DLTI Impulse Response (1A)

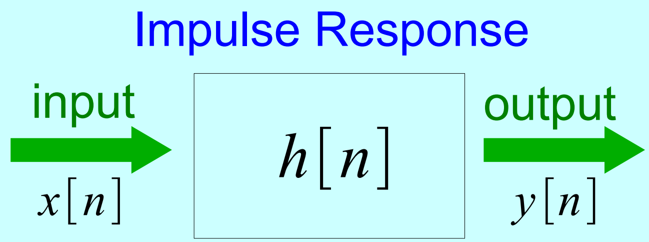
-

Please send corrections (or suggestions) to youngwlim@hotmail.com.

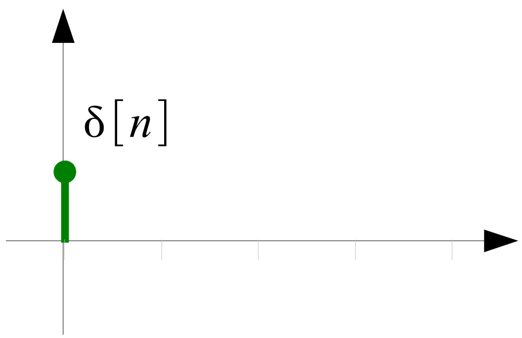
This document was produced by using OpenOffice and Octave.

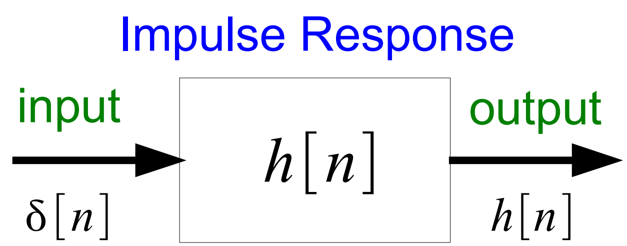
Finite Impulse Response (1)

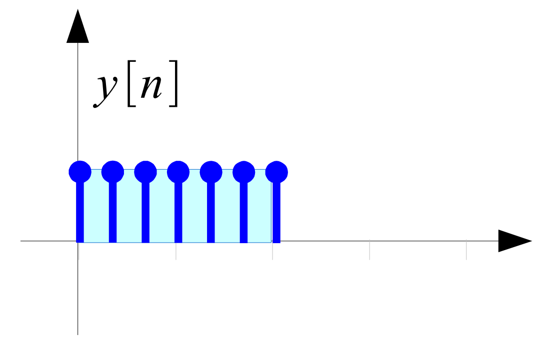
y[n] as a
convolution sum



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$



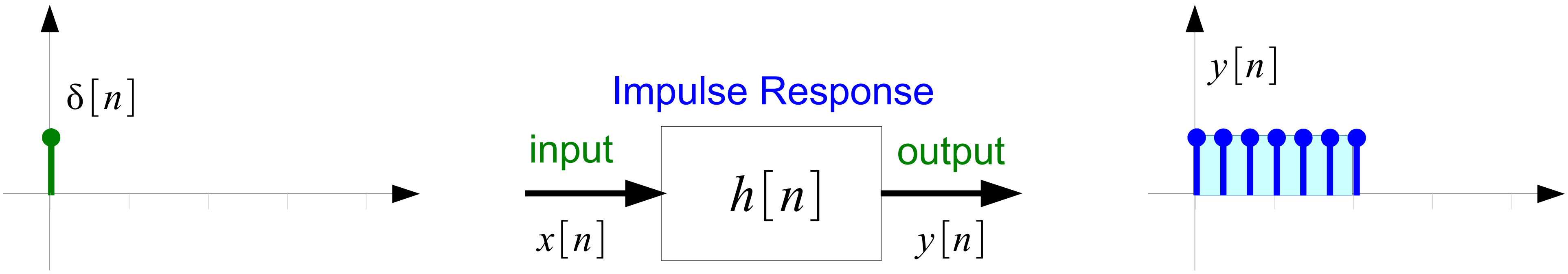




Special Case: ***h*[*n*]** has a finite duration

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

Finite Impulse Response (2)



Special Case: ***h*[*n*]** has a finite duration

y[n] as a convolution sum

$$y[n] = \sum_{k=0}^{M} h[k]\, x[n-k]$$

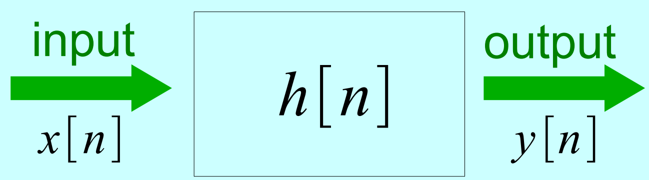
y[n] as a difference equation

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

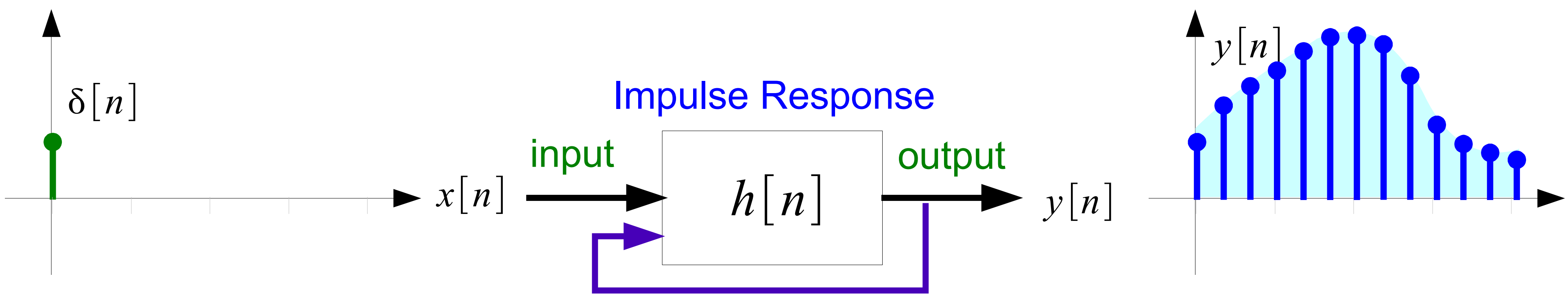
FIR (Finite Impulse Response) Filter

Infinite Impulse Response (1)

y[n] as a **convolution sum**



$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

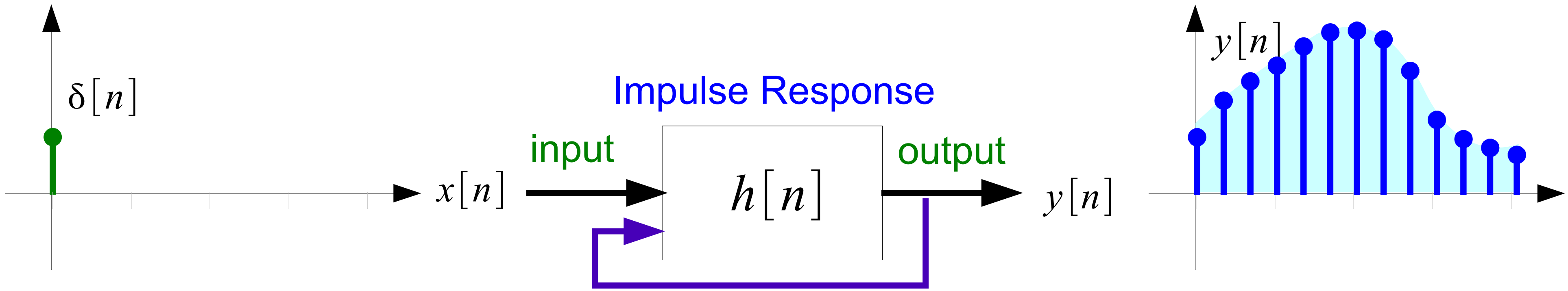


Special Case: ***Feedback***

***h*[*n*]** has a <u>infinite</u> duration

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

Infinite Impulse Response (2)



Special Case: ***Feedback***

h[n] has a <u>infinite</u> duration

y[n] as a **convolution sum**

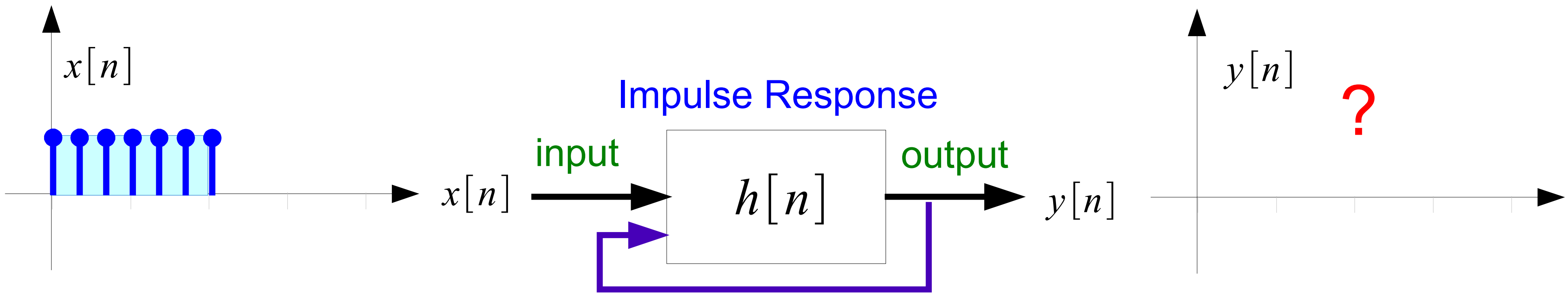
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

y[n] as a **difference equation**

$$\begin{aligned} y[n] &= a_1 y[n-1] + a_2 y[n-2] + \cdots + a_N y[n-N] \\ &\quad + b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M] \end{aligned}$$

IIR (Infinite Impulse Response) Filter

Infinite Impulse Response (3)



$$y[n] = a_1\, y[n-1] + b_0\, x[n]$$

$$\begin{aligned}
y[0] &= a_1\, \overset{0}{y[-1]} + b_0\, x[0] &&= b_0 &&= b_0 \\
y[1] &= a_1\, y[0] + b_0\, x[1] &&= a_1 b_0 + b_0 &&= b_0(a_1 + 1) \\
y[2] &= a_1\, y[1] + b_0\, x[2] &&= a_1(a_1 b_0 + b_0) + b_0 &&= b_0(a_1^2 + a_1 + 1) \\
y[3] &= a_1\, y[2] + b_0\, x[3] &&= a_1(a_1^2 b_0 + a_1 b_0 + b_0) + b_0 &&= b_0(a_1^3 + a_1^2 + a_1 + 1)
\end{aligned}$$

$$y[M] = a_1\, y[M-1] + b_0\, x[M] \qquad = b_0(a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

$$\begin{cases} S_N = (a_1^M + a_1^{M-1} + \cdots + a_1 + 1) \\ a_1 S_N = (a_1^{M+1} + a_1^M + \cdots + a_1^2 + a_1) \end{cases}$$

$$S_N = (a_1^M + a_1^{M-1} + \cdots + a_1 + 1)$$

Infinite Impulse Response (4)

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$y[M] = a_1 y[M-1] + b_0 x[M] = b_0(\underline{a_1^M + a_1^{M-1} + \cdots + a_1 + 1})$$

Geometric Sequence

$$\begin{aligned} S_N &= (a_1^M + a_1^{M-1} + \cdots + a_1 + 1) \\ a_1 S_N &= (a_1^{M+1} + a_1^M + \cdots + a_1^2 + a_1) \end{aligned}$$

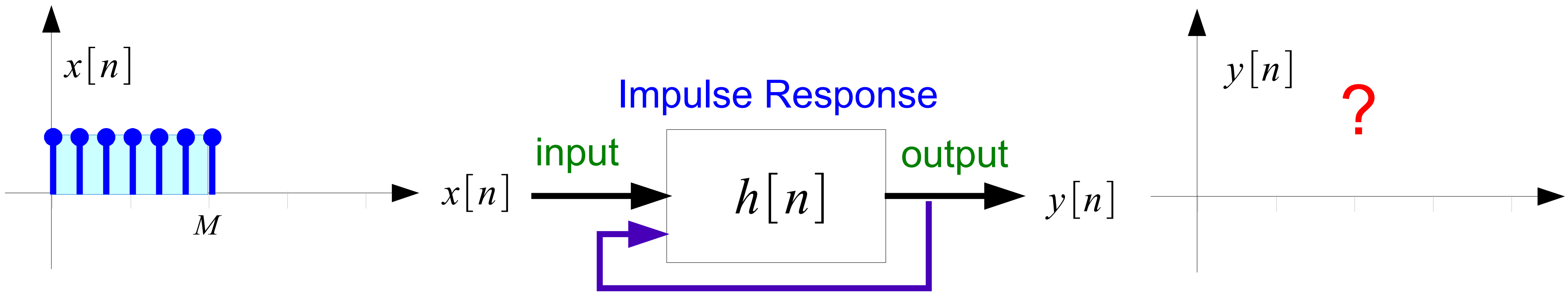
$$(1-a_1) S_N = 1 - a_1^{M+1}$$

$$S_N = \begin{cases} \dfrac{1 - a_1^{M+1}}{1 - a_1} & (a_1 \neq 1) \\ M & (a_1 = 1) \end{cases}$$

$$\lim_{N \to \infty} S_N = \frac{1}{1 - a_1} \qquad (\,|a_1| < 1\,)$$

$$y[M] = b_0(a_1^M + a_1^{M-1} + \cdots + a_1 + 1) = b_0 \frac{1 - a_1^{M+1}}{1 - a_1}$$

Infinite Impulse Response (4)



$$y[n] = a_1\, y[n-1] + b_0\, x[n]$$

$$x[n] = \delta[n] \;\rightarrow\; y[n] = h[n]$$

$$h[n] = a_1\, h[n-1] + b_0\, \delta[n]$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] R.D. Strum, et al., Discrete Systems and Digital Signal Processing