

# DLTI Impulse Response (1A)

---

-

Copyright (c) 2011 Young W. Lim.

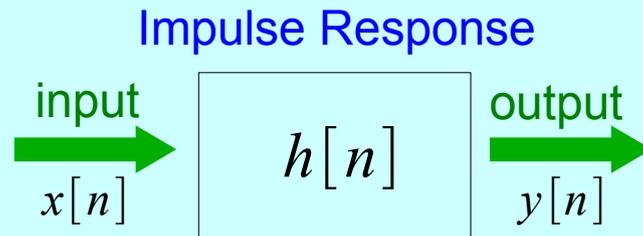
Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

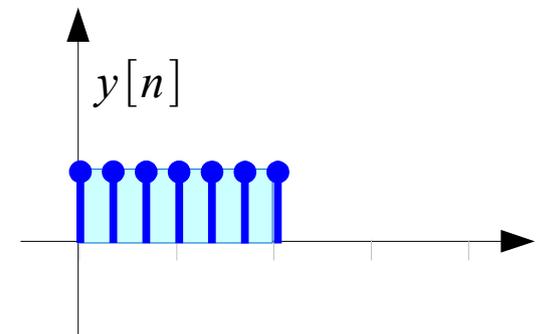
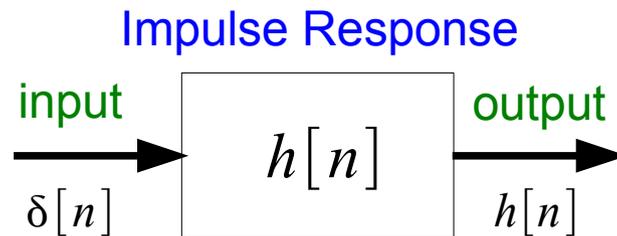
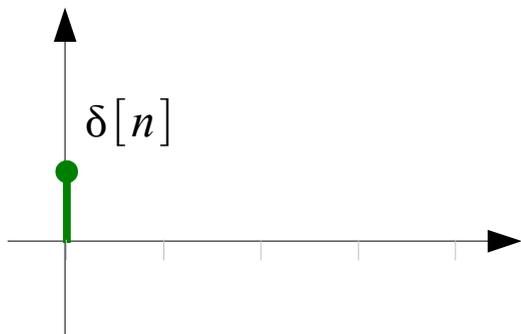
This document was produced by using OpenOffice and Octave.

# Finite Impulse Response (1)

$y[n]$  as a convolution sum



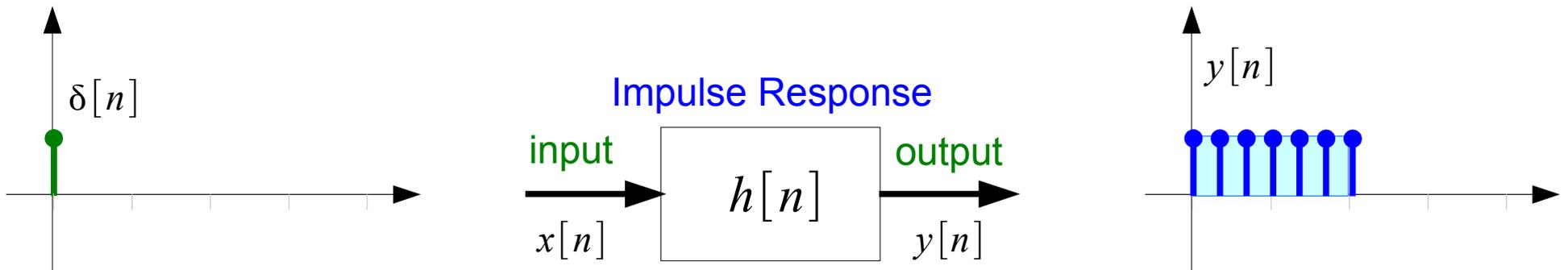
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$



Special Case:  $h[n]$  has a finite duration

$$y[n] = \sum_{k=0}^M h[k] x[n-k]$$

# Finite Impulse Response (2)



Special Case:  $h[n]$  has a finite duration

$y[n]$  as a  
convolution sum



$$y[n] = \sum_{k=0}^M h[k] x[n-k]$$

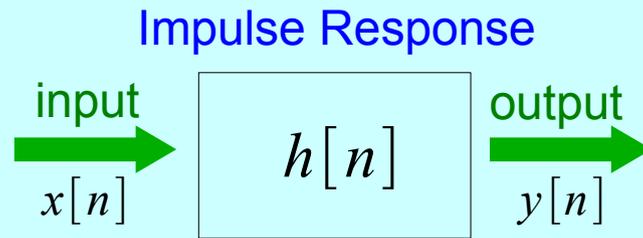
$y[n]$  as a  
difference equation

$$y[n] = b_0 x[n] + b_1 x[n-1] + \dots + b_M x[n-M]$$

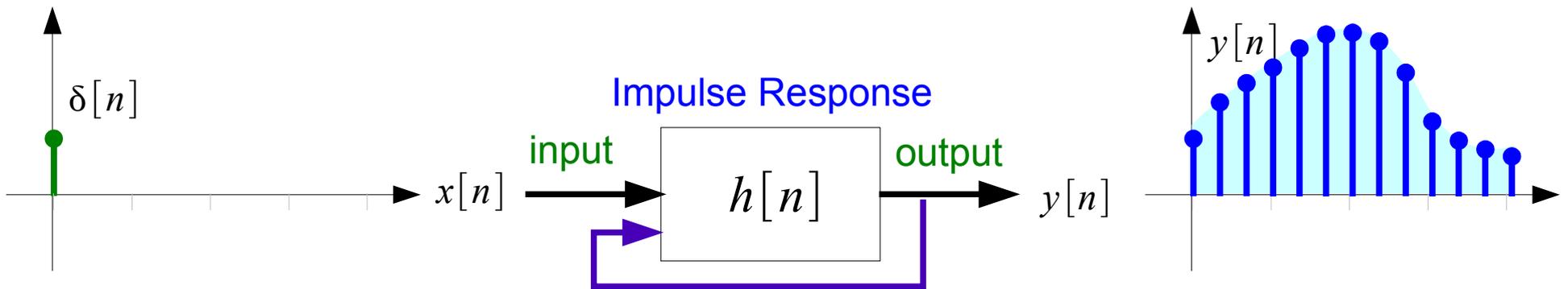
FIR (Finite Impulse Response) Filter

# Infinite Impulse Response (1)

$y[n]$  as a convolution sum



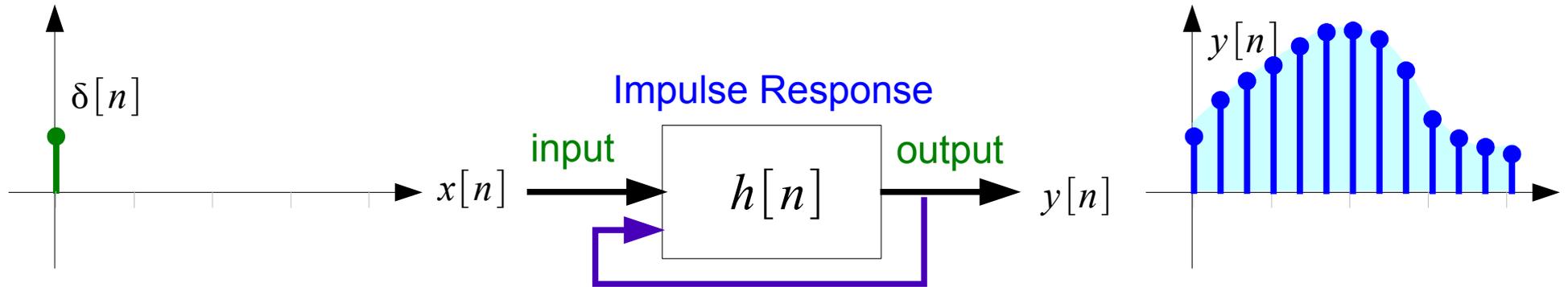
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$



Special Case: **Feedback**  
 $h[n]$  has a infinite duration

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

# Infinite Impulse Response (2)



Special Case: **Feedback**  
 $h[n]$  has a infinite duration

$y[n]$  as a  
convolution sum

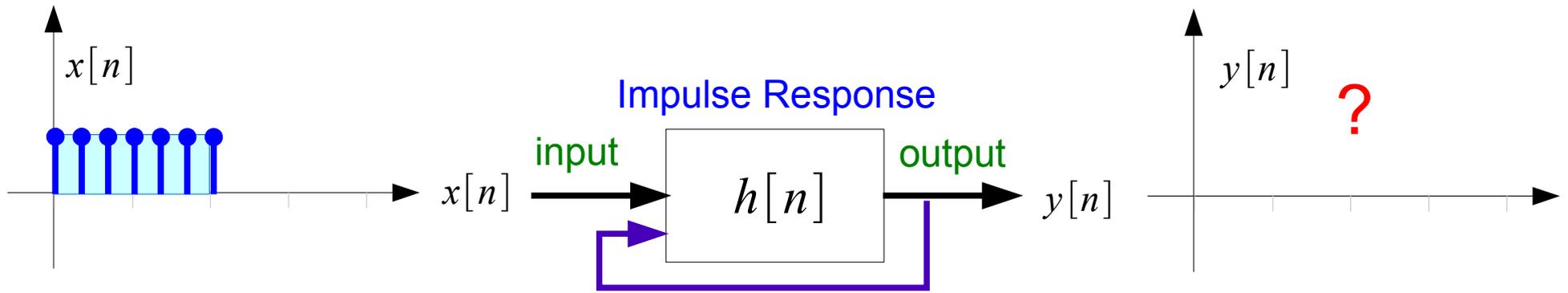
$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

$y[n]$  as a  
difference equation

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + \cdots + a_N y[n-N] \\ + b_0 x[n] + b_1 x[n-1] + \cdots + b_M x[n-M]$$

IIR (Infinite Impulse Response) Filter

# Infinite Impulse Response (3)



$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$y[0] = a_1 y[-1] + b_0 x[0] = b_0 = b_0$$

$$y[1] = a_1 y[0] + b_0 x[1] = a_1 b_0 + b_0 = b_0(a_1 + 1)$$

$$y[2] = a_1 y[1] + b_0 x[2] = a_1(a_1 b_0 + b_0) + b_0 = b_0(a_1^2 + a_1 + 1)$$

$$y[3] = a_1 y[2] + b_0 x[3] = a_1(a_1^2 b_0 + a_1 b_0 + b_0) + b_0 = b_0(a_1^3 + a_1^2 + a_1 + 1)$$

$$y[M] = a_1 y[M-1] + b_0 x[M] = b_0(a_1^M + a_1^{M-1} + \dots + a_1 + 1)$$

$$\begin{cases} S_N = (a_1^M + a_1^{M-1} + \dots + a_1 + 1) \\ a_1 S_N = (a_1^{M+1} + a_1^M + \dots + a_1^2 + a_1) \end{cases}$$

$$S_N = (a_1^M + a_1^{M-1} + \dots + a_1 + 1)$$

# Infinite Impulse Response (4)

$$y[n] = a_1 y[n-1] + b_0 x[n]$$

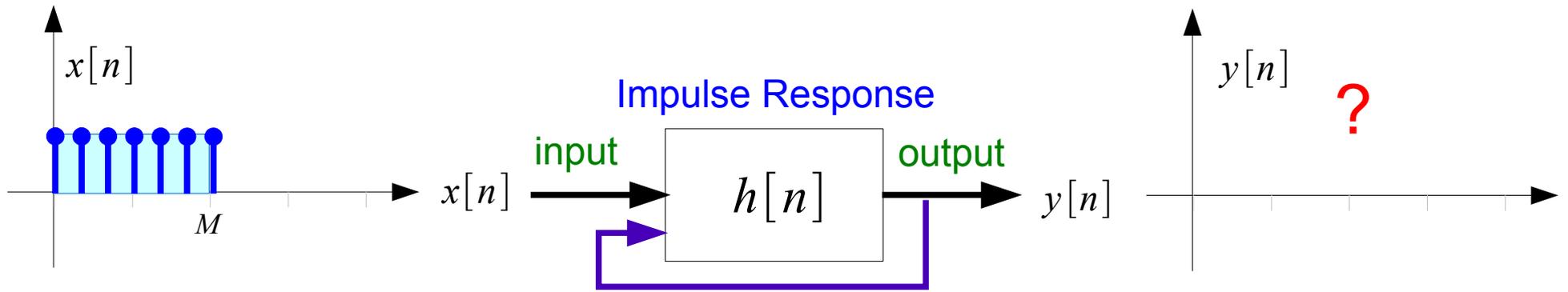
$$y[M] = a_1 y[M-1] + b_0 x[M] = b_0 (a_1^M + a_1^{M-1} + \dots + a_1 + 1)$$

## Geometric Sequence

$$\begin{array}{l} S_N = (a_1^M + a_1^{M-1} + \dots + a_1 + 1) \\ a_1 S_N = (a_1^{M+1} + a_1^M + \dots + a_1^2 + a_1) \\ \hline (1-a_1)S_N = 1 - a_1^{M+1} \end{array} \quad \Rightarrow \quad S_N = \begin{cases} \frac{1 - a_1^{M+1}}{1 - a_1} & (a_1 \neq 1) \\ M & (a_1 = 1) \end{cases}$$
$$\lim_{N \rightarrow \infty} S_N = \frac{1}{1 - a_1} \quad (|a_1| < 1)$$

$$y[M] = b_0 (a_1^M + a_1^{M-1} + \dots + a_1 + 1) = b_0 \frac{1 - a_1^{M+1}}{1 - a_1}$$

# Infinite Impulse Response (4)



$$y[n] = a_1 y[n-1] + b_0 x[n]$$

$$x[n] = \delta[n] \rightarrow y[n] = h[n]$$

$$h[n] = a_1 h[n-1] + b_0 \delta[n]$$

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] R.D. Strum, et al., Discrete Systems and Digital Signal Processing