

# DFT Matrix Examples (DFT.2.A)

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# DFT Matrix Elements

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

$$\begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} & & k \\ & \xrightarrow{n} & \\ & & \end{bmatrix} \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ \vdots \\ x[N-1] \end{bmatrix}$$

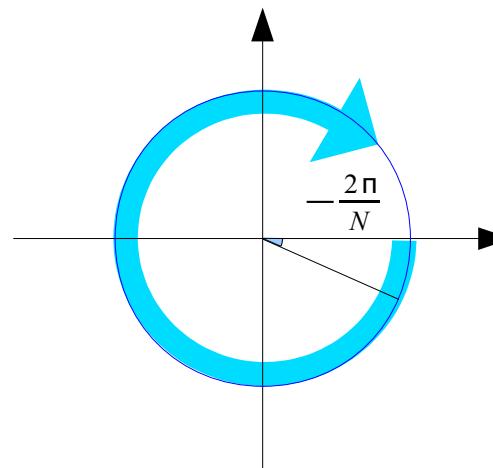
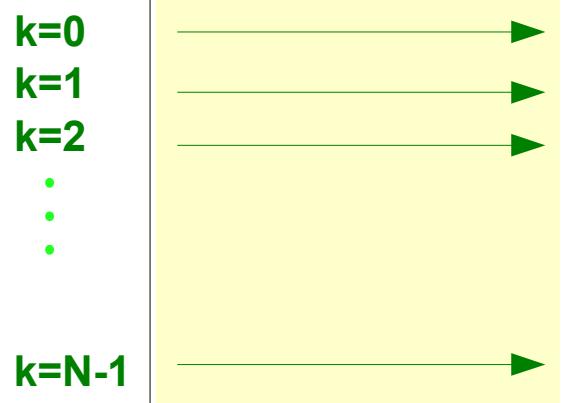
$$\begin{aligned} e^{-j\left(\frac{2\pi}{N}\right)kn} &= e^{-j\left(\frac{2\pi}{N}\right)(kn \bmod N)} \\ &= \underbrace{\cos\left(-\frac{2\pi}{N}kn\right)}_{\text{N multiples of the smallest angle}} + j \underbrace{\sin\left(-\frac{2\pi}{N}kn\right)}_{\left(-\frac{2\pi}{N}\right)} \end{aligned}$$

**N multiples of the smallest angle**  $\left(-\frac{2\pi}{N}\right)$

$$\left\{ -\frac{2\pi}{N} \cdot 0, -\frac{2\pi}{N} \cdot 1, \dots, -\frac{2\pi}{N} \cdot (N-1) \right\}$$

# Rows of a DFT Matrix

$$e^{-j\left(\frac{2\pi}{N}\right)k n} \in \left\{ e^{-j\left(\frac{2\pi}{N}\right)\cdot 0}, e^{-j\left(\frac{2\pi}{N}\right)\cdot 1}, e^{-j\left(\frac{2\pi}{N}\right)\cdot 2}, \dots, e^{-j\left(\frac{2\pi}{N}\right)(N-1)} \right\}$$



Negative Angles

Negative Frequency

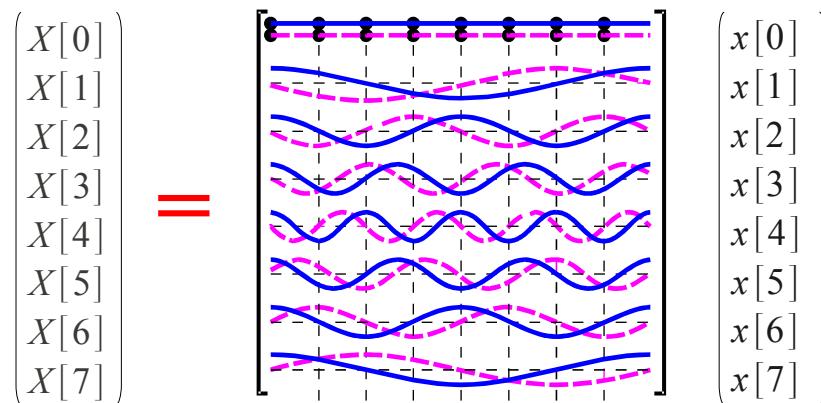
when $k=0$	N samples	→ 0 round	→ 0 cycles
when $k=1$	N samples	→ 1 round	→ -1 cycles
when $k=2$	N samples	→ 2 rounds	→ -2 cycles
⋮	⋮	⋮	⋮
when $k=N-2$	N samples	→ N-2 rounds	→ +2 cycles
when $k=N-1$	N samples	→ N-1 rounds	→ +1 cycles

*complex  
conjugate*

# Graphical Representation of a DFT Matrix

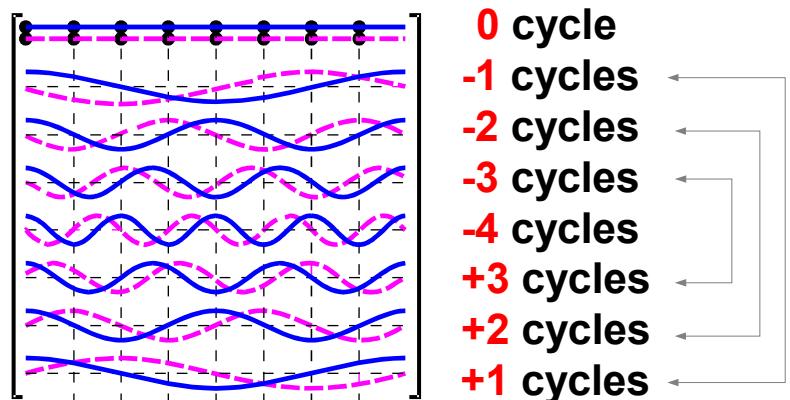
$$X[k] = \sum_{n=0}^7 W_8^{kn} x[n]$$

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn}$$



$$\text{Re}\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \cos\left(-\frac{2\pi}{8}kn\right)$$

$$\text{Im}\left\{ e^{-j\frac{2\pi}{8}kn} \right\} = \sin\left(-\frac{2\pi}{8}kn\right)$$



*complex  
conjugate*

## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., *Signal Processing First*, Pearson Prentice Hall, 2003