

# DT Correlation (1B)

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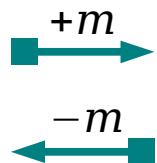
# Correlation of Energy Signals

## Discrete Time LTI System

### Energy Signals

$$\begin{aligned} R_{xy}[m] &= \sum_{n=-\infty}^{+\infty} x[n] y^*[n+m] \\ &= \sum_{n=-\infty}^{+\infty} x[n-m] y^*[n] \end{aligned}$$

$$\begin{aligned} R_{xy}[m] &= \sum_{n=-\infty}^{+\infty} x[n] y[n+m] \quad (\text{real}) \\ &= \sum_{n=-\infty}^{+\infty} x[n-m] y[n] \quad (\text{real}) \end{aligned}$$

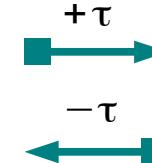


## Continuous Time LTI System

### Energy Signals

$$\begin{aligned} R_{xy}(\tau) &= \int_{-\infty}^{+\infty} x(t) y^*(t+\tau) dt \\ &= \int_{-\infty}^{+\infty} x(t-\tau) y^*(t) dt \end{aligned}$$

$$\begin{aligned} R_{xy}(\tau) &= \int_{-\infty}^{+\infty} x(t) y(t+\tau) dt \quad (\text{real}) \\ &= \int_{-\infty}^{+\infty} x(t-\tau) y(t) dt \quad (\text{real}) \end{aligned}$$



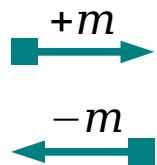
# Correlation of Power Signals

## Discrete Time LTI System

### Power Signals

$$\begin{aligned} R_{xy}[m] &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] y^*[n+m] \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] y^*[n] \end{aligned}$$

$$\begin{aligned} R_{xy}[m] &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] y[n+m] \quad (\text{real}) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] y[n] \quad (\text{real}) \end{aligned}$$

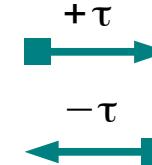


## Continuous Time LTI System

### Power Signals

$$\begin{aligned} R_{xy}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) y^*(t+\tau) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t-\tau) y^*(\tau) dt \end{aligned}$$

$$\begin{aligned} R_{xy}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) y(t+\tau) dt \quad (\text{real}) \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t-\tau) y(\tau) dt \quad (\text{real}) \end{aligned}$$



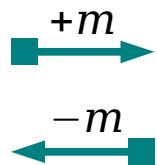
# Correlation of Periodic Power Signals

## Discrete Time LTI System

### Periodic Power Signals

$$\begin{aligned} R_{xy}[m] &= \frac{1}{N} \sum_{n=(N)} x[n] y^*[n+m] \\ &= \frac{1}{N} \sum_{n=(N)} x[n-m] y^*[n] \end{aligned}$$

$$\begin{aligned} R_{xy}[m] &= \frac{1}{N} \sum_{n=(N)} x[n] y[n+m] \quad (\text{real}) \\ &= \frac{1}{N} \sum_{n=(N)} x[n-m] y[n] \quad (\text{real}) \end{aligned}$$

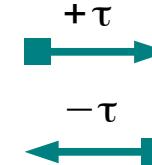


## Continuous Time LTI System

### Periodic Power Signals

$$\begin{aligned} R_{xy}(\tau) &= \frac{1}{T} \int_T x(t) y^*(t+\tau) dt \\ &= \frac{1}{T} \int_T x(t-\tau) y^*(\tau) dt \end{aligned}$$

$$\begin{aligned} R_{xy}(\tau) &= \frac{1}{T} \int_T x(t) y(t+\tau) dt \quad (\text{real}) \\ &= \frac{1}{T} \int_T x(t-\tau) y(t) dt \quad (\text{real}) \end{aligned}$$



# Correlation & Convolution : Energy Signals

## Discrete Time LTI System

### Energy Signals

$$\left\{ \begin{array}{l} R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]y[n+m] \quad (\text{real}) \\ x[n] * y[n] = \sum_{m=-\infty}^{+\infty} x[n-m]y[m] \end{array} \right.$$

$$R_{xy}[m] = x[-m] * y[m]$$

$$R_{xy}[m] \quad \xleftrightarrow{\text{DTFT}} \quad X^*(F)Y(F)$$

$$x[-\textcolor{violet}{n}] * y[\textcolor{violet}{n}] = \sum_{\textcolor{green}{m}=-\infty}^{+\infty} x[-\textcolor{violet}{n} + \textcolor{green}{m}]y[\textcolor{green}{m}]$$

$$x[-\textcolor{violet}{m}] * y[\textcolor{violet}{m}] = \sum_{\textcolor{green}{n}=-\infty}^{+\infty} x[\textcolor{green}{n} - \textcolor{violet}{m}]y[\textcolor{violet}{n}]$$

$$x[-\textcolor{violet}{m}] * y[\textcolor{violet}{m}] = \sum_{\textcolor{green}{n}=-\infty}^{+\infty} x[\textcolor{green}{n}]y[\textcolor{green}{n} + \textcolor{violet}{m}]$$

## Continuous Time LTI System

### Energy Signals

$$\left\{ \begin{array}{l} R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t) y(t+\tau) dt \quad (\text{real}) \\ x(t) * y(t) = \int_{-\infty}^{+\infty} x(t-\tau) y(\tau) d\tau \end{array} \right.$$

$$R_{xy}(\tau) = x(-\tau) * y(\tau)$$

$$R_{xy}(\tau) \quad \xleftrightarrow{\text{CTFT}} \quad X^*(f)Y(f)$$

$$x(-\textcolor{violet}{t}) * y(\textcolor{violet}{t}) = \int_{-\infty}^{+\infty} x(-\textcolor{violet}{t} + \textcolor{green}{\tau}) y(\textcolor{green}{\tau}) d\tau$$

$$x(-\textcolor{violet}{\tau}) * y(\textcolor{violet}{\tau}) = \int_{-\infty}^{+\infty} x(\textcolor{green}{t} - \textcolor{violet}{\tau}) y(\textcolor{green}{t}) dt$$

$$x(-\textcolor{violet}{\tau}) * y(\textcolor{violet}{\tau}) = \int_{-\infty}^{+\infty} x(\textcolor{green}{t}) y(\textcolor{green}{t} + \textcolor{violet}{\tau}) dt$$

# Time Reversal Fourier Transforms

## Discrete Time LTI System

$$x[m] = \int_1 X(F) e^{+j2\pi F n} dF$$

$$x[-m] = \int_1 X(F) e^{-j2\pi F n} dF$$

$$x[-m] = - \int_{-\infty}^1 X(-v) e^{+j2\pi v m} dv$$

$$x[-m] \quad \xrightarrow{\text{DTFT}} \quad X(-F)$$

$-m$

$-F$

## Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t} df$$

$$x(-t) = \int_{-\infty}^{+\infty} X(f) e^{-j2\pi f t} df$$

$$x(-t) = - \int_{+\infty}^{-\infty} X(-v) e^{+j2\pi v t} dv$$

$$x(-t) \quad \xrightarrow{\text{CTFT}} \quad X(-f)$$

$-t$

$-f$

# Conjugate Fourier Transforms

## Discrete Time LTI System

$$x[m] = \int_1 X(F) e^{+j2\pi F n} dF$$

$$x^*[m] = \int_1 X^*(F) e^{-j2\pi F m} dF$$

$$x^*[m] = - \int_{-\infty}^1 X^*(-v) e^{+j2\pi v m} dv$$

$$x^*[m] \quad \xrightarrow{\text{DTFT}} \quad X^*(-F)$$

\*  $m$

\*  $-F$

## Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t} df$$

$$x^*(t) = \int_{-\infty}^{+\infty} X^*(f) e^{-j2\pi f t} df$$

$$x^*(t) = - \int_{+\infty}^{-\infty} X^*(-v) e^{+j2\pi v t} dv$$

$$x^*(t) \quad \xrightarrow{\text{CTFT}} \quad X^*(-f)$$

\*  $t$

\*  $-f$

$$x^*[-m] \quad \xrightarrow{\text{DTFT}} \quad X^*(F)$$

\*  $(-m)$

\*  $-(-F)$

$$x^*(-t) \quad \xrightarrow{\text{CTFT}} \quad X^*(f)$$

\*  $(-t)$

\*  $-(-f)$

# Fourier Transforms of Real Signals

## Discrete Time LTI System

$$x[m] = \int_1 X(F) e^{+j2\pi F n} dF$$

## Continuous Time LTI System

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{+j2\pi f t} df$$

$$x^*[m] \quad \xleftrightarrow{\text{DTFT}} \quad X^*(-F)$$

$$x^*[-m] \quad \xleftrightarrow{\text{DTFT}} \quad X^*(F)$$

||

||

$$x[-m] \quad \xleftrightarrow{\text{DTFT}} \quad X(-F)$$

A real signal

Hermitian Symmetry

$$x^*(t) \quad \xleftrightarrow{\text{CTFT}} \quad X^*(-f)$$

$$x^*(-t) \quad \xleftrightarrow{\text{CTFT}} \quad X^*(f)$$

||

||

$$x(-t) \quad \xleftrightarrow{\text{CTFT}} \quad X(-f)$$

$$x[-m]$$

$\xleftrightarrow{\text{DTFT}}$

$$X^*(F)$$

(real)

(real)

$$x(-t)$$

$\xleftrightarrow{\text{CTFT}}$

$$X^*(f)$$

# Correlation & Convolution : Energy Signals

## Discrete Time LTI System

### Energy Signals

#### Correlation Definition A

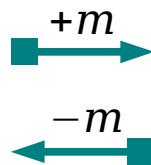
$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n] y^*[n+m]$$

conjugate the second

#### Correlation Definition B

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x^*[n] y[n+m]$$

conjugate the first



## Continuous Time LTI System

### Energy Signals

#### Correlation Definition A

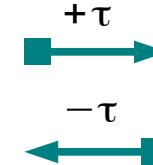
$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t) y^*(t+\tau) dt$$

conjugate the second

#### Correlation Definition B

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x^*(t) y(t+\tau) dt$$

conjugate the first



# Correlation & Convolution : Energy Signals

## Discrete Time LTI System

### Energy Signals

#### Correlation Definition A

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x[n]y^*[n+m]$$

#### Convolution

$$x[n] * y^*[n] = \sum_{m=-\infty}^{+\infty} x[n-m]y^*[m]$$

$$x[-m] * y^*[m] = \sum_{n=-\infty}^{+\infty} x[n-m]y^*[n]$$

$$R_{xy}[m] = x[-m] * y^*[m]$$

$$R_{xy}[m] \xrightarrow{\text{DTFT}} X(-F)Y^*(-F)$$

$$R_{xy}[m] \xrightarrow{\text{DTFT}} X(F)Y^*(F) \quad (\text{even}) \quad (\text{even})$$

## Continuous Time LTI System

### Energy Signals

#### Correlation Definition A

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t)y^*(t+\tau) dt$$

#### Convolution

$$x(t) * y^*(t) = \int_{-\infty}^{+\infty} x(t-\tau)y^*(\tau)d\tau$$

$$x(-\tau) * y^*(\tau) = \int_{-\infty}^{+\infty} x(\tau-t)y^*(t)dt$$

$$R_{xy}(\tau) = x(-\tau) * y^*(\tau)$$

$$R_{xy}(\tau) \xrightarrow{\text{CTFT}} X(-f)Y^*(-f)$$

$$R_{xy}(\tau) \xrightarrow{\text{CTFT}} X(f)Y^*(f)$$

# Correlation & Convolution : Energy Signals

## Discrete Time LTI System

### Energy Signals

#### Correlation Definition B

$$R_{xy}[m] = \sum_{n=-\infty}^{+\infty} x^*[n]y[n+m]$$

#### Convolution

$$x^*[n] * y[n] = \sum_{m=-\infty}^{+\infty} x^*[n-m]y[m]$$

$$x^*[-m] * y[m] = \sum_{n=-\infty}^{+\infty} x^*[n-m]y[n]$$

$$R_{xy}[m] = [x^*[-m] * y[m]]$$

$$R_{xy}[m] \xrightarrow{\text{DTFT}} X^*(F)Y(F)$$

## Continuous Time LTI System

### Energy Signals

#### Correlation Definition B

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x^*(t) y(t+\tau) dt$$

#### Convolution

$$x^*(t) * y(t) = \int_{-\infty}^{+\infty} x^*(t-\tau)y(\tau)d\tau$$

$$x^*(-\tau) * y(\tau) = \int_{-\infty}^{+\infty} x^*(\tau)y(\tau)d\tau$$

$$R_{xy}(\tau) = [x^*(-\tau) * y(\tau)]$$

$$R_{xy}(\tau) \xrightarrow{\text{CTFT}} X^*(f)Y(f)$$

# Correlation Functions

## Discrete Time LTI System

Energy Signals

$$R_{xy}[m] = \sum_{n=-\infty}^{\infty} x[n]y[n+m]$$

Power Signals

Power Signal + Energy Signal

$$R_{xy}[m] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n]y[n+m]$$

## Continuous Time LTI System

Energy Signals

$$R_{xy}(\tau) = \int_{-\infty}^{+\infty} x(t) y(t+\tau) dt$$

Power Signals

Power Signal + Energy Signal

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) y(t+\tau) dt$$

# AutoCorrelation of Energy Signals

## Discrete Time LTI System

### Energy Signals

$$\begin{aligned} R_{xx}[m] &= \sum_{n=-\infty}^{+\infty} x[n] x^*[n+m] \\ &= \sum_{n=-\infty}^{+\infty} x[n-m] x^*[n] \end{aligned}$$

$$R_{xx}[0] = \sum_{n=-\infty}^{+\infty} x^2[n] \quad \text{total energy}$$

## Continuous Time LTI System

### Energy Signals

$$\begin{aligned} R_{xx}(\tau) &= \int_{-\infty}^{+\infty} x(t) x^*(t+\tau) dt \\ &= \int_{-\infty}^{+\infty} x(t) x^*(t+\tau) dt \end{aligned}$$

$$R_{xx}(0) = \int_{-\infty}^{+\infty} x^2(t) dt \quad \text{total energy}$$

# AutoCorrelation of Power Signals

## Discrete Time LTI System

### Power Signals

$$\begin{aligned} R_{xx}[m] &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] x^*[n+m] \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] x^*[n] \end{aligned}$$

$$R_{xx}[0] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x^2[n] \text{ total energy}$$

## Continuous Time LTI System

### Power Signals

$$\begin{aligned} R_{xx}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t) x^*(t+\tau) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x(t-\tau) x^*(\tau) dt \end{aligned}$$

$$R_{xx}(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_T x^2(t) dt \text{ total energy}$$

# AutoCorrelation of Periodic Power Signals

## Discrete Time LTI System

### Periodic Power Signals

$$\begin{aligned} R_{xx}[m] &= \frac{1}{N} \sum_{n=(N)} x[n] x^*[n+m] \\ &= \frac{1}{N} \sum_{n=(N)} x[n-m] x^*[n] \end{aligned}$$

$$R_{xx}[m] = \frac{1}{N} \sum_{n=(N)} x[n] x^*[n+m]$$

## Continuous Time LTI System

### Periodic Power Signals

$$\begin{aligned} R_{xx}(\tau) &= \frac{1}{T} \int_T x(t) x^*(t+\tau) dt \\ &= \frac{1}{T} \int_T x(t-\tau) x^*(\tau) dt \end{aligned}$$

$$R_{xx}(\tau) = \frac{1}{T} \int_T x(t) x^*(t+\tau) dt$$



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## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M. J. Roberts, Fundamentals of Signals and Systems
- [4] S. J. Orfanidis, Introduction to Signal Processing
- [5] B. P. Lathi, Signals and Systems
- [6] <http://www.cs.unm.edu/~williams/cs530/symmetry.pdf>