Background (DFT.0B)

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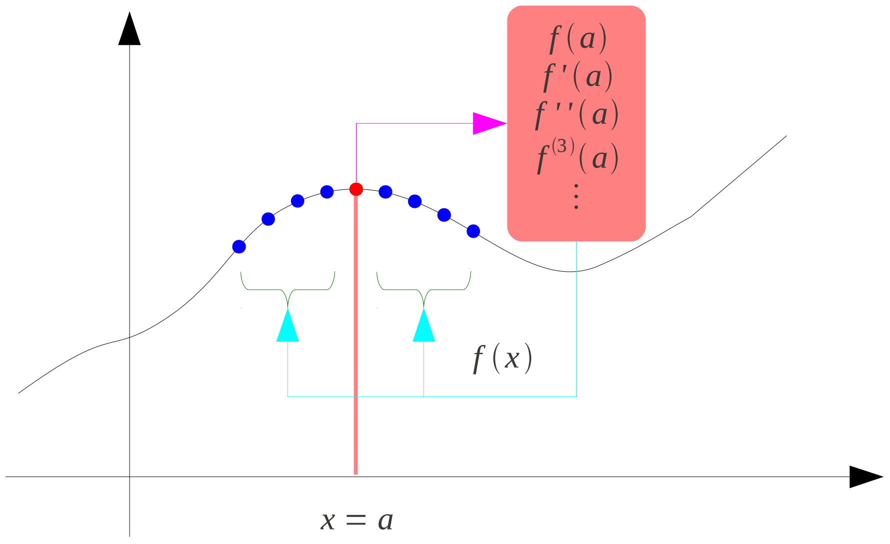
Taylor Series

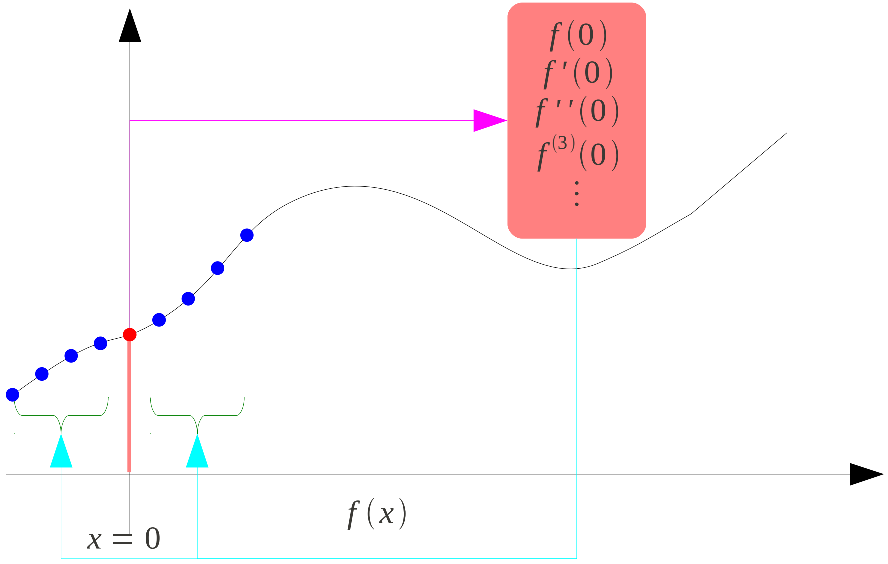
$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$



Maclaurin Series

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$



Power Series Expansion

$$f(x) = f(0) + f'(0)\,x + \frac{f''(0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

The Euler Constant e

$$\frac{d}{dx} a^x = \lim_{h \to 0} \frac{a^{(x+h)} - a^x}{h} = a^x \lim_{h \to 0} \frac{a^h - 1}{h}$$

$$\lim_{h \to 0} \frac{a^h - 1}{h} = 1 \quad iif \quad a = e$$

$$\frac{d}{dx} e^x = e^x$$

$$e = 2.71828\cdots$$

$$f(x) = e^x \Rightarrow f'(x) = e^x \Rightarrow f''(x) = e^x \quad \cdots$$

$$f'(0) = 1 \qquad \lim_{h \to 0} \frac{e^h - e^0}{h - 0} = 1$$

Complex Number

$$i = \sqrt{-1}$$

$$i^2 = -1$$

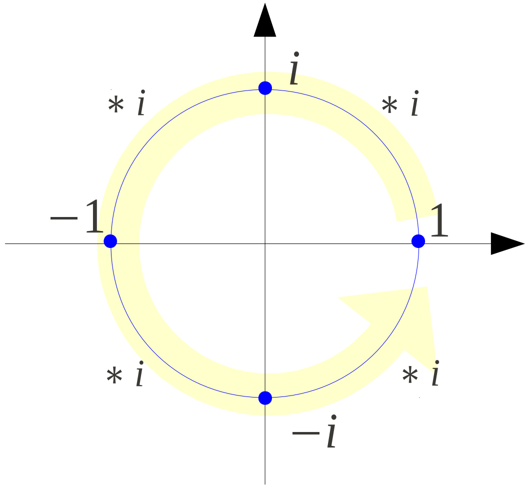
$$i^3 = -i$$

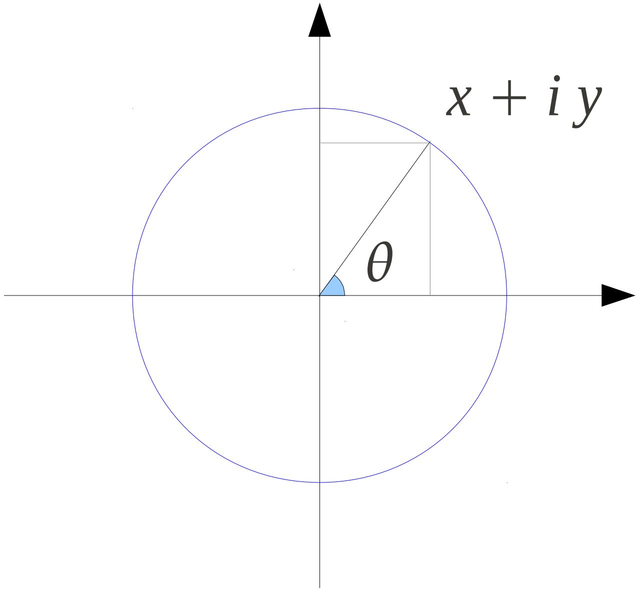
$$i^4 = +1$$

$$x + i\,y = r\cos\theta + i\,r\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$





Complex Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$\begin{aligned} e^{i\theta} &= 1 + (i\theta) + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots \\ &= 1 + i\theta - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} + \cdots \\ &= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \cdots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots\right) \end{aligned}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler Series (1)

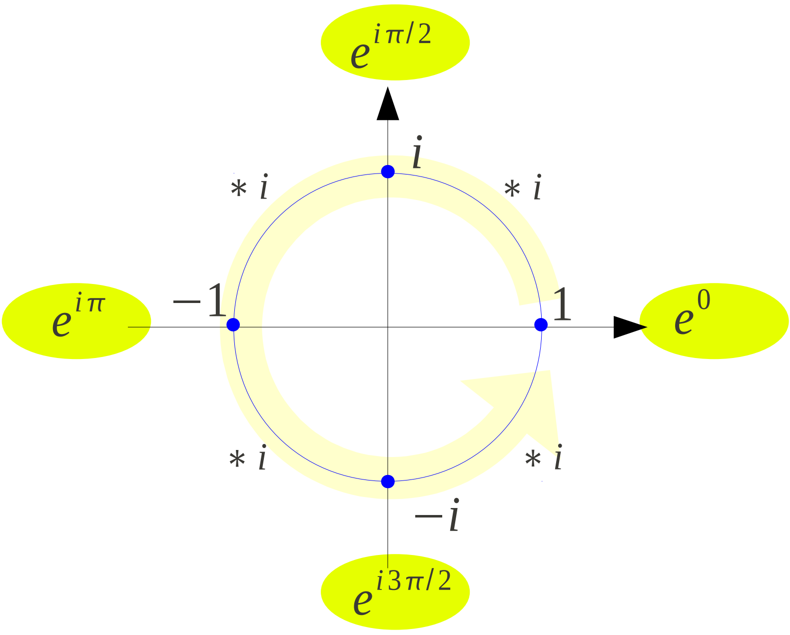
$$e^{i\theta} = \cos\theta + i\sin\theta$$

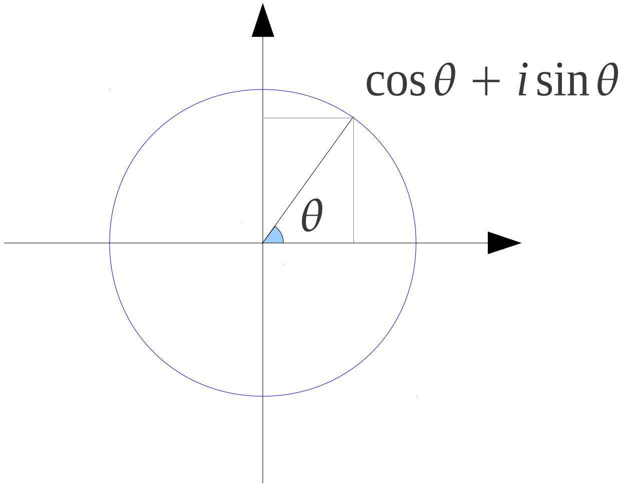
$$\Re\{e^{i\theta}\} = \cos\theta$$

$$\Im\{e^{i\theta}\} = \sin\theta$$

$$|e^{i\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$$

$$arg(e^{i\theta}) = \tan^{-1}\left(\frac{\sin\theta}{\cos\theta}\right) = \theta$$





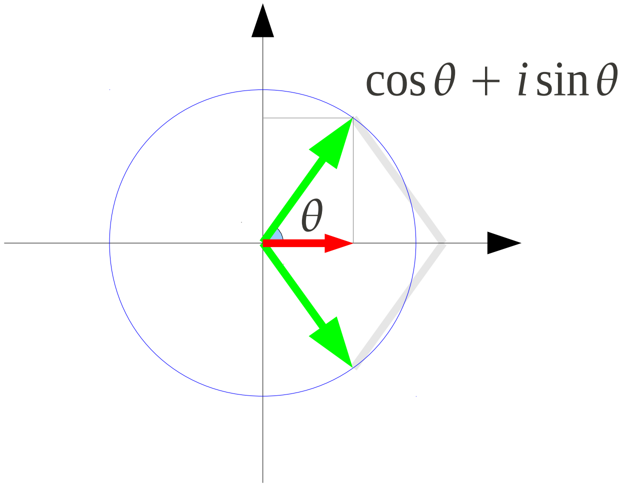
Euler Series (2)

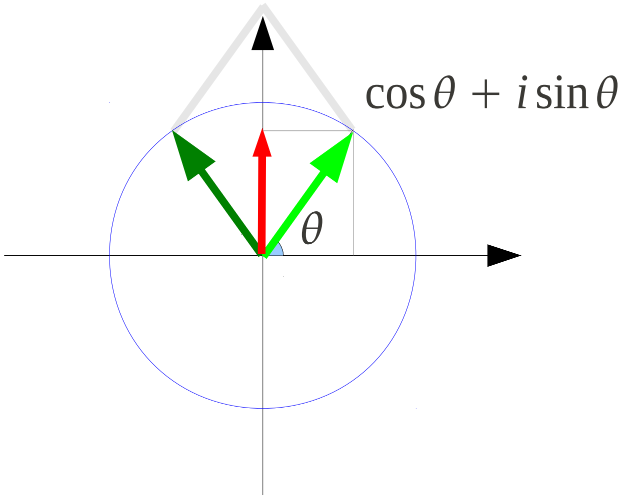
$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$\Re\{e^{i\theta}\} = \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\Im\{e^{i\theta}\} = \sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$





DeMoivre's Theorem

$$e^{i\theta} = \cos\theta + i\sin\theta$$

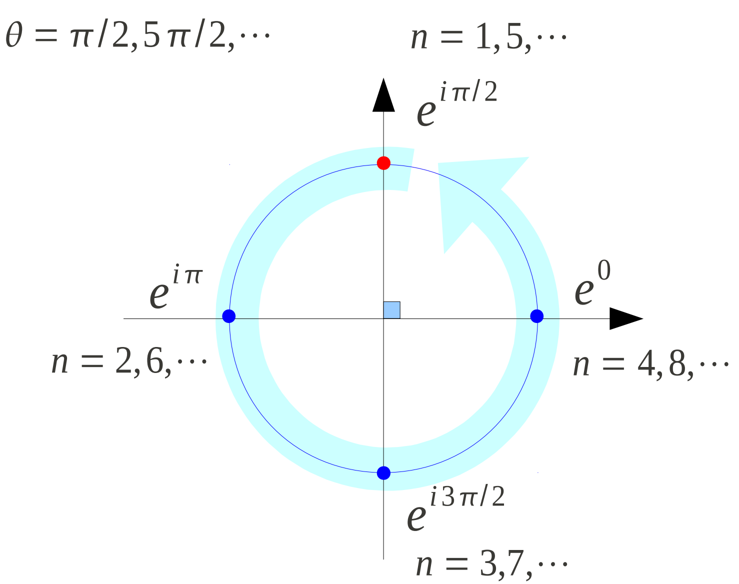
$$\left(e^{i\theta}\right)^n = \left(\cos\theta + i\sin\theta\right)^n$$

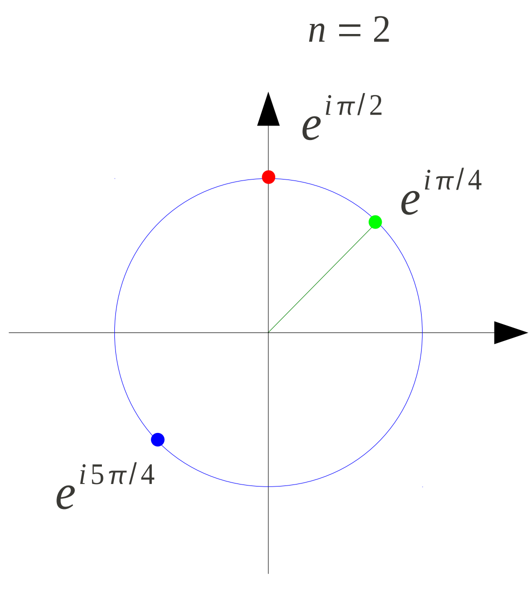
$$e^{in\theta} = \cos n\theta + i\sin n\theta$$

$$\left(e^{i\theta}\right)^{1/n} = \left(\cos\theta + i\sin\theta\right)^{1/n}$$

$$\left(e^{i\theta}\right)^{1/n} \Rightarrow \cos\frac{\theta}{n} + i\sin\frac{\theta}{n}$$

$$= \cos\left(\frac{\theta + 2k\pi}{n}\right) + i\,\sin\left(\frac{\theta + 2k\pi}{n}\right)$$





$$\begin{aligned} &\Rightarrow\ e^{i\pi/2} \rightarrow e^{i\pi/4} \\ &=\ e^{i5\pi/2} \rightarrow e^{i5\pi/4} \\ &=\ e^{i9\pi/2} \rightarrow e^{i9\pi/4} \\ &=\ e^{i13\pi/2} \rightarrow e^{i13\pi/4} \end{aligned}$$

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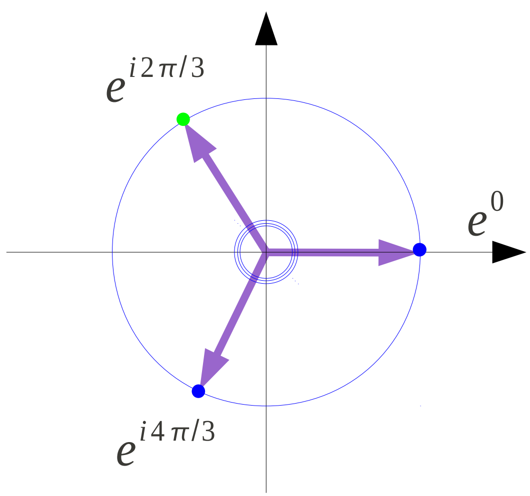
Complex Roots

$$z = r e^{i\theta} \Rightarrow z^{1/n} = r^{1/n} (e^{i\theta})^{1/n} \Rightarrow \sqrt[n]{r} \left(\cos \frac{\theta}{n} + i \sin \frac{\theta}{n} \right)$$

$$= \sqrt[n]{r} \left(\cos \left(\frac{\theta + 2k\pi}{n} \right) + i \sin \left(\frac{\theta + 2k\pi}{n} \right) \right)$$

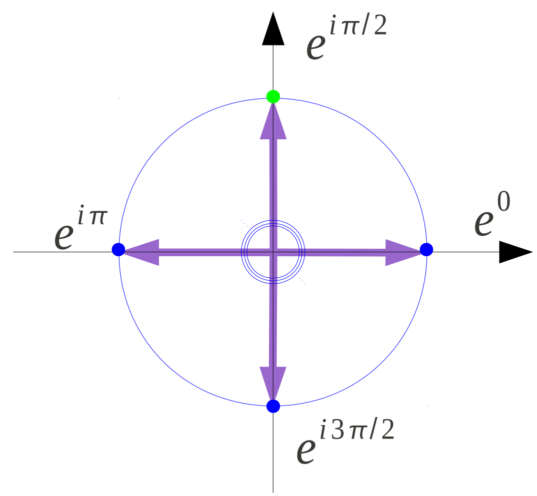
$z^3 = 1$



$n = 3$

$$\Rightarrow e^{2\pi} \rightarrow e^{2\pi/3}$$
$$= e^{4\pi} \rightarrow e^{4\pi/3}$$
$$= e^{6\pi} \rightarrow e^{6\pi/3}$$

$z^4 = 1$



$n = 4$

$$\Rightarrow e^{2\pi} \rightarrow e^{2\pi/4}$$
$$= e^{4\pi} \rightarrow e^{4\pi/4}$$
$$= e^{6\pi} \rightarrow e^{6\pi/4}$$
$$= e^{8\pi} \rightarrow e^{8\pi/4}$$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003