(n-2) + (n-2)^2)$

$= A(2)(n^2 + n^2 - 2n + n^2 - 4n + 4) = 2A(3n^2 - 6n + 4)$

$B(n^2 - (n-2)^2) = B(n-(n-2))(n+(n-2)) = B(2)(2n-2) = 4B(n-1)$

$C(n-(n-1)) = C(1) = C$

$6An^2 + (-12A + 4B)n + (8A - 4B + c) = 5n^2$

$6A = 5 \qquad A = \frac{5}{6}$

$-12A + 4B = 0 \qquad B = 3A = 3 \cdot \frac{5}{6} = \frac{5}{2}$

$8A - 4B + C = 0 \qquad 8 \cdot \frac{5}{6} - 4 \cdot \frac{5}{2} + c = 0 \qquad \frac{20}{3} - 10 + C = 0 \quad C = \frac{10}{3}$

$y_p = \frac{5}{6}n^2 + \frac{5}{2}n + \frac{10}{3}$