CMOS Delay-6 (H.6)
Multi-Stage Delay

20161231

References

Some Figures from the following sites

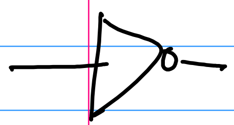
[1] http://pages.hmc.edu/harris/cmosvlsi/4e/index.html
Weste & Harris Book Site

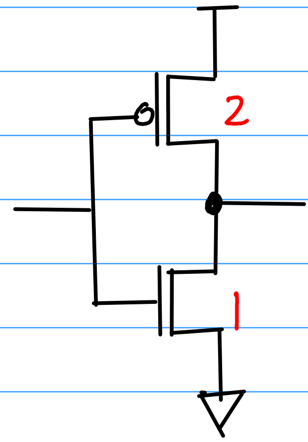
[2] en.wikipedia.org

[3] http://www.ee.ic.ac.uk/pcheung/teaching/ee4_asic/notes/Topic%

* C_{in}, g, p

$R_p = R_n$

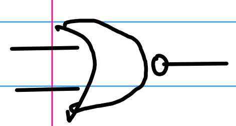


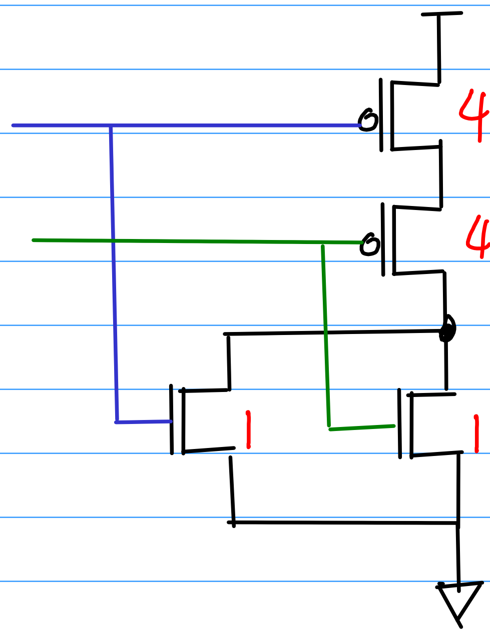


$C_{in} = 3$

$g = 3/3 = 1$

$p = \frac{3}{3} = \frac{2+1}{3}$

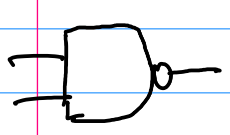


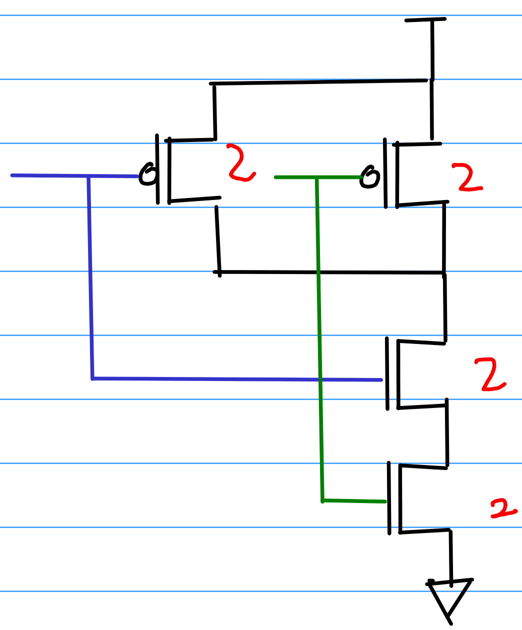


$C_{in} = 5$

$g = 5/3$

$p = \frac{6}{3} = \frac{4+1+1}{3} = 2$



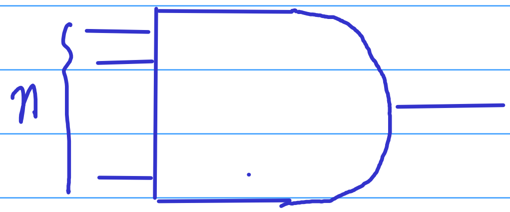


$C_{in} = 4$

$g = 4/3$

$p = \frac{6}{3} = \frac{2+2+2}{3} = 2$

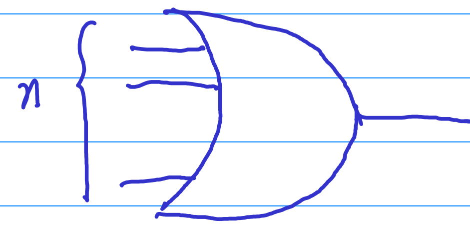
NAND



$$g = \frac{(2+n)}{3}$$

$$p = (2n+n)/3 = n$$

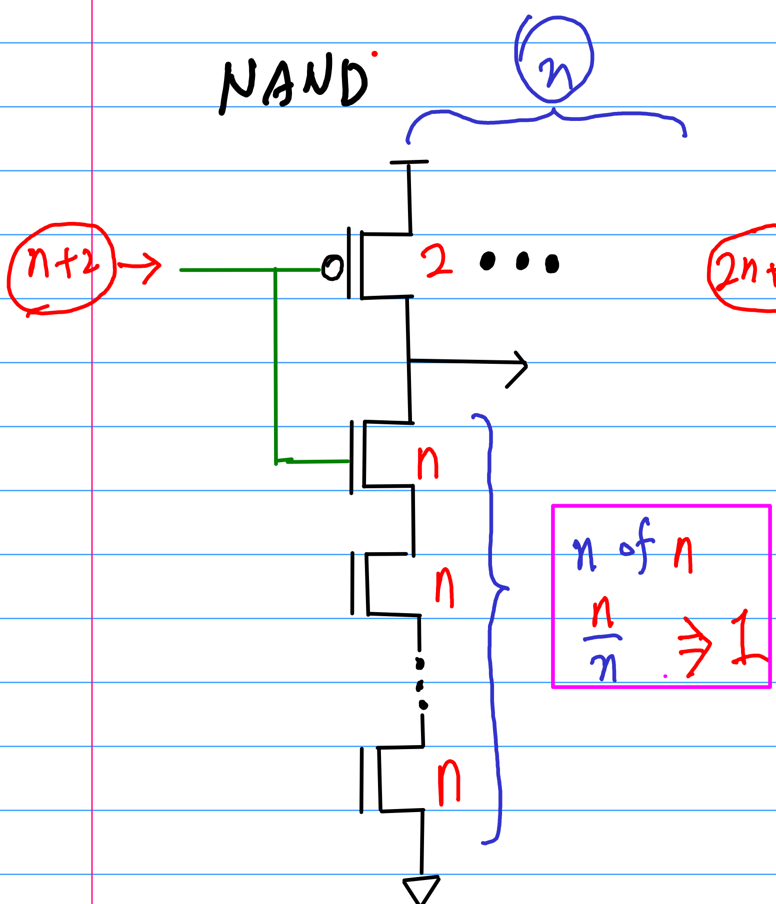
NOR



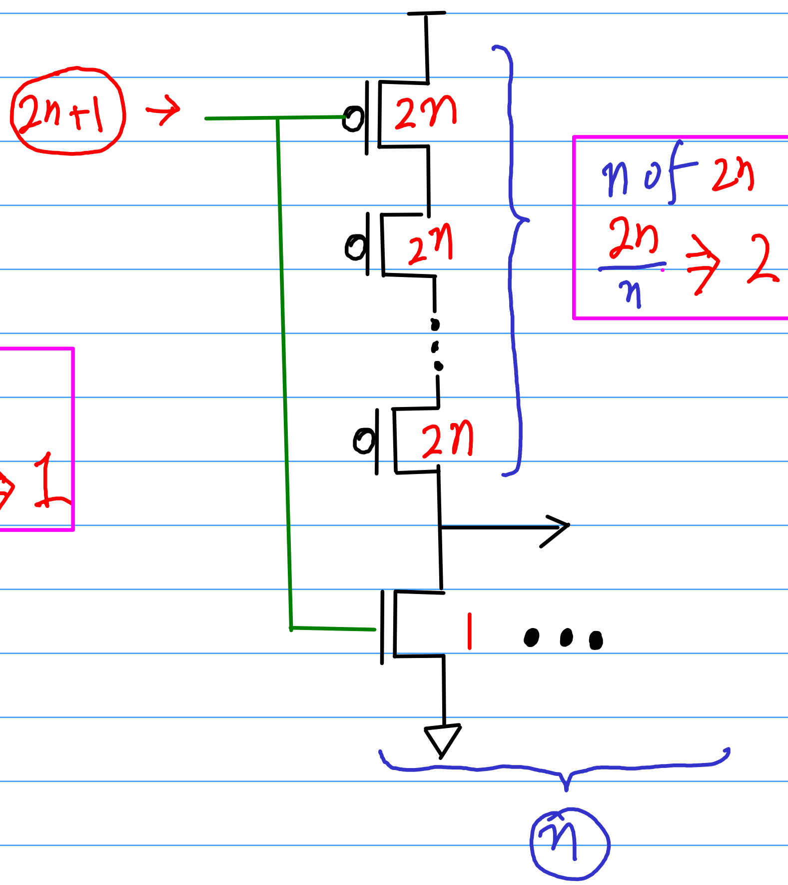
$$g = \frac{(2n+1)}{3}$$

$$p = (2n+n)/3 = n$$

NAND



NOR



$$D = N \sqrt[N]{F} + N$$

Path Delay

$$\boxed{\begin{aligned} D = \sum_i d_i &= D_F + P \\ &= \sum_i f_i + \sum_i p_i \end{aligned}}$$

$$\forall i \qquad \hat{f} = f_i = g_i h_i = F^{1/N}$$ equal delay

$$\boxed{\begin{aligned} f_1 + f_2 + \cdots + f_N &= g_1 h_1 + g_2 h_2 + \cdots + g_N h_N \\ &\geq (g_1 h_1)(g_2 h_2) \cdots (g_N h_N) \end{aligned}}$$

Path Effort $\quad F = GBH \qquad\qquad F = GH$

$$\boxed{\begin{aligned} G &= \prod g_i \\ H &= \frac{C_{out}}{C_{in}} \\ B &= \prod b_i \end{aligned}} \qquad \boxed{\begin{aligned} G &= \prod g_i \\ H &= \frac{C_{out}}{C_{in}} \\ B &= 1 \end{aligned}}$$

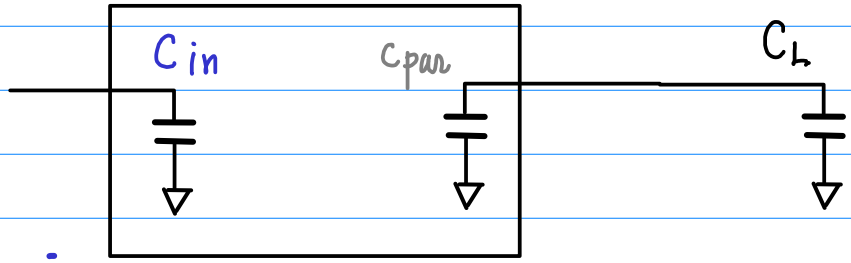
$$\boxed{\begin{aligned} D = \sum_i d_i &= \sum_i f_i + \sum_i p_i \\ &= N F^{1/N} + N \end{aligned}}$$

$$\boxed{C_{out} = C_{par} + C_L}$$

drain parasitic cap.

$C_{par} \rightarrow p$

$C_L \rightarrow h$



In computing h (electrical effort)

$$\boxed{C_{out} = C_L}$$

only external cap
excluding para cap.

In computing p (parasitic capacitance)

$$\boxed{p = \frac{C_{p,ref}}{C_{ref}}}$$

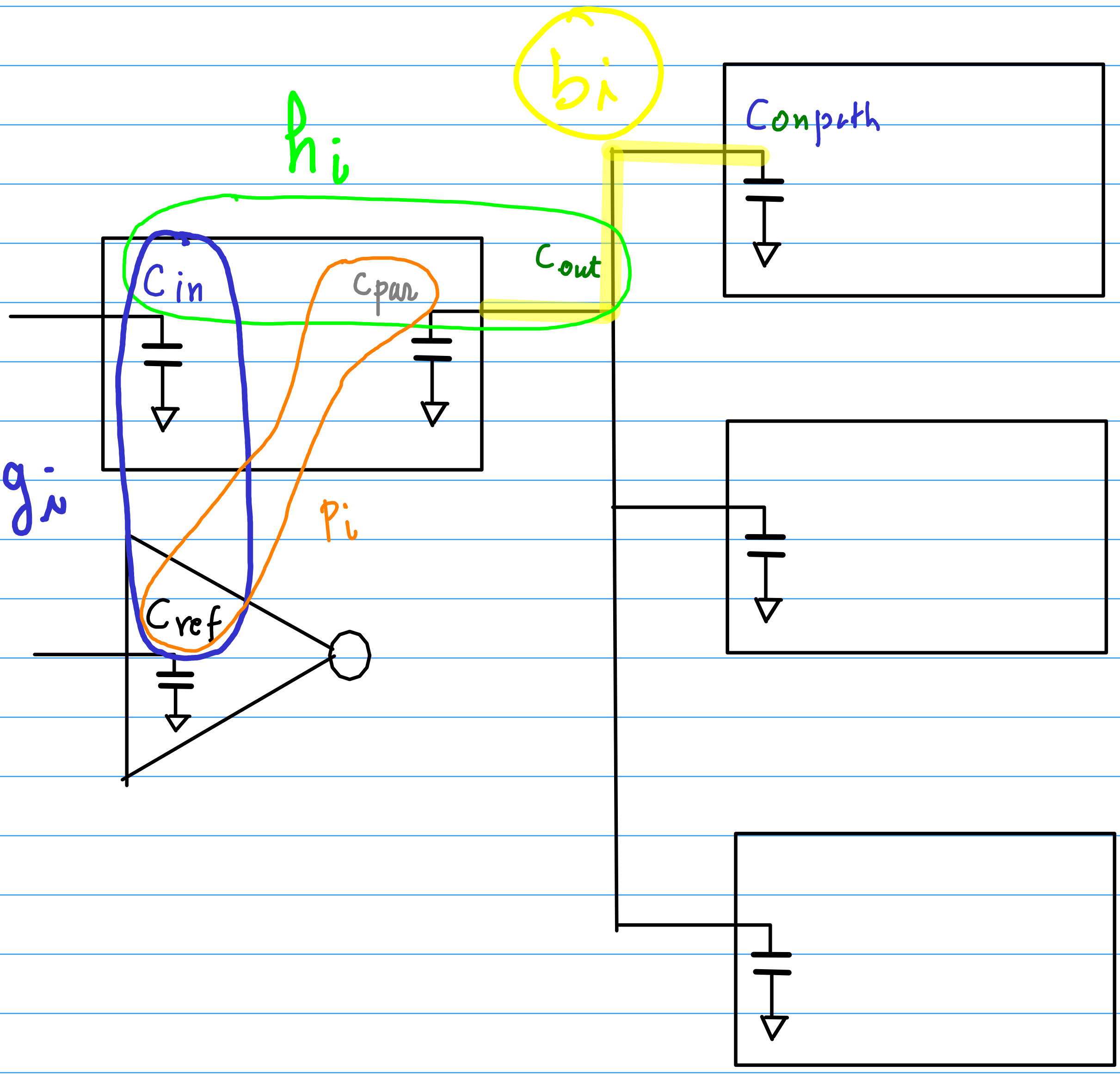
$C_{dp} + C_{dn}$ drain parasitic cap

C_{in} of the ref inverter
(symmetric Inverter)

Logical Effort $g_i = \frac{C_{in}}{C_{ref}}$

Electrical Effort $h_i = \frac{C_{out}}{C_{in}}$

Branching Effort $b_i = \frac{C_{onpath} + C_{offpath}}{C_{onpath}}$

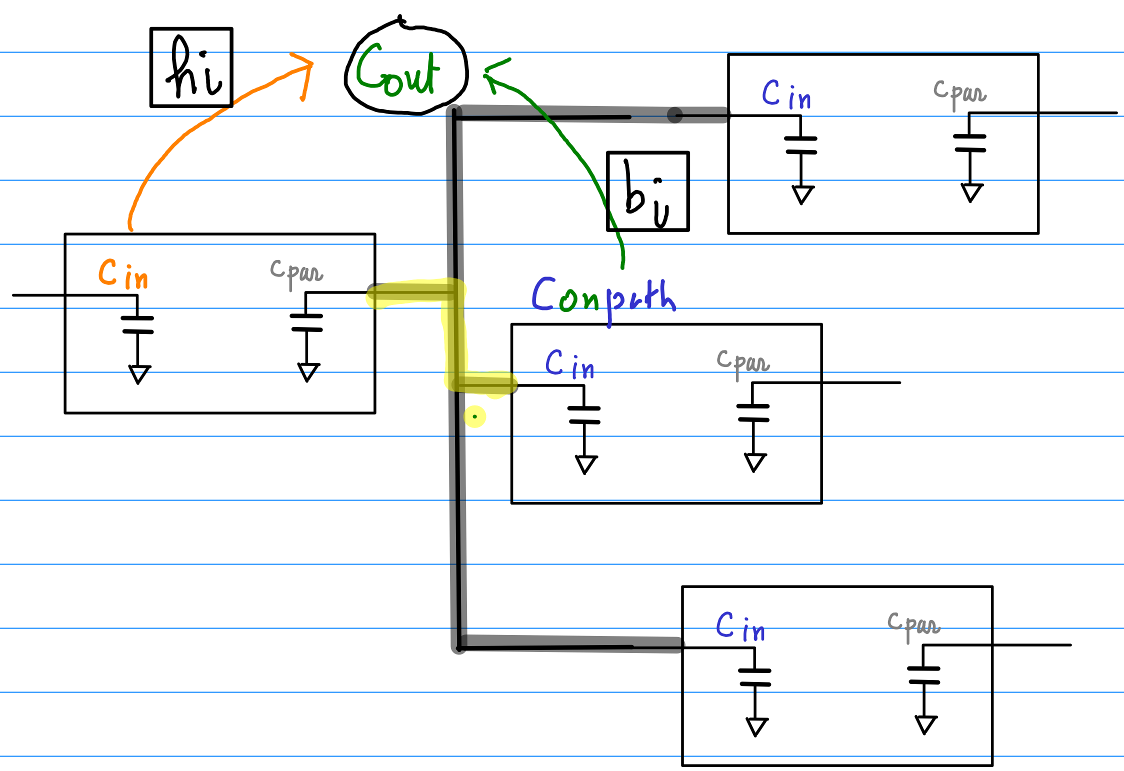


Logical Effort $\quad g_i = \dfrac{C_{in}}{C_{ref}}$

Electrical Effort $\quad h_i = \dfrac{C_{out}}{C_{in}}$

Branching Effort $\quad b_i = \dfrac{C_{onpath} + C_{offpath}}{C_{onpath}}$

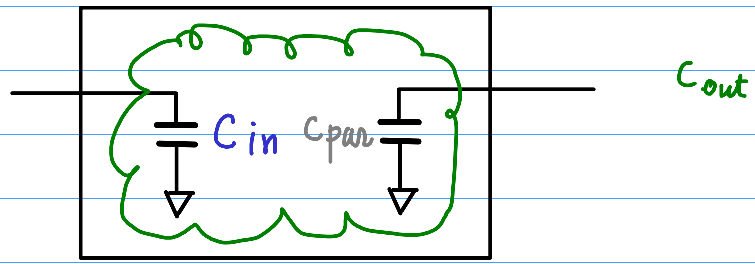
$$\boxed{\begin{aligned} C_{out} &= h_i \, C_{in} \\ &= b_i \cdot C_{onpath} = C_{onpath} + C_{offpath} \end{aligned}}$$



* Intra Connection → C_{in} , C_{par}

logic gate's topology → Logical Effort f

parasitic p

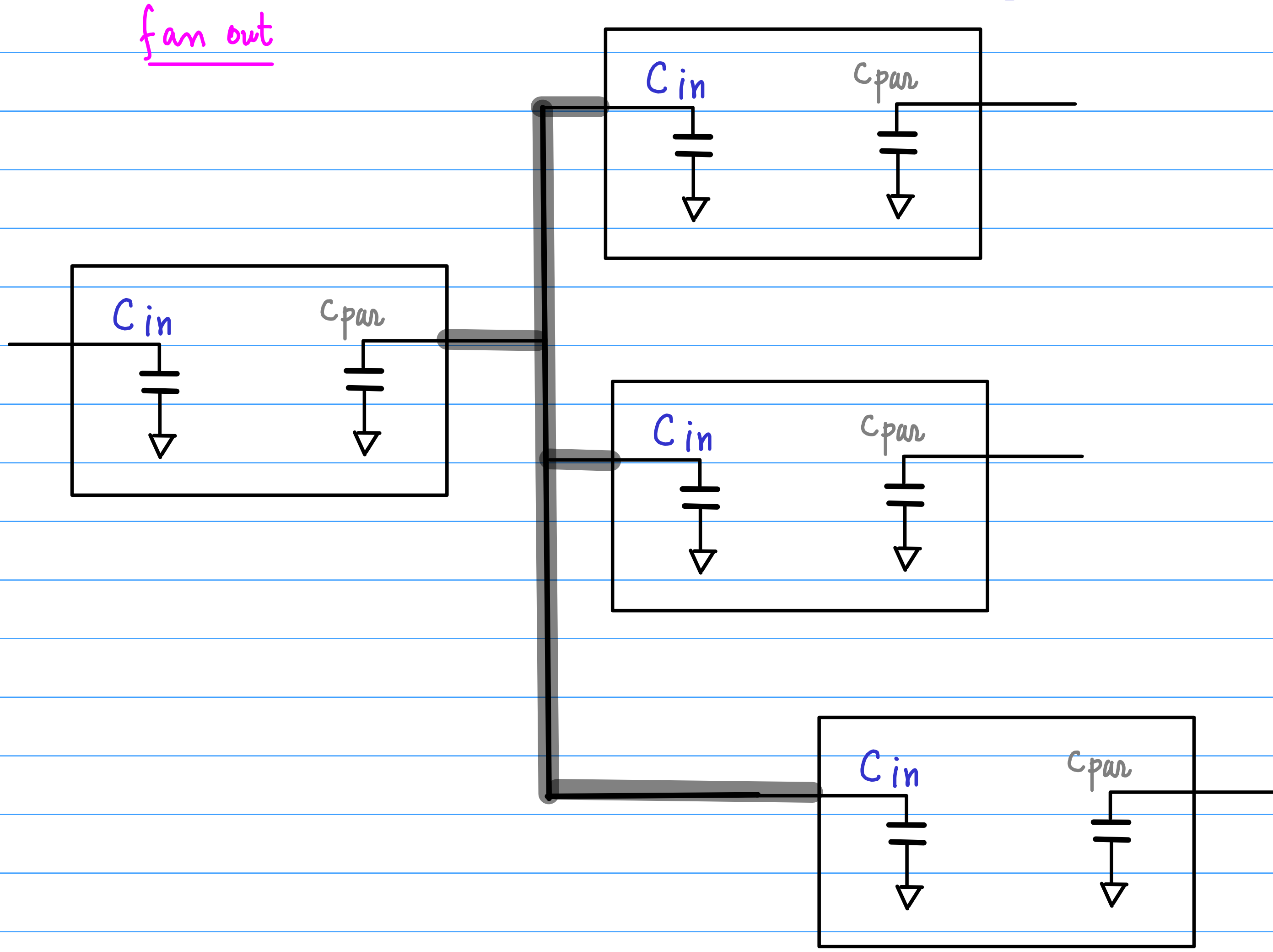


NAND, NOR, INV, XOR

* Inter Connection → C_{out}

electrical design connection → electrical effort h

fan out



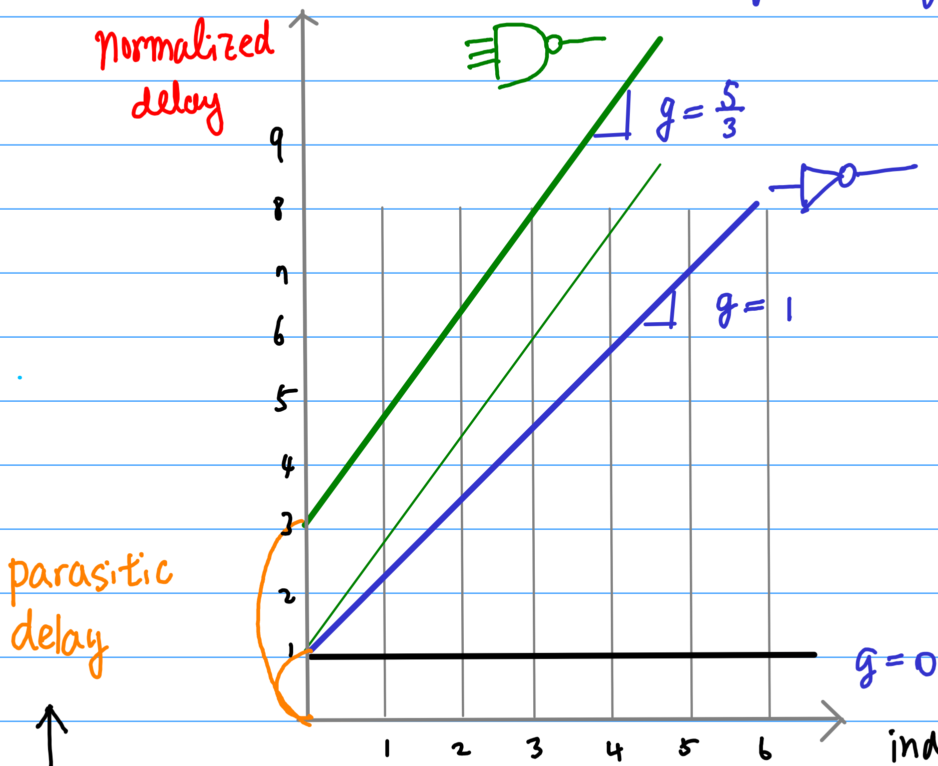
* No Gate Sizing assumed

For a given logic gate

electrical effort (f)
parasitic (p)

fixed value

slope : logical effort

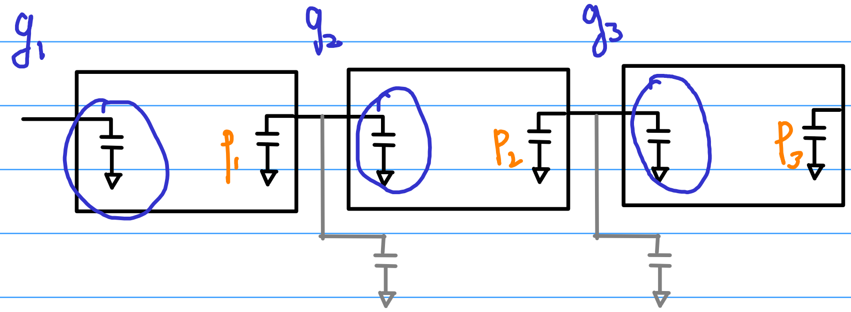


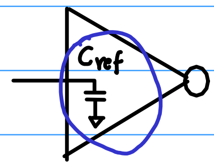
delay due to
only internal cap
without external load cap

electrical effort $h = \frac{C_{out}}{C_{in}}$

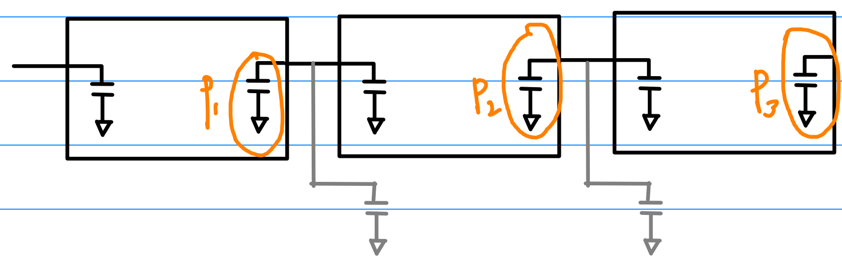
$$\boxed{d = g \cdot h + p}$$

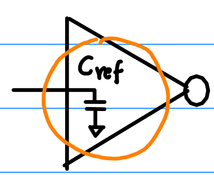
k c c



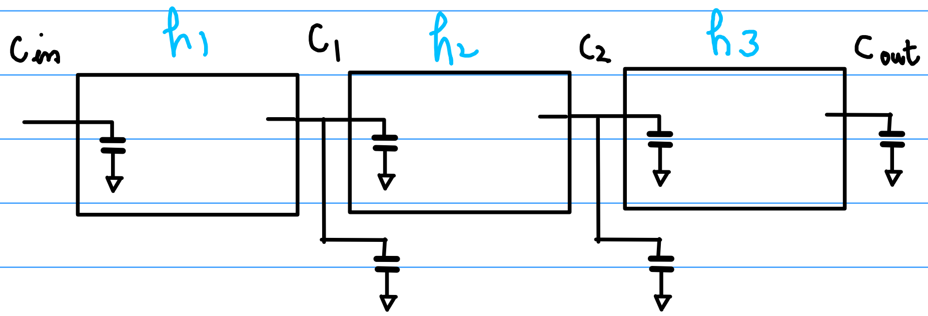


$$\boxed{g_i = \frac{C_{in}}{C_{ref}}}$$





$$\boxed{p = \frac{C_{p,ref}}{C_{ref}}}$$



$$H = \prod_i h_i = h_1 h_2 h_3 = \frac{\cancel{C_1}}{C_{in}} \frac{\cancel{C_2}}{\cancel{C_1}} \frac{C_{out}}{\cancel{C_2}} = \frac{C_{out}}{C_{in}}$$

because we don't know h_i's
until the design is done.

$H = \frac{C_{out}}{C_{in}}$ instead of $H = \prod_i h_i$

$$f_i = \hat{f} = \sqrt[N]{F} = \sqrt[N]{GH}$$

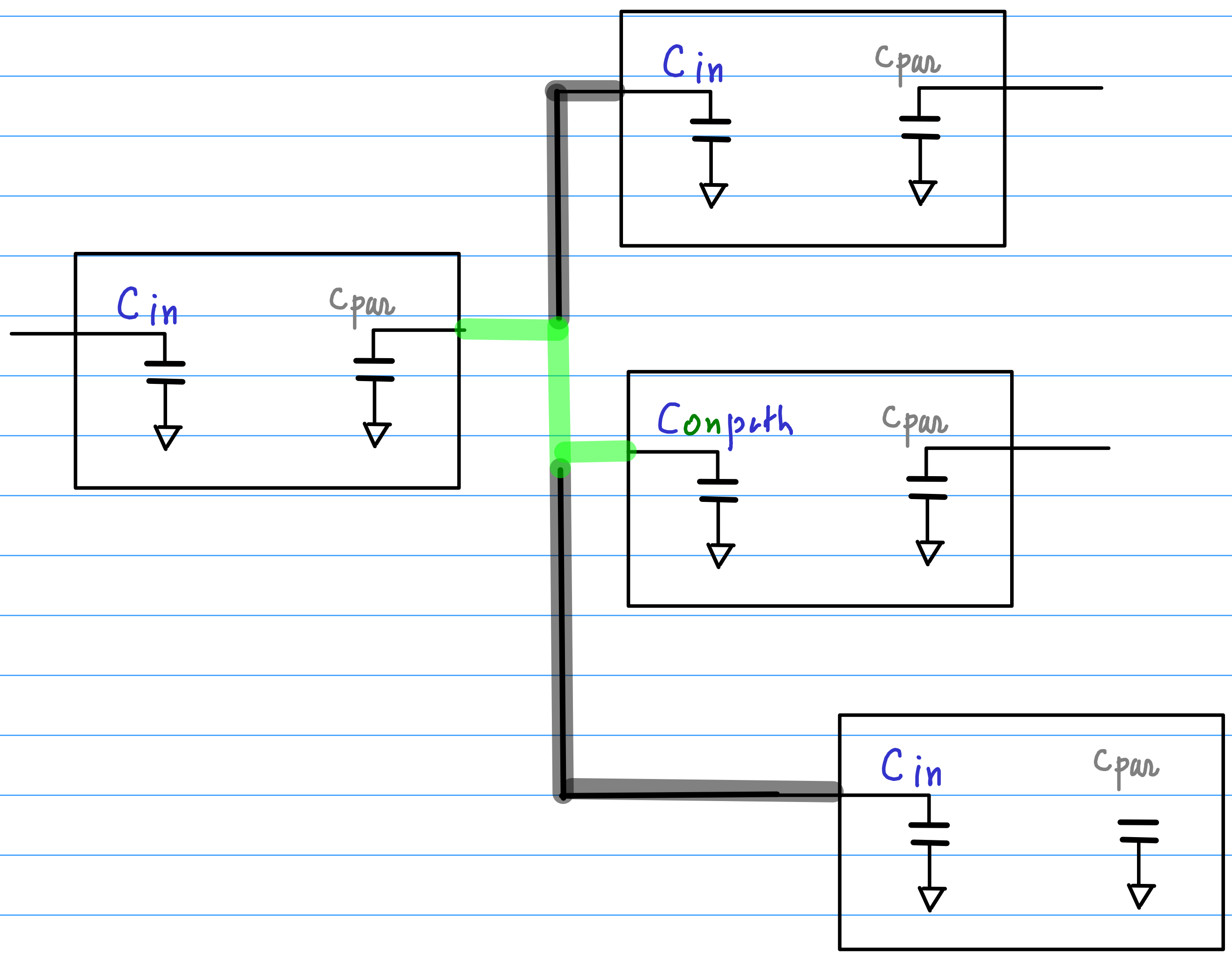
$$g_3 h_3 = \hat{f} \qquad \boxed{h_3} = \hat{f} / g_3$$

$$g_2 h_2 = \hat{f} \qquad \boxed{h_2} = \hat{f} / g_2$$

$$g_1 h_1 = \hat{f} \qquad \boxed{h_1} = \hat{f} / g_1$$

Branching Effort

$$b_i = \frac{C_{onpath} + C_{offpath}}{C_{onpath}}$$



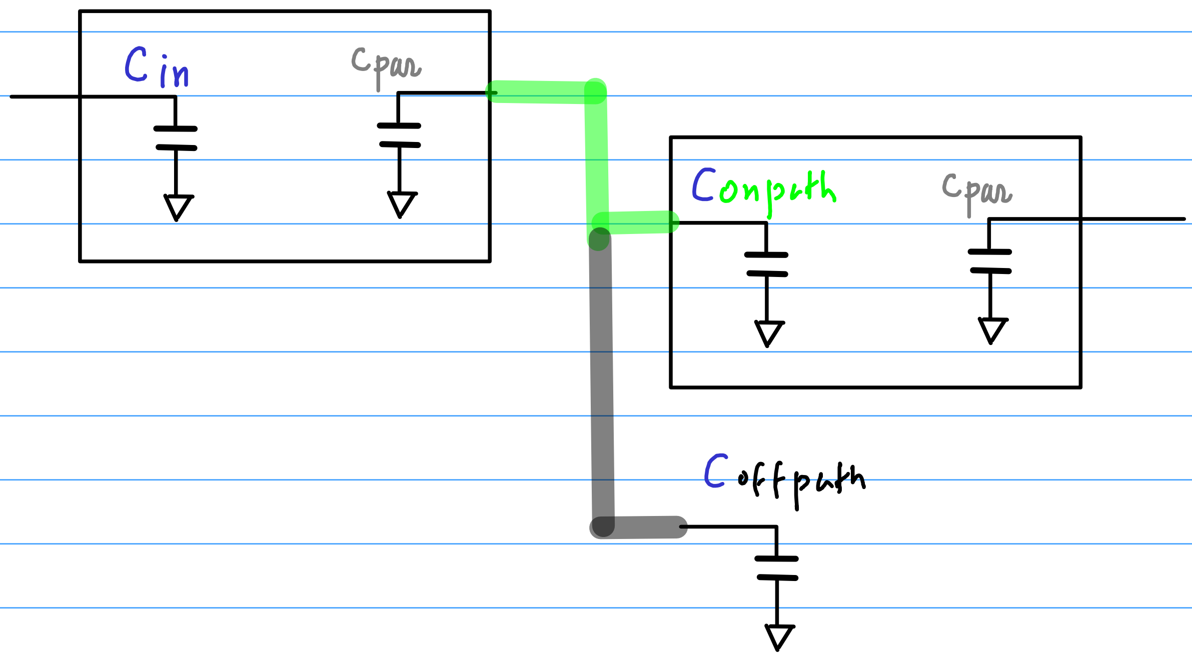
$\tau = RC$

must be $C_{onpath} + C_{offpath}$

$$B\, C_{onpath} = C_{onpath} \cdot \frac{(C_{onpath} + C_{offpath})}{C_{onpath}}$$

* total fanout Cap in term of C_{onpath}

How many C_{onpath}'s in total



Branching Effort $\quad b_i = \dfrac{C_{onpath} + C_{offpath}}{C_{onpath}}$.

total fanout capacitance normalized by C_{onpath}

in terms of C_{onpath}

Logical Effort $g_i = \frac{C_{in}}{C_{ref}}$

Electrical Effort $h_i = \frac{C_{out}}{C_{in}}$

Branching Effort $b_i = \frac{C_{onpath} + C_{offpath}}{C_{onpath}}$

path logical effort $G = \prod g_i$

path electrical effort $H = \frac{C_{out(path)}}{C_{in(path)}}$

path branching effort $B = \prod b_i$

path **effort**

No Branch $F = \prod f_i = \prod g_i h_i = GH$

with Branch $F = \prod f_i = \prod b_i g_i h_i = BGH$

Path Delay $D = \sum d_i = D_F + D_P$

$= \sum f_i + \sum p_i$

Path Effort Delay $D_F = \sum f_i \geqslant N \sqrt[N]{\prod f_i}$

path Parasitic Delay $D_P = \sum p_i$

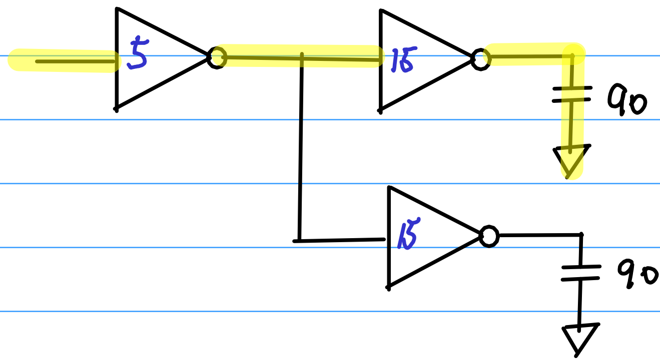
$N\hat{f} = \sum \hat{f} = N \sqrt[N]{F}$

$\min D_F \quad \hat{f} = g_i h_i = F^{1/N}$

$\min D = N F^{1/N} + P$

$\underline{\frac{a+b}{2} \geqslant \sqrt[2]{ab}} \geqslant \frac{2ab}{a+b}$

Branching Effort



$$G = g_1 \cdot g_2 = 1 \cdot 1 = 1$$

$$H = h_1 \cdot h_2 = \frac{15}{5} \cdot \frac{90}{15} = \frac{90}{5} = 18$$

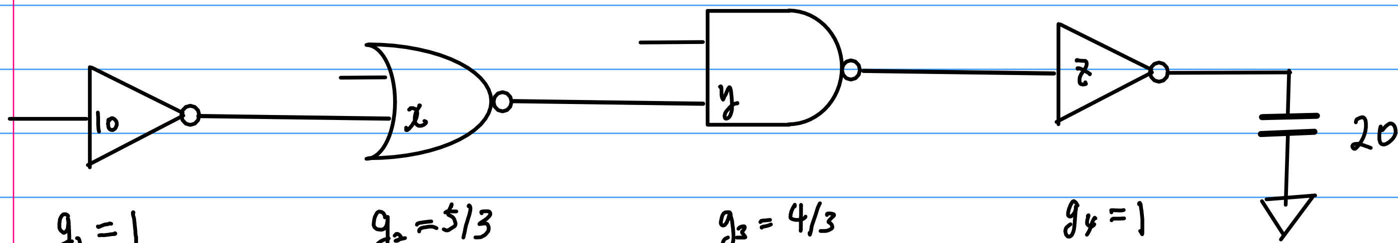
$$GH = 1 \times 18 = 18$$

$$F = f_1 \cdot f_2 = g_1 \cdot h_1 \cdot g_2 \cdot h_2 = \left(1 \cdot \frac{15+15}{5}\right) \cdot \left(1 \cdot \frac{90}{15}\right) = 2 \cdot 18$$
$$= 36$$

$$F \neq GH$$

$$F = 2\,GH$$ Branching Effort

Ex 1)



$g_1 = 1$ $\quad$ $h_1 = x/10$

$g_2 = 5/3$ $\quad$ $h_2 = y/x$

$g_3 = 4/3$ $\quad$ $h_3 = z/y$

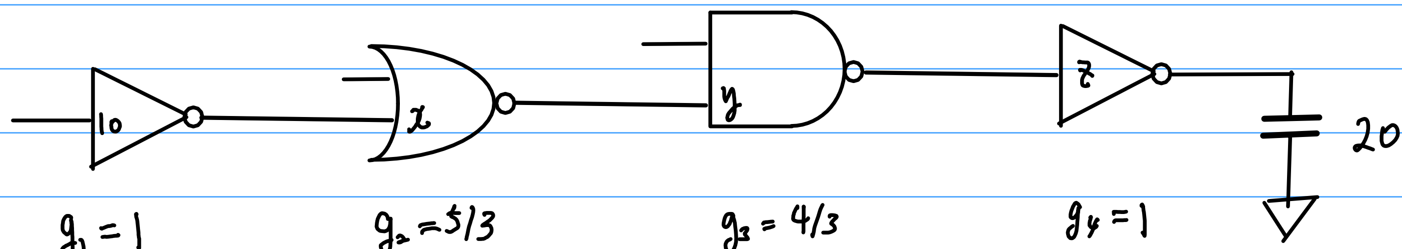
$g_4 = 1$ $\quad$ $h_4 = 20/z$

$$G = \prod g_i$$

$$H = \frac{C_{out\,(path)}}{C_{in\,(path)}} \quad \text{instead of } \prod h_i$$

$$F = \prod g_i h_i = GH \quad \Leftarrow B = 1$$

Ex 1) G H



$g_1 = 1$ $\quad h_1 = x/10$

$g_2 = 5/3$ $\quad h_2 = y/x$

$g_3 = 4/3$ $\quad h_3 = z/y$

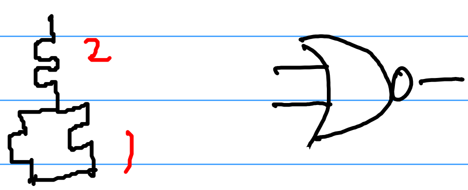
$g_4 = 1$ $\quad h_4 = 20/z$

path logical effort

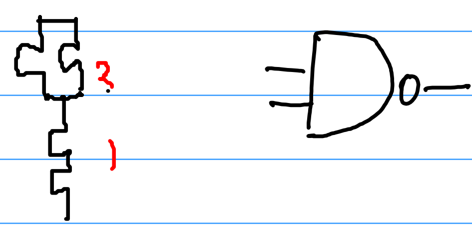
$$G = \prod g_i = 1 \times \frac{5}{3} \times \frac{4}{3} \times 1 = \frac{20}{9}$$

path electrical effort

$$H = \frac{C_{out}(path)}{C_{in}(path)} = \frac{20}{10}$$



$$\frac{(2 \cdot n + 1)}{3} = \frac{5}{3}$$

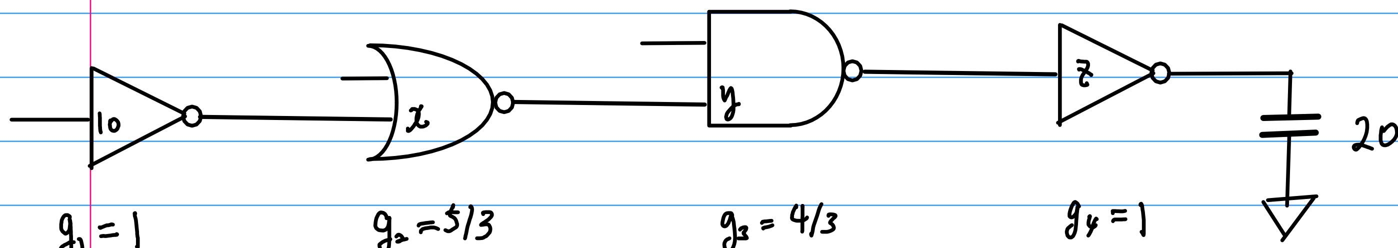


$$\frac{(2 + n)}{3} = \frac{4}{3}$$

$$F = G \cdot H = \frac{20}{9} \times \frac{20}{10} = \frac{20^2}{90}$$

$$\hat{f} = F^{1/4} = \left(\frac{400}{90}\right)^{1/4} = 1.452$$

$$\hat{f} = F^{1/N} = g_i h_i \qquad \boxed{\hat{f},\ g_i \rightarrow h_i}$$



$g_1 = 1$, $h_1 = x/10$

$g_2 = 5/3$, $h_2 = y/x$

$g_3 = 4/3$, $h_3 = z/y$

$g_4 = 1$, $h_4 = 20/z$

$$\hat{f}_1 = \hat{f}_2 = \hat{f}_3 = \hat{f}_4 = F^{1/4} = 1.45$$

$$\longleftarrow$$

$g_1 h_1 = \hat{f}$	$g_2 h_2 = \hat{f}$	$g_3 h_3 = \hat{f}$	$g_4 h_4 = \hat{f}$
$g_1 \frac{x}{10} = 1.45$	$g_2 \frac{y}{x} = 1.45$	$g_3 \cdot \frac{z}{y} = 1.45$	$g_4 \frac{20}{z} = 1.45$
$1 \cdot \frac{14.6}{10} \approx 1.46$	$\left(\frac{5}{3}\right) \frac{12.7}{x} = 1.45$	$\left(\frac{4}{3}\right) \frac{13.8}{y} = 1.45$	$1 \cdot \left(\frac{20}{z}\right) = 1.45$

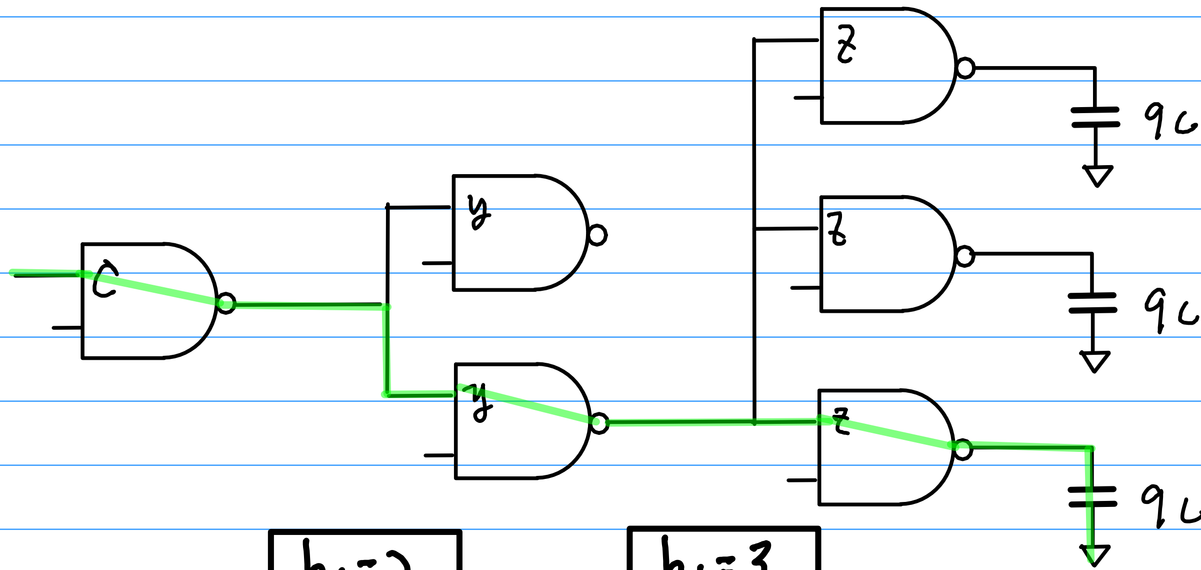
$x = \left(\frac{5}{3}\right) \frac{12.7}{1.45} = 14.6$

$y = \left(\frac{4}{3}\right) \frac{13.8}{1.45} = 12.7$

$z = 1 \cdot \frac{20}{1.45} \approx 13.8$

@ Don't define $H = \prod h_i$ because we don't know h_i until the design is done.

Ex 2)



$\boxed{b_1 = 2}$ $\boxed{b_1 = 3}$

path Logical Effort $G = \left(\frac{4}{3}\right)^3$

path Electrical Effort $H = \frac{9c}{c} = 9$

path Branching Effort $B = 2 \cdot 3 = 6$

Path Effort $F = GHB = \left(\frac{4}{3}\right)^3 \cdot 9 \cdot 6 = 128$

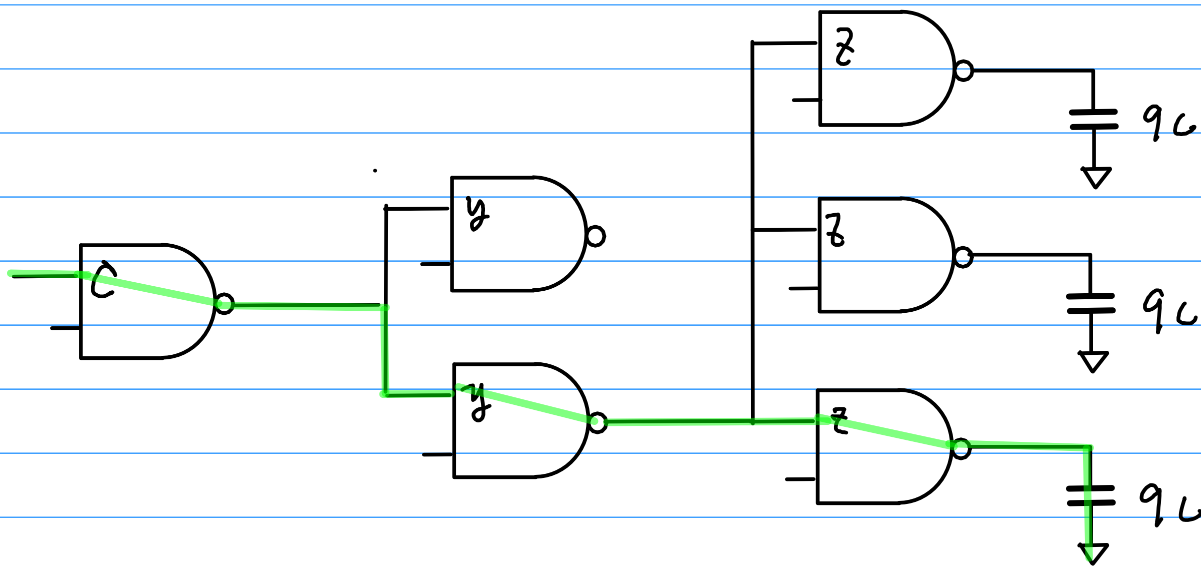
Best Stage Effort $\hat{f} = F^{1/3} = \sqrt[3]{128} \doteqdot 5$

Delay $D = (5+2) + (5+2) + (5+2) = 21$

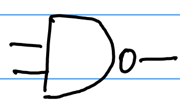
$\left(\frac{4}{3}\right) \frac{9c}{z} = \hat{f} = 5$ $\quad z = \left(\frac{4}{3}\right) \frac{9c}{5} \doteqdot 2.4c$

$\left(\frac{4}{3}\right) \frac{3z}{y} = \hat{f} = 5$ $\quad y = \left(\frac{4}{3}\right) \frac{3z}{5} \doteqdot 1.92c$

Ex 2) B G H

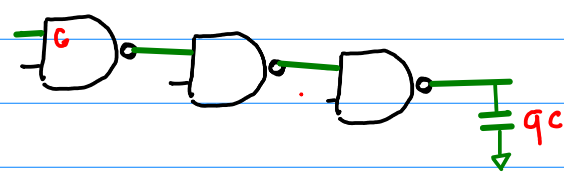


$$G = \left(\frac{4}{3}\right)\left(\frac{4}{3}\right)\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3$$

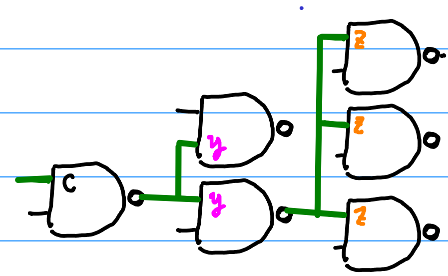


$$\frac{(2+n)}{3} = \frac{4}{3}$$

$$H = \frac{9c}{c} = 9$$



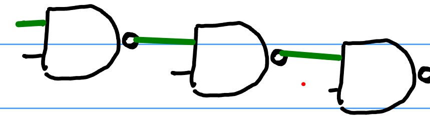
$$B = \left(\frac{2y}{y}\right)\left(\frac{3z}{z}\right) = 2\cdot 3 = 6$$

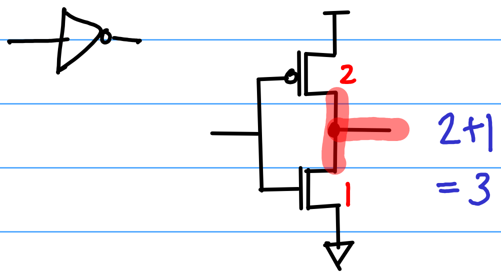


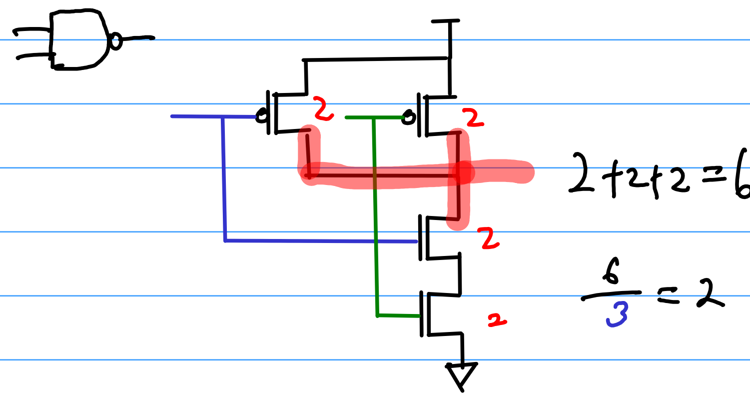
$$F = BGH = 6\cdot\left(\frac{4}{3}\right)^3\cdot 9 = 2\cdot\cancel{3}\,\frac{4^3}{\cancel{3^3}}\,\cancel{9} = 128$$

Ex 2) p

$p = \left(\frac{2+2+2}{3}\right) + \left(\frac{2+2+2}{3}\right) + \left(\frac{2+2+2}{3}\right)$







min $D = N F^{1/N} + p$

$F = 128$

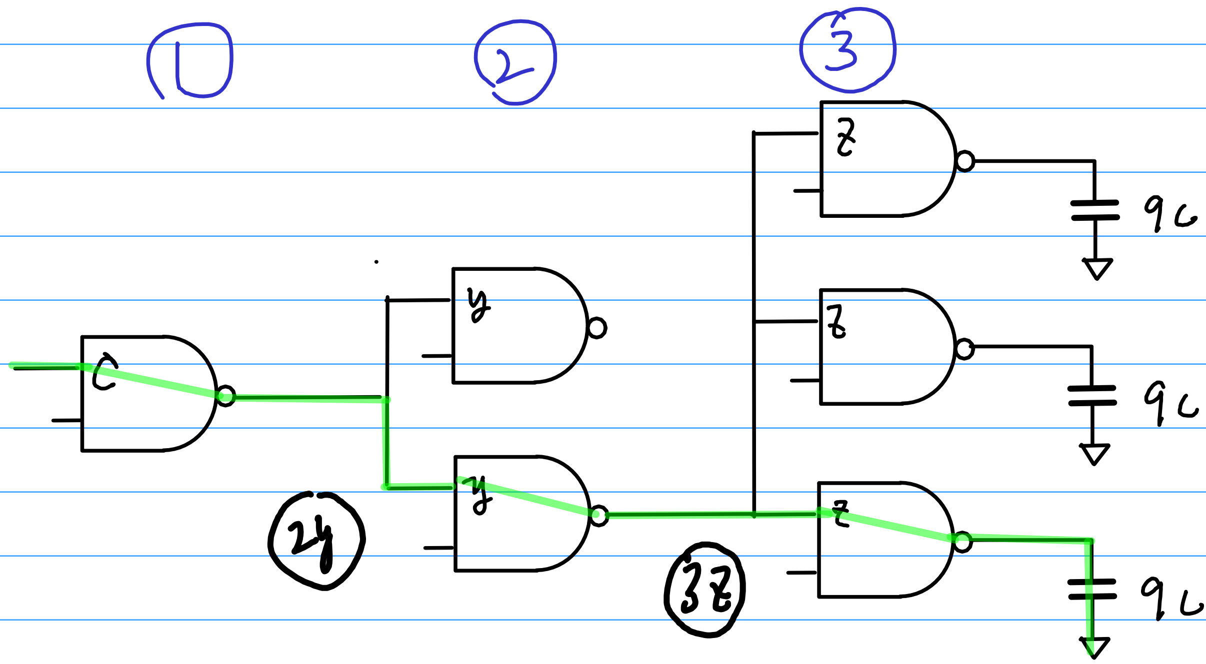
$\hat{f} = \sqrt[3]{128} = 5.0396$

min $D = (5+2)+(5+2)+(5+2) = \sum_i g_i h_i + p_i = \sum_i \hat{f} + p_i$

$= 3 \cdot 5 + 3 \cdot 2$

$= 3 \cdot 5 + 6 = 21$

Ex 2) y, z



$g_1 = \frac{4}{3}$ $\quad g_2 = \frac{4}{3}$ $\quad g_3 = \frac{4}{3}$

$h_1 = \frac{2y}{C}$ $\quad h_2 = \frac{3z}{y}$ $\quad h_3 = \frac{9c}{z}$

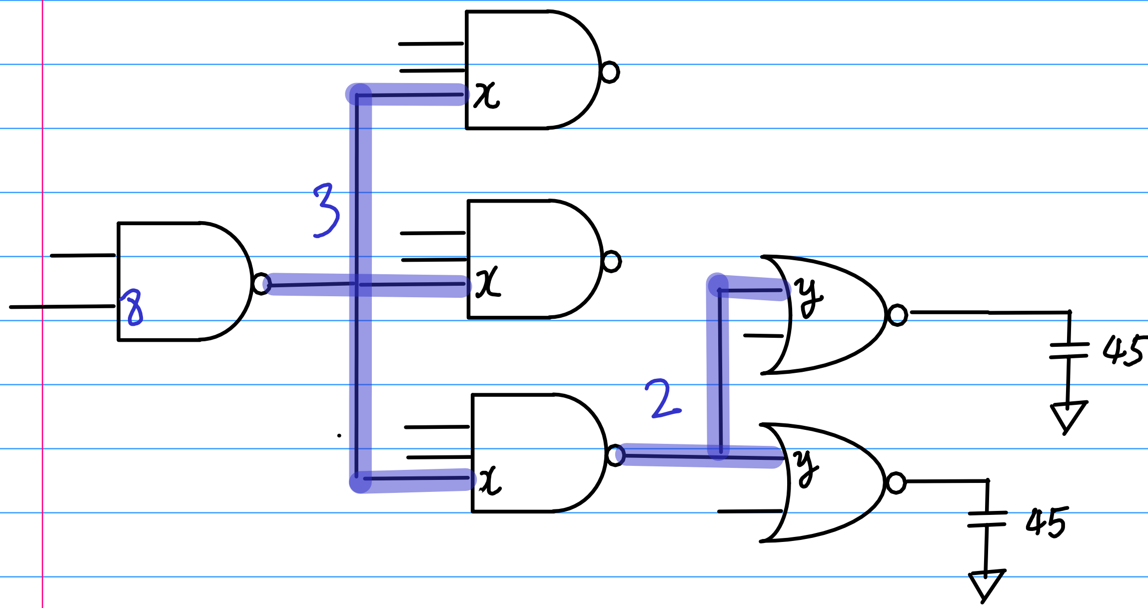
$$\boxed{\hat{f} = g_1 h_1} = \boxed{\hat{f} = g_2 h_2} = \boxed{\hat{f} = g_3 h_3} = \sqrt[3]{128} = 5.0$$

$5 = \left(\frac{4}{3}\right)\frac{2y}{c}$ $\quad 5 = \left(\frac{4}{3}\right)\frac{3z}{y}$ $\quad 5 = \left(\frac{4}{3}\right)\frac{9c}{z}$

$= \left(\frac{4}{3}\right)\frac{2}{c}\, 1.92c$ $\quad y = \left(\frac{4}{3}\right)\frac{3z}{5}$ $\quad z = \left(\frac{4}{3}\right) 9c/5$

$= 5.12$ $\quad \boxed{y = 1.92C}$ $\quad \boxed{z = 2.4C}$

Ex 3) $F = GBH$



$g_1 = \frac{2+2}{1+2} = \frac{4}{3}$ $g_2 = \frac{3+2}{1+2} = \frac{5}{3}$ $g_3 = \frac{1+2\cdot2}{1+2} = \frac{5}{3}$

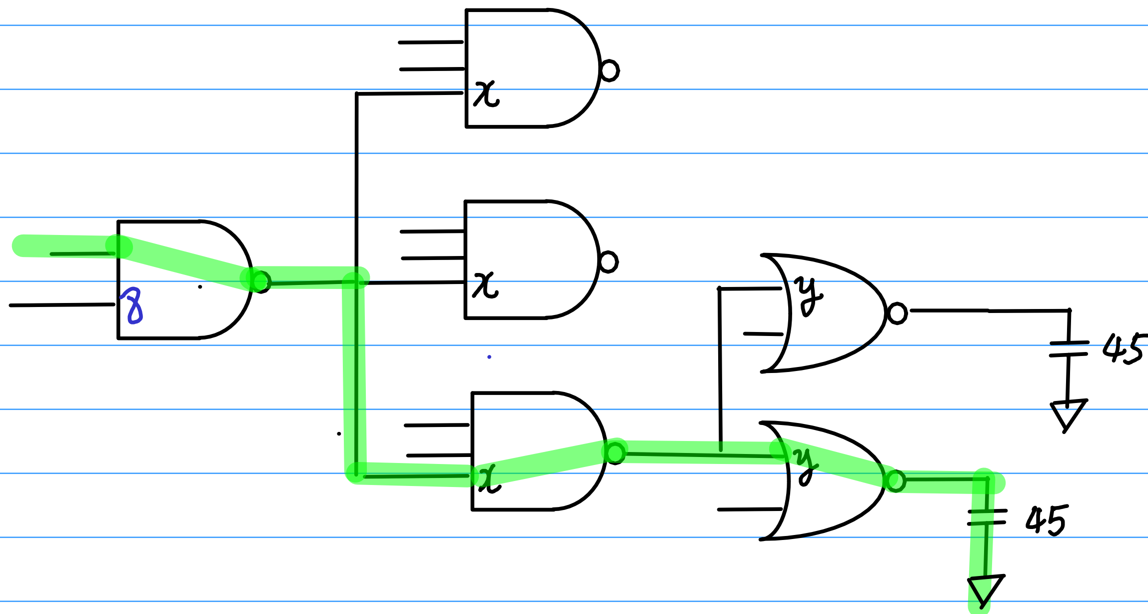
$G = g_1 g_2 g_3 = \frac{4}{3} \cdot \frac{5}{3} \cdot \frac{5}{3} = \frac{100}{27}$

$H = \frac{45}{8}$

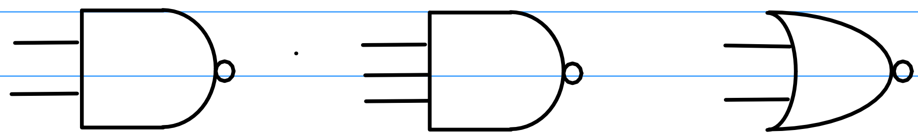
$B = 3 \times 2 = 6$

$F = GBH = \frac{100}{27} \frac{45}{8} 6 = 125$ $\hat{f} = \sqrt[3]{125} = 5$

Ex3) P Parasitic Delay



$P = 2 + 3 + 2 = 7$

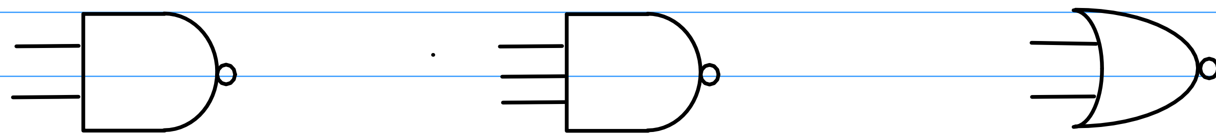


2 inputs
$\Rightarrow p = 2$

3 inputs
$\Rightarrow p = 3$

2 inputs
$\Rightarrow p = 2$

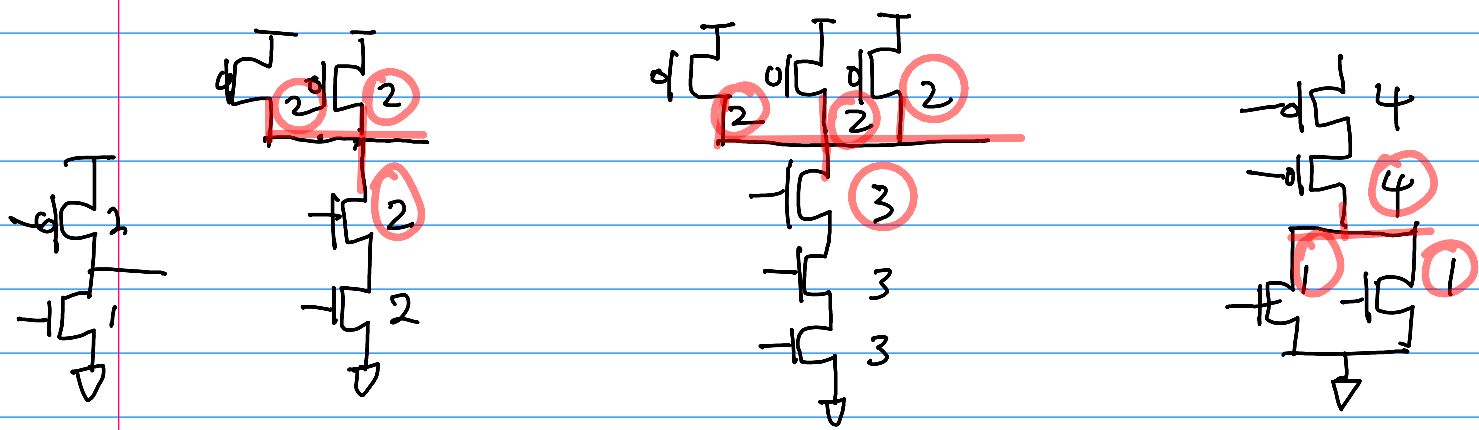
Ex3) P Parasitic Delay



2 inputs $\Rightarrow p=2$

3 inputs $\Rightarrow p=3$

2 inputs $\Rightarrow p=2$



$\frac{3}{3}=1$ $\frac{6}{3}=2$ $\frac{9}{3}=3$ $\frac{6}{3}=2$

Ex3) Min Path Delay

min path delay $D = NF^{1/N} + P$

$= 3\sqrt[3]{125} + 7 = 3 \cdot 5 + 7 = 22$

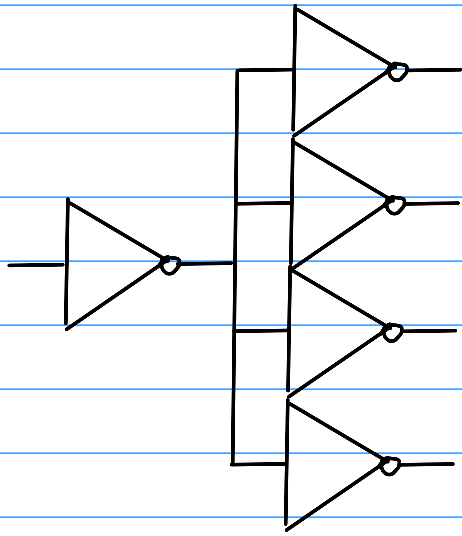
$22 / 5 = 4.4 \times$ FO4 delay

FO4 : $d = h + 1 = 5$

FO4 (Invater with fanout = 4)

$g \cdot h + p = h + 1 = 5$

Ex 3) FO4 Inverter Delay



Fanout-of-4

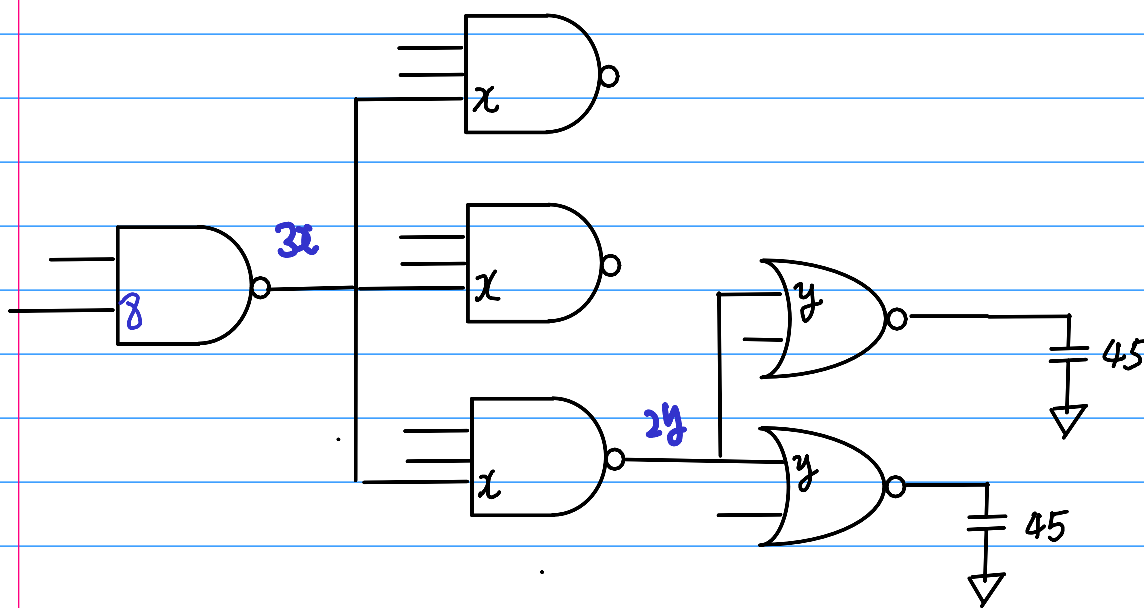
$g = 1$
$h = 4$
$p = p_{inv} = 1$

$$\boxed{d = gh + p = 5}$$

Useful metric to
characterize process

5τ

Ex3) Input Cap x, y



$g_1 = \frac{2+2}{1+2} = \frac{4}{3}$ $\quad g_2 = \frac{3+2}{1+2} = \frac{5}{3}$ $\quad g_3 = \frac{1+2\cdot 2}{1+2} = \frac{5}{3}$ $\quad \hat{f} = \sqrt[3]{125} = 5$

$$\boxed{\begin{gathered} g_3 h_3 = \hat{f} \\ \frac{5}{3} \cdot \frac{45}{y} = 5 \end{gathered}}$$

$y = 45 \cdot (5/3)/5 = 15$

$$\boxed{\begin{gathered} g_2 h_2 = \hat{f} \\ \frac{5}{3} \cdot \frac{2y}{x} = 5 \end{gathered}}$$

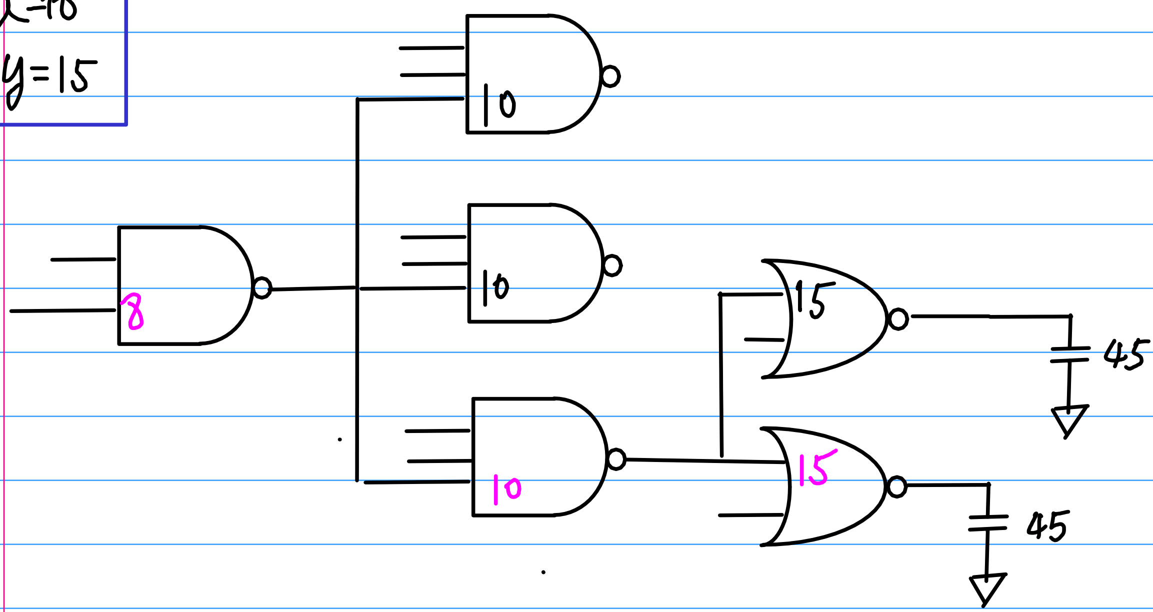
$x = 2 \cdot 15 \cdot (5/3)/5 = 10$

$$\boxed{\begin{gathered} g_1 h_1 = \hat{f} \\ \frac{4}{3} \cdot \frac{3 \cdot x}{8} = 5 \end{gathered}}$$

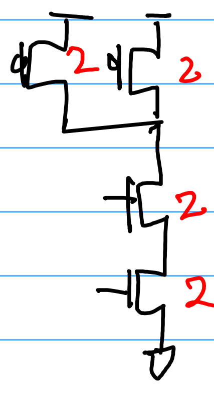
$= \frac{4}{\cancel{3}} \cdot \frac{\cancel{3} \cdot 10}{\cancel{8}\, 2}$ o.k.

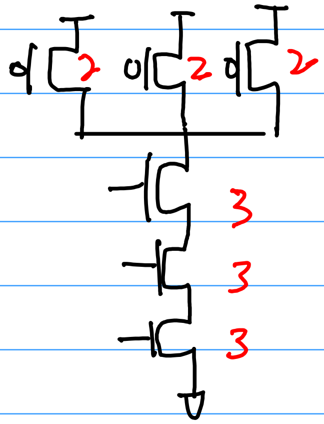
Ex 3) Tr Sizing Info

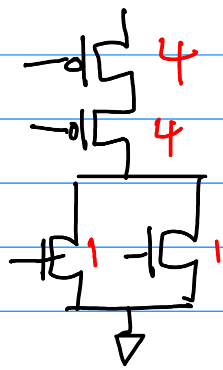
$\boxed{\begin{array}{l} x = 10 \\ y = 15 \end{array}}$



$g_1 = \frac{2+2}{1+2} = \frac{4}{3}$ $\qquad g_2 = \frac{3+2}{1+2} = \frac{5}{3}$ $\qquad g_3 = \frac{1+2\cdot 2}{1+2} = \frac{5}{3}$







2 : 2 P/N ratio $\qquad$ 2 : 3 P/N ratio $\qquad$ 4 : 1 P/N ratio

$C_{in} = 8$ $\qquad C_{in} = 10$ $\qquad C_{in} = 15$

4 : 4 $\qquad$ 4 : 6 $\qquad$ 12 : 3

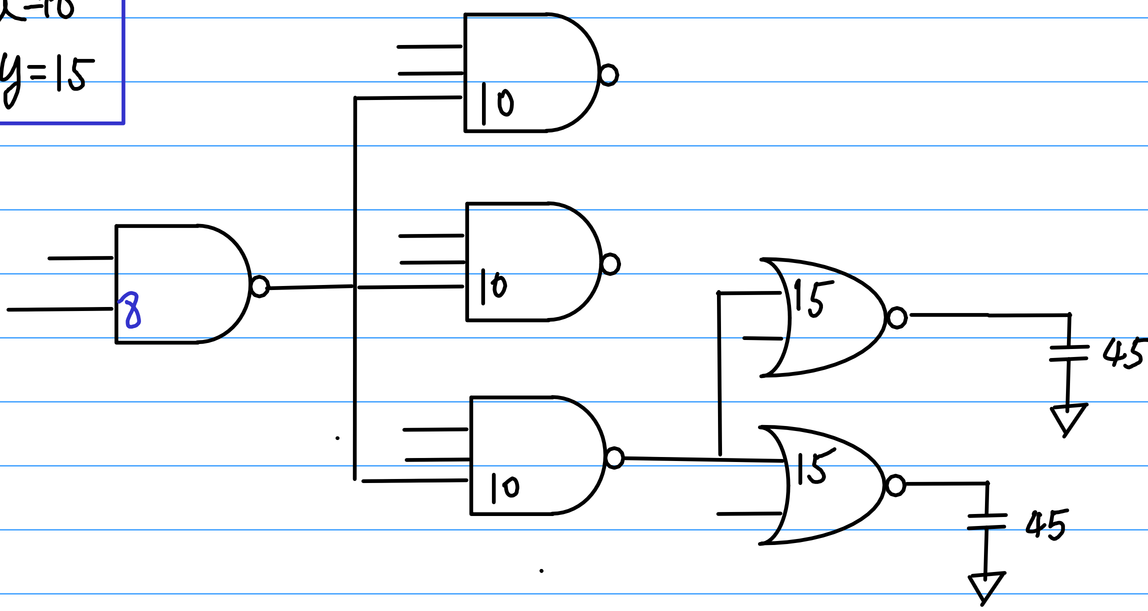
↑ ↑ $\qquad$ ↑ ↑ $\qquad$ ↑ ↑

P N $\qquad$ P N $\qquad$ P N

$8 = 4 + 4$ $\qquad 10 = 4 + 6$ $\qquad 15 = 12 + 3$

Ex3) Verification

$x = 10$
$y = 15$



$g_1 = \frac{2+2}{1+2} = \frac{4}{3}$ $\quad g_2 = \frac{3+2}{1+2} = \frac{5}{3}$ $\quad g_3 = \frac{1+2\cdot 2}{1+2} = \frac{5}{3}$

$h_1 = \frac{10\times 3}{8}$ $\quad h_2 = \frac{15\times 2}{10}$ $\quad h_3 = \frac{45}{15}$

$p_1 = 2$ $\quad p_2 = 3$ $\quad p_3 = 2$

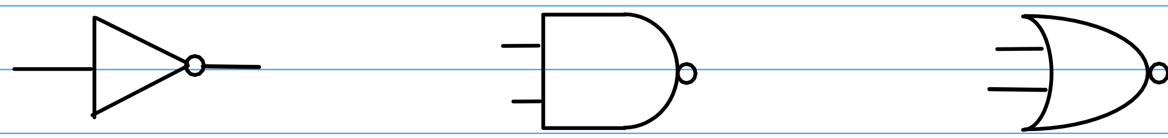
$d_1 = g_1 h_1 + p_1 = \frac{4}{3}\times\frac{30}{8} + 2 = 7$

$d_2 = g_2 h_2 + p_2 = \frac{5}{3}\times\frac{15\cdot 2}{10} + 3 = 8$

$d_3 = g_3 h_3 + p_3 = \frac{5}{3}\times\frac{45}{15} + 2 = 7$

$d_1 + d_2 + d_3 = 7 + 8 + 7 = 22$

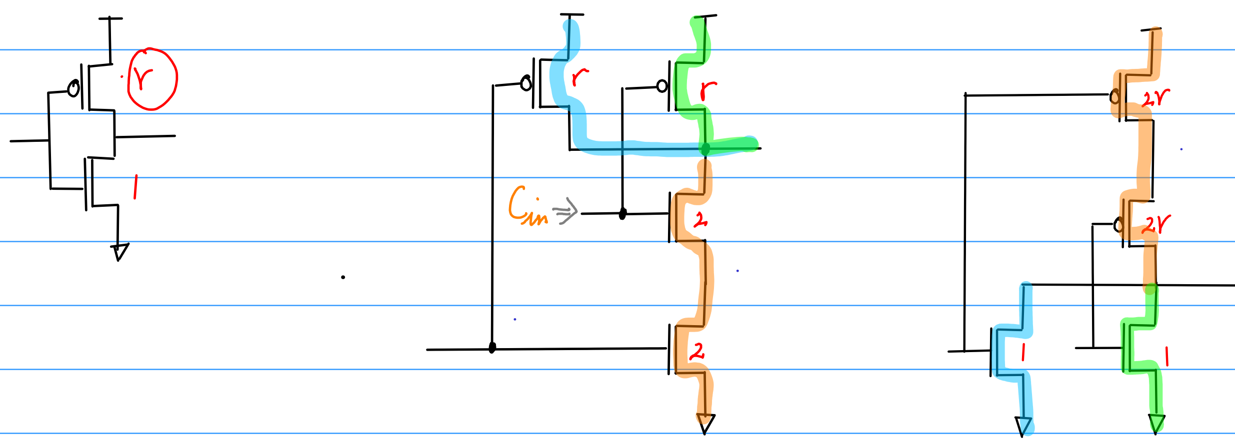
C_{in} using r & C_{Gn}



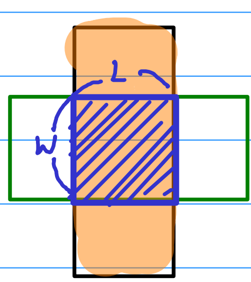
$\boxed{C_{in} = (1+r)\,C_{Gn}}$ $\approx 3.5\,C_{Gn}$

$\boxed{C_{in} = (2+r)\,C_{Gn}}$ $\approx 4.5\,C_{Gn}$

$\boxed{C_{in} = (1+2r)\,C_{Gn}}$ $\approx 6\,C_{Gn}$

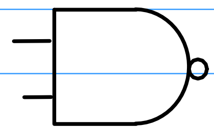


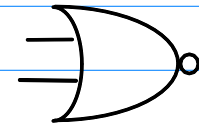
$$\boxed{C_{Gn} = C_{ox}\,L\,W_n}$$



1 $\rightarrow$ W_n

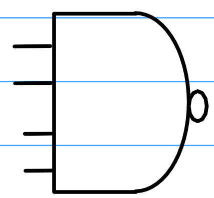
Logical Efforts using γ

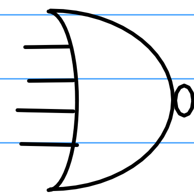




$$g = \frac{2+\gamma}{1+\gamma}$$

$$g = \frac{1+2\gamma}{1+\gamma}$$

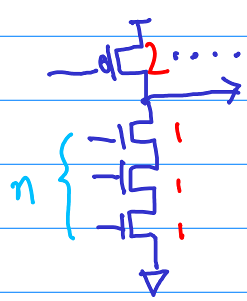




$$g = \frac{n+\gamma}{1+\gamma}$$

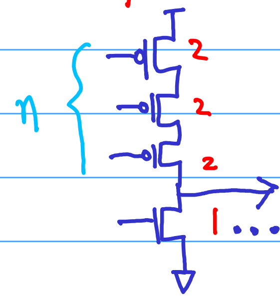
$$g = \frac{1+n\gamma}{1+\gamma}$$

n-input NAND



$(1 \cdot n + 2)$

n-input NOR



$(1 + 2 \cdot n)$

Bigger Inverter

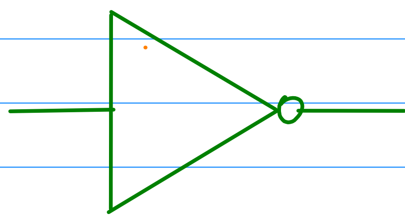
to minimize t_s (generic switching Delay)

$$t_s = t_0 + \alpha C_L \Rightarrow t_0 + \overset{R}{\frac{\alpha}{S}}\,\overset{C}{C_L}$$

bigger size

$C_{in}\uparrow$ $\qquad$ $\alpha\downarrow \Rightarrow R\downarrow \Rightarrow \beta\uparrow$

$\boxed{S\,C_{in}}$



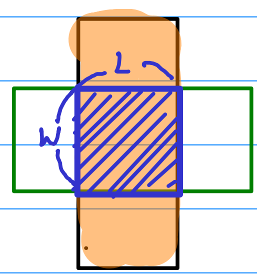
$\boxed{\frac{\alpha}{S}}$ $\boxed{\frac{R}{S}}$ $\boxed{S\,\beta}$

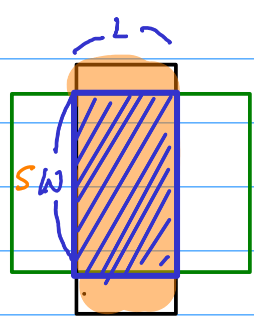
Scaling Factor S

$\Rightarrow$

$$\boxed{\begin{aligned} W_n' &= S\,W_n \\ C_{in}' &= S\,C_{in} \end{aligned}}$$

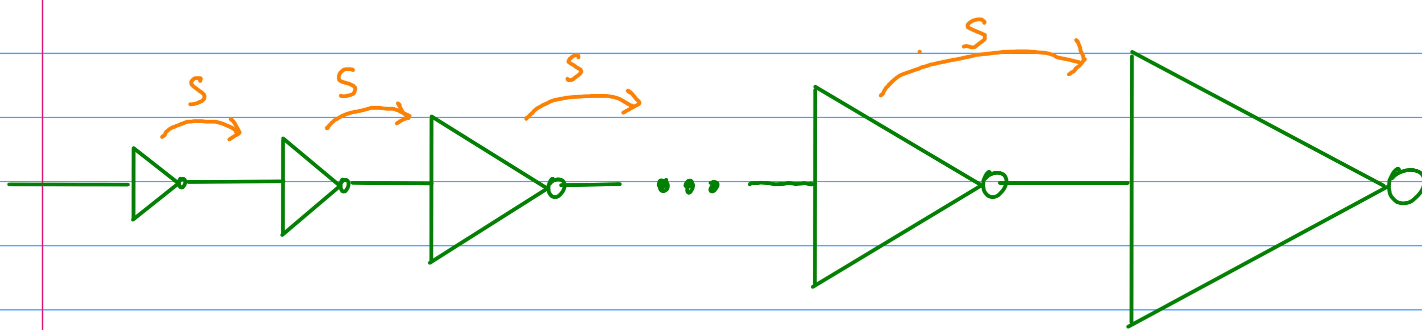
cause problem to the driver.





$$\boxed{\begin{aligned} \beta' &= S\,\beta \\ R' &= \frac{R}{S} \\ \alpha' &= \frac{\alpha}{S} \end{aligned}}$$

think G.P. (Geometric Progression)
in size factors



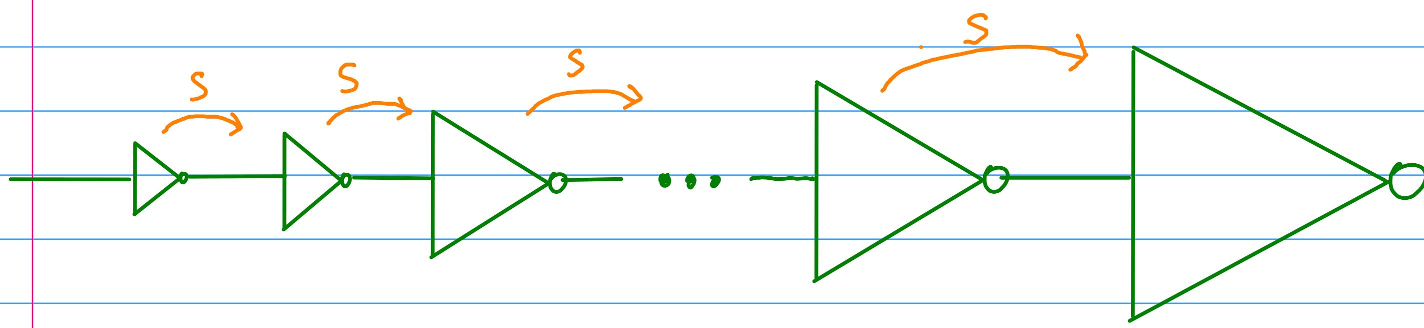
* Arithmetic Progression $\geq$ Geometric Progression

Σ path delay

↓
min delay.
when all equal
path delay

Delay Minimization in an inverter cascade

* increasing size



$$\beta_1 < \beta_2 < \beta_3 < \cdots < \beta_{N-1} < \beta_N$$

$$\beta_2 = s\beta_1$$
$$\beta_3 = s\beta_2 = s^2\beta_1$$
$$\beta_4 = s\beta_3 = s^3\beta_1$$

Geometric progress

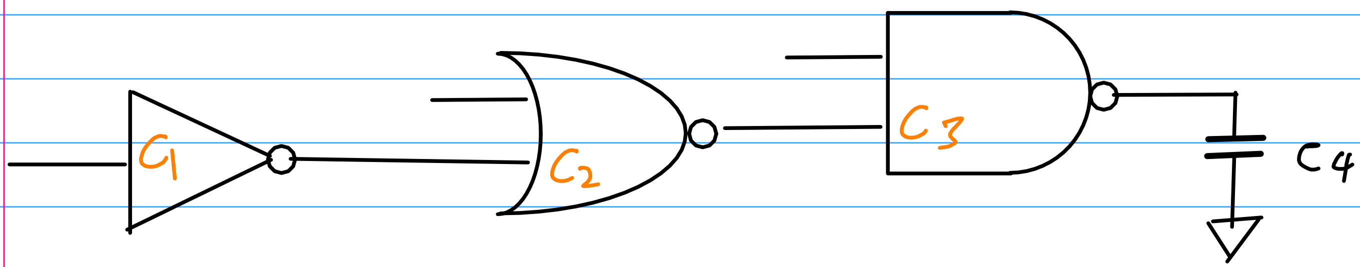
$$\beta_j = s\beta_{j-1} = s^{j-1}\beta_1$$

$$\boxed{\beta_j = s^{j-1}\beta_1}$$

$$C_j = s^{j-1} C_1$$
$$R_j = \frac{R_1}{s^{j-1}}$$

common ratio = $s \rightarrow e$ for min delay

Ex4) $F = GH$



$$G = (1)\left(\frac{1+2r}{1+r}\right)\left(\frac{2+r}{1+r}\right) = 2.2 \quad (r = 2.5)$$

$$H = \frac{C_4}{C_1} = \frac{500}{20} = 25$$

$$F = GH = 2.2 \times 25 = 55$$

the optimum stage effort

$$\hat{f} = F^{1/N} = (55)^{1/3} = 3.8$$

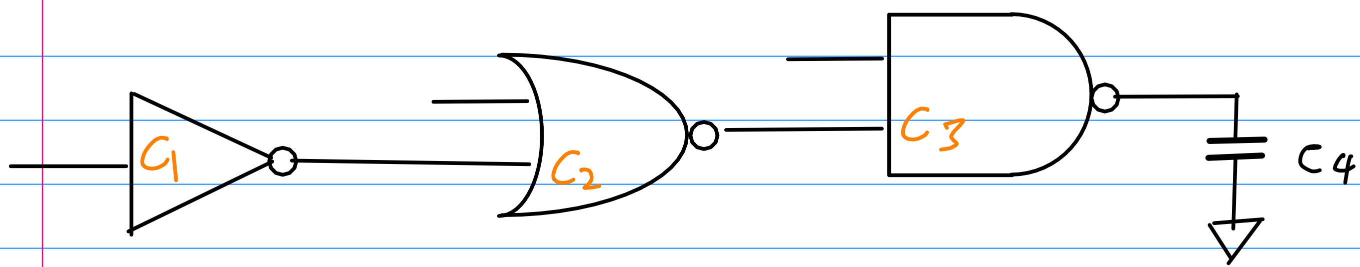
parasitic delay term

$$P = P_1 + P_2 + P_3$$

the total path delay

$$\hat{D} = NF^{1/N} + P = 3(3.8) + P = 11.41 + P$$

Ex4) Input Capacitance C_1, C_2, C_3



$$\boxed{g_3 h_3 = \hat{f} = 3.8}$$

$$\frac{C_4}{C_3} = h_3 = \hat{f} / g_3$$

$$\boxed{C_{in} = (2+r)\, C_{Gn}}$$

$g_3 = (2+r)/(1+r) = 4.5/3.5 = 1.29$ (NAND)

$h_3 = \hat{f} / g_3 = 3.8 / 1.29 = 2.95 = C_4 / C_3$

$C_3 = 500 / 2.95 = \boxed{169.5}$

$$C_3 = S_3 (2+r)\, C_{Gn}$$
$$= S_3 (4.5\, C_{GN})$$

$$\boxed{g_2 h_2 = \hat{f} = 3.8}$$

$$\frac{C_3}{C_2} = h_2 = \hat{f} / g_2$$

$$\boxed{C_{in} = (1+2r)\, C_{Gn}}$$

$g_3 = (1+2r)/(1+r) = 6/3.5 = 1.71$ (NOR)

$h_2 = \hat{f} / g_2 = 3.8 / 1.71 = 2.22 = C_3 / C_2$

$C_2 = 169.5 / 2.22 = \boxed{76.35}$

$$C_2 = S_2 (1+2r)\, C_{Gn}$$
$$= S_2 (6\, C_{GN})$$

$$\boxed{g_1 h_1 = \hat{f} = 3.8}$$

$$\frac{C_2}{C_1} = h_1 = \hat{f} / g_1$$

$$\boxed{C_{in} = (1+r)\, C_{Gn}}$$

$g_1 = 1$ (INV)

$h_1 = \hat{f} / g_1 = 3.8 / 1 = 3.8 = C_2 / C_1$

$C_1 = 76.35 / 3.8 = \boxed{20}$

$$C_1 = S_1 (1+r)\, C_{Gn}$$
$$= S_1 (3.5\, C_{Gn})$$

Ex4) Scaling Factors S_1, S_2, S_3

NOT	$C_{in} = C_{Gn}(1+r) = 3.5\,C_{Gn}$
NAND	$C_{in} = C_{Gn}(2+r) = 4.5\,C_{Gn}$
NOR	$C_{in} = C_{Gn}(1+2r) = 6\,C_{Gn}$
	$C_{Gn} = C_{ox} W_n L$

$(r = 2.5)$

$\boxed{20}$ $\quad C_1 = C_{ref} = C_{GN}(1+r) = 3.5\,C_{GN} = 20$

$$C_1 = S_1\,C_{GN}(1+r) = S_1\,(3.5\,C_{GN})$$

$$S_1 = \frac{C_1}{3.5\,C_{GN}} = \frac{20}{3.5\,C_{GN}} = \frac{5.71}{C_{Gn}}$$

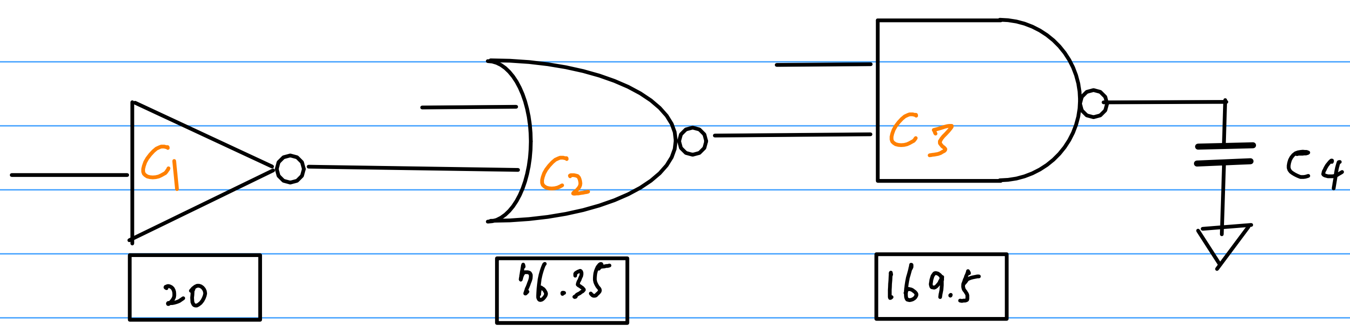
$\boxed{76.35}$ $\quad C_2 = S_2\,C_{GN}(1+2r) = S_2\,(6\,C_{GN})$

$$S_2 = \frac{C_2}{6\,C_{Gn}} = \frac{76.35}{6\,C_{Gn}} = \frac{12.725}{C_{Gn}}$$

$\boxed{169.5}$ $\quad C_3 = S_3\,C_{GN}(2+r) = S_3\,(4.5\,C_{GN})$

$$S_3 = \frac{C_3}{4.5\,C_{Gn}} = \frac{169.5}{4.5\,C_{Gn}} = \frac{37.67}{C_{Gn}}$$

$C_1 = S_1\,C_{Gn}(1+r)$ $\qquad$ $C_2 = S_2\,C_{Gn}(1+2r)$ $\qquad$ $C_3 = S_3\,C_{Gn}(2+r)$



Ex4) Scaling Factors S_1', S_2', S_3'

$$S_1 = \frac{C_1}{3.5\,C_{GN}} = \frac{20}{3.5\,C_{GN}} = \frac{5.71}{C_{Gn}}$$

$$S_2 = \frac{C_2}{6\,C_{Gn}} = \frac{76.35}{6\,C_{Gn}} = \frac{12.725}{C_{Gn}}$$

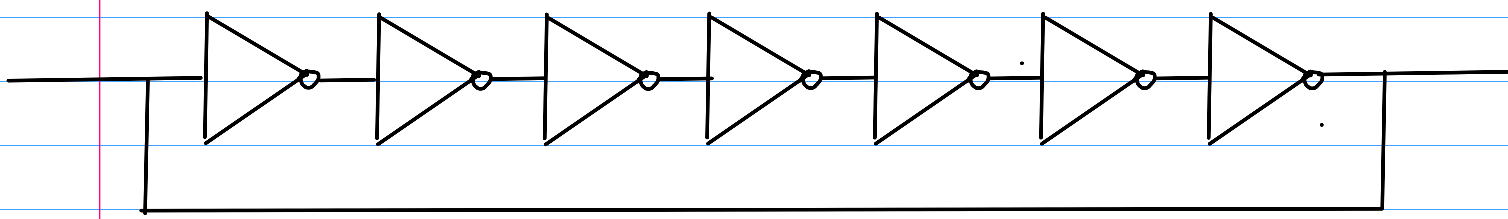
$$S_3 = \frac{C_3}{4.5\,C_{Gn}} = \frac{169.5}{4.5\,C_{Gn}} = \frac{37.67}{C_{Gn}}$$

$$S_1' = \frac{5.71}{C_{Gn}} \Big/ \frac{5.71}{C_{Gn}} = 1$$

$$S_2' = \frac{12.725}{C_{Gn}} \Big/ \frac{5.71}{C_{Gn}} = 2.23$$

$$S_3' = \frac{37.67}{C_{Gn}} \Big/ \frac{5.71}{C_{Gn}} = 6.60$$

N inverters (odd number)



$$g = \frac{C_{in}}{C_{ref}} = \frac{C_{ref}}{C_{ref}} = 1$$

$$h = \frac{C_{out}}{C_{in}} = \frac{C_{ref}}{C_{ref}} = 1$$

$$p = \left(\frac{C_{p,ref}}{C_{ref}}\right) = \left(\frac{\text{internal diffusion cap}}{\text{gate cap of ref inv}}\right) = \frac{3}{3} = 1$$

$$d = gh + p = 2 \qquad d_{abs} = 2\tau$$

$$freq = \frac{1}{2N\,d_{abs}} = \frac{1}{4N\tau}$$