

Truth Table (2A)

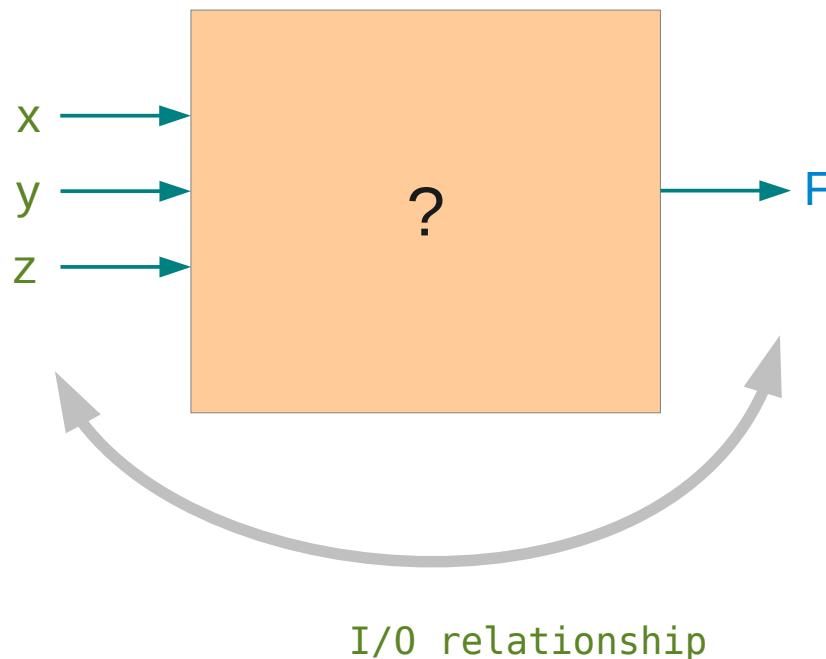
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Truth Table



x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

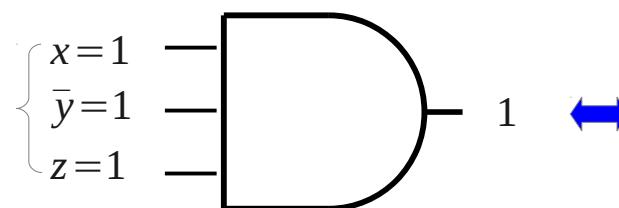
Truth Table and minterms (1)

x	y	z			
0	0	0	the case when x=0 and y=0 and z=0	\leftrightarrow	$\bar{x}\bar{y}\bar{z} = 1$
0	0	1	the case when x=0 and y=0 and z=1	\leftrightarrow	$\bar{x}\bar{y}z = 1$
0	1	0	the case when x=0 and y=1 and z=0	\leftrightarrow	$\bar{x}y\bar{z} = 1$
0	1	1	the case when x=0 and y=1 and z=1	\leftrightarrow	$\bar{x}yz = 1$
1	0	0	the case when x=1 and y=0 and z=0	\leftrightarrow	$x\bar{y}\bar{z} = 1$
1	0	1	the case when x=1 and y=0 and z=1	\leftrightarrow	$x\bar{y}z = 1$
1	1	0	the case when x=1 and y=1 and z=0	\leftrightarrow	$xy\bar{z} = 1$
1	1	1	the case when x=1 and y=1 and z=1	\leftrightarrow	$xyz = 1$

inputs

All possible combination of inputs

$$x\bar{y}z = 1 \quad \leftrightarrow$$



$$\begin{cases} x=1 \\ y=0 \\ z=1 \end{cases}$$

For the output of an **and** gate to be 1,
all inputs must be 1

Truth Table and minterms (2)

	x	y	z
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

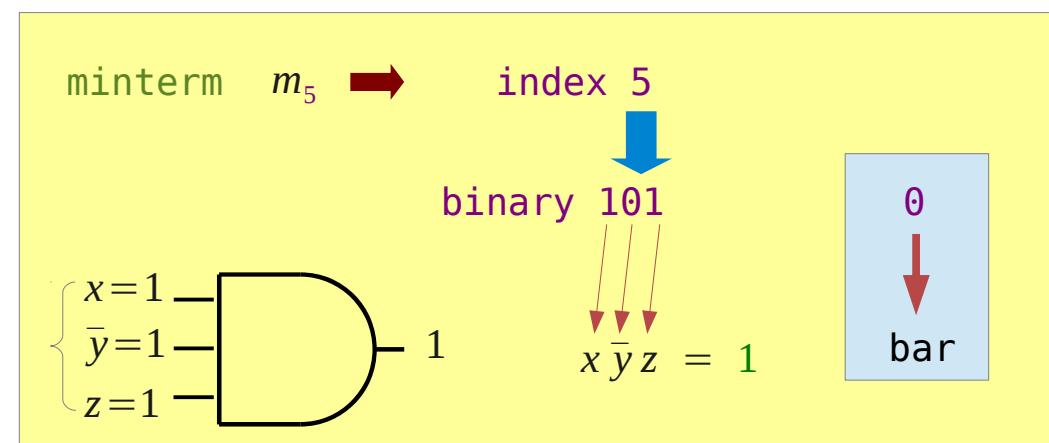
index

inputs

All possible combination of inputs

- the case when the minterm $m_0 = \bar{x}\bar{y}\bar{z} = 1$
- the case when the minterm $m_1 = \bar{x}\bar{y}z = 1$
- the case when the minterm $m_2 = \bar{x}y\bar{z} = 1$
- the case when the minterm $m_3 = \bar{x}yz = 1$
- the case when the minterm $m_4 = x\bar{y}\bar{z} = 1$
- the case when the minterm $m_5 = x\bar{y}z = 1$
- the case when the minterm $m_6 = xy\bar{z} = 1$
- the case when the minterm $m_7 = xyz = 1$

$m_0 = \bar{x}\bar{y}\bar{z} = 1$
$m_1 = \bar{x}\bar{y}z = 1$
$m_2 = \bar{x}y\bar{z} = 1$
$m_3 = \bar{x}yz = 1$
$m_4 = x\bar{y}\bar{z} = 1$
$m_5 = x\bar{y}z = 1$
$m_6 = xy\bar{z} = 1$
$m_7 = xyz = 1$



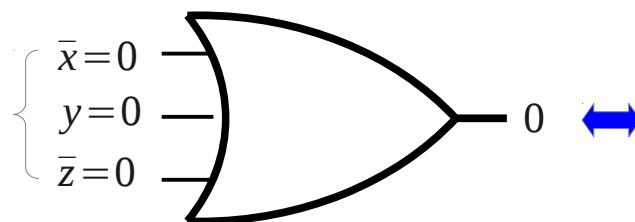
Truth Table and MAXterms (1)

x	y	z			
0	0	0	the case when x=0 and y=0 and z=0	\leftrightarrow	$x+y+z = 0$
0	0	1	the case when x=0 and y=0 and z=1	\leftrightarrow	$x+y+\bar{z} = 0$
0	1	0	the case when x=0 and y=1 and z=0	\leftrightarrow	$x+\bar{y}+z = 0$
0	1	1	the case when x=0 and y=1 and z=1	\leftrightarrow	$x+\bar{y}+\bar{z} = 0$
1	0	0	the case when x=1 and y=0 and z=0	\leftrightarrow	$\bar{x}+y+z = 0$
1	0	1	the case when x=1 and y=0 and z=1	\leftrightarrow	$\bar{x}+y+\bar{z} = 0$
1	1	0	the case when x=1 and y=1 and z=0	\leftrightarrow	$\bar{x}+\bar{y}+z = 0$
1	1	1	the case when x=1 and y=1 and z=1	\leftrightarrow	$\bar{x}+\bar{y}+\bar{z} = 0$

inputs

All possible combination of inputs

$$\bar{x}+y+\bar{z} = 0 \quad \leftrightarrow$$



$$\begin{cases} x=1 \\ y=0 \\ z=1 \end{cases}$$

For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and MAXterms (2)

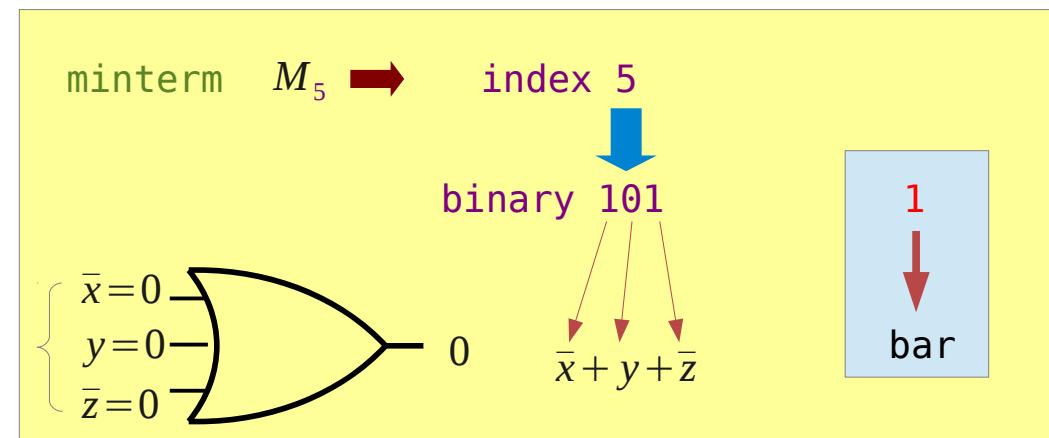
	x	y	z	
0	0	0	0	
1	0	0	1	
2	0	1	0	
3	0	1	1	
4	1	0	0	
5	1	0	1	
6	1	1	0	
7	1	1	1	

index

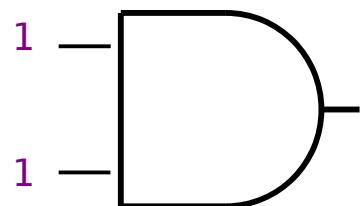
inputs

All possible combination of inputs

- the case when the MAXterm $M_0 = x + y + z = 0$
- the case when the MAXterm $M_1 = x + y + \bar{z} = 0$
- the case when the MAXterm $M_2 = x + \bar{y} + z = 0$
- the case when the MAXterm $M_3 = x + \bar{y} + \bar{z} = 0$
- the case when the MAXterm $M_4 = \bar{x} + y + z = 0$
- the case when the MAXterm $M_5 = \bar{x} + y + \bar{z} = 0$
- the case when the MAXterm $M_6 = \bar{x} + \bar{y} + z = 0$
- the case when the MAXterm $M_7 = \bar{x} + \bar{y} + \bar{z} = 0$

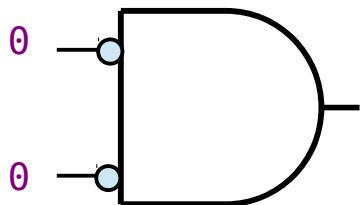
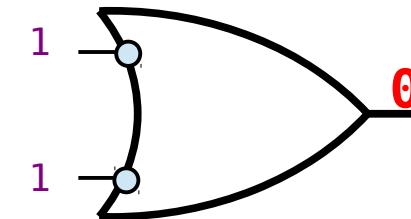


Maxterm and minterm Conditions



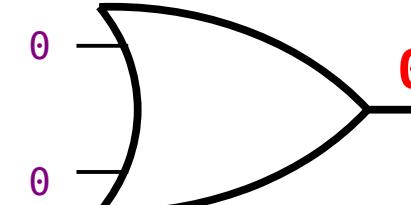
	x	y	xy
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

	\bar{x}	\bar{y}	$\bar{x}+\bar{y}$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	0



	\bar{x}	\bar{y}	$\bar{x}\bar{y}$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0

	x	y	x+y
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



Boolean Function with minterms (1)

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index { } inputs output

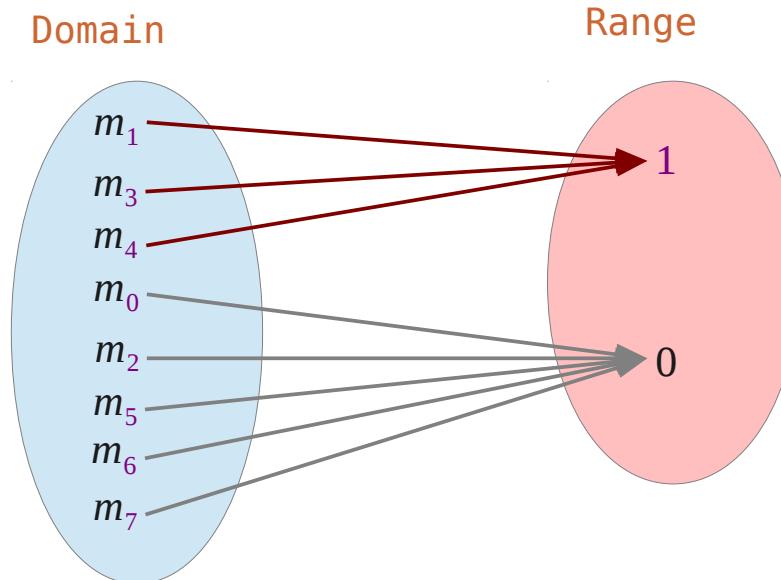
All possible combination of inputs

The output F becomes 1, for one of the three following cases

(the case when x=0 and y=0 and z=1) \leftrightarrow $m_1 = \bar{x}\bar{y}z = 1$

or (the case when x=0 and y=1 and z=1) \leftrightarrow $m_3 = \bar{x}yz = 1$

or (the case when x=1 and y=0 and z=0) \leftrightarrow $m_4 = x\bar{y}\bar{z} = 1$



Boolean Function with minterms (2)

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

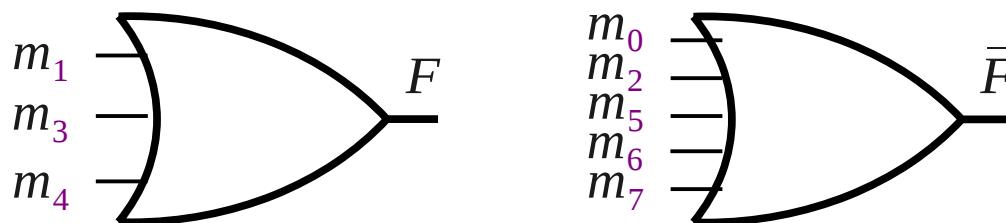
index inputs output

All possible combination of inputs

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \Rightarrow \quad F = 1$$

$$\Leftarrow \quad F = m_1 + m_3 + m_4$$



For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with minterms (3)

x	y	z	F
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	1
5	1	0	0
6	1	1	0
7	1	1	0

index



inputs output

All possible
combination of inputs

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

$$m_1 + m_3 + m_4 = 1 \quad \overrightarrow{\quad} \quad F = 1$$

$$\overleftarrow{\quad} \quad F = m_1 + m_3 + m_4$$

The output F becomes 0,
either $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$$m_0 + m_2 + m_5 + m_6 + m_7 = 1 \quad \overrightarrow{\quad} \quad F = 0$$

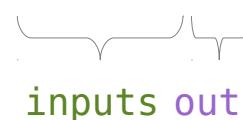
$$\overleftarrow{\quad} \quad \bar{F} = m_0 + m_2 + m_5 + m_6 + m_7$$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with Maxterms (1)

	\rightarrow	x	y	z	F
0	→ 0	0	0	0	0
1	1	0	0	1	
2	→ 2	0	1	0	0
3	3	0	1	1	
4	4	1	0	0	
5	→ 5	1	0	1	0
6	→ 6	1	1	0	0
7	→ 7	1	1	1	0

index



All possible combination of inputs

The output F becomes 0,
for one of the five following cases

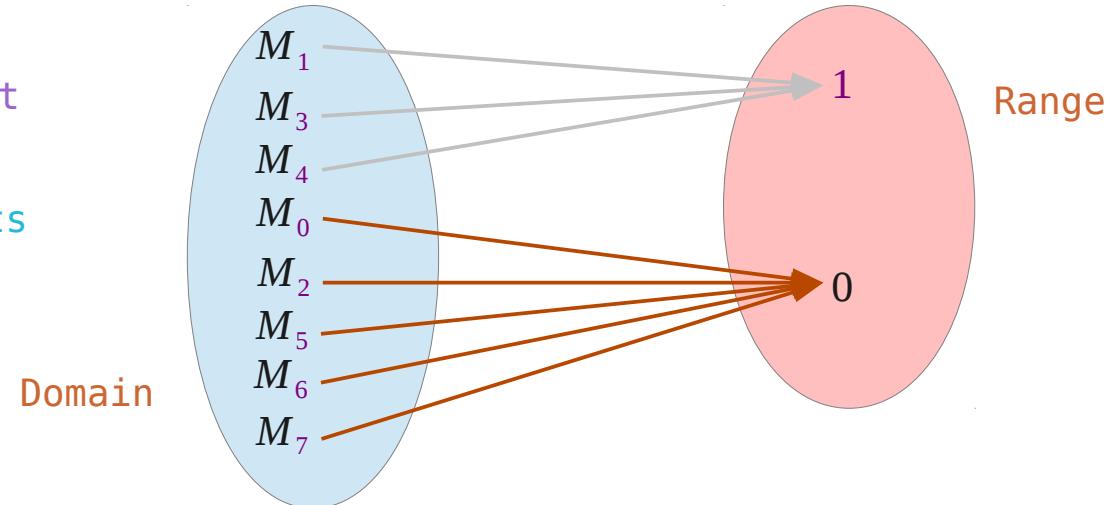
(the case when $x=0$ and $y=0$ and $z=0$) \leftrightarrow $x+y+z = 0$

or (the case when $x=0$ and $y=1$ and $z=0$) \leftrightarrow $x+\bar{y}+z = 0$

or (the case when $x=1$ and $y=0$ and $z=1$) \leftrightarrow $\bar{x}+y+\bar{z} = 0$

or (the case when $x=1$ and $y=1$ and $z=0$) \leftrightarrow $\bar{x}+\bar{y}+z = 0$

or (the case when $x=1$ and $y=1$ and $z=1$) \leftrightarrow $\bar{x}+\bar{y}+\bar{z} = 0$



Boolean Function with Maxterms (2)

	x	y	z	F
→ 0	0	0	0	0
1	0	0	1	1
→ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
→ 5	1	0	1	0
→ 6	1	1	0	0
→ 7	1	1	1	0

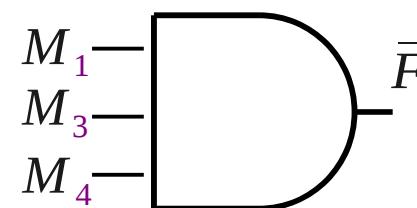
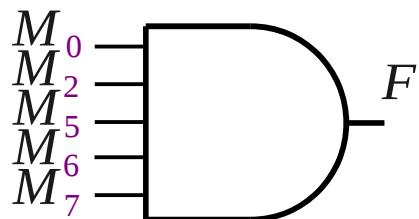
index { inputs } output

All possible combination of inputs

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \Leftrightarrow F = 0$$

$$\Leftrightarrow F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$



For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with Maxterms (2)

	x	y	z	F
→ 0	0	0	0	0
1	0	0	1	1
→ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
→ 5	1	0	1	0
→ 6	1	1	0	0
→ 7	1	1	1	0

index ↗ ↘ ↗ ↘
inputs output

All possible
combination of inputs

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \quad \Leftrightarrow F = 0$$

$$\Leftrightarrow F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,
either $M_1=0$ or $M_3=0$ or $M_4=0$

$$M_1 \cdot M_3 \cdot M_4 = 0 \quad \Leftrightarrow F = 1$$

$$\Leftrightarrow \bar{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

Complimentary Relations

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

{ } { }

inputs output

All possible combination of inputs

$$m_i = \overline{M}_i$$

$$M_i = \overline{m}_i$$

$$F(x, y, z) = m_1 + m_3 + m_4$$

The output F becomes 1,
either $m_1=1$ or $m_3=1$ or $m_4=1$

For the output of an **or** gate to be 1,
at least one must be 1

$$\bar{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7$$

$$\begin{aligned} \Leftrightarrow F(x, y, z) &= \overline{m_0 + m_2 + m_5 + m_6 + m_7} \\ &= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_5} \cdot \overline{m_6} \cdot \overline{m_7} \end{aligned}$$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,
either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 1,

for the cases

1) when $m_1=1$ or $m_3=1$ or $m_4=1$

$$F(x, y, z) = m_1 + m_3 + m_4 \rightarrow F=1$$

2) when $M_1=0$ or $M_3=0$ or $M_4=0$

$$\bar{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \rightarrow F=1 (\bar{F}=0)$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 0,

for the cases

1) when $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$$\bar{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \rightarrow F=0 (\bar{F}=1)$$

2) when $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \rightarrow F=0$$

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$F(x, y, z) = m_1 + m_3 + m_4 \rightarrow F=1$$

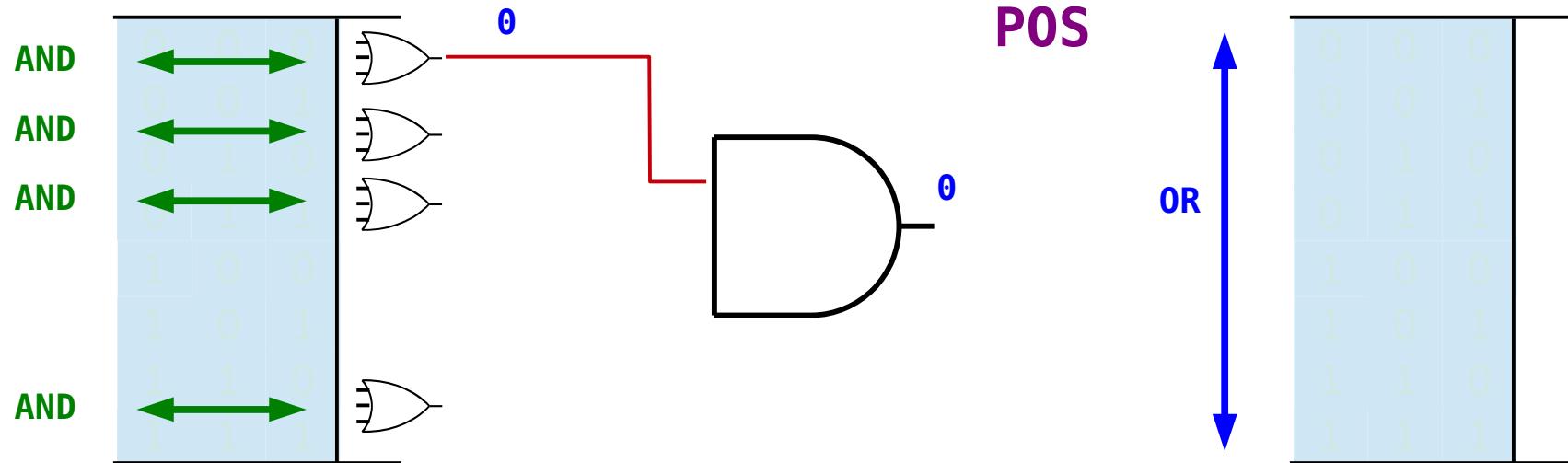
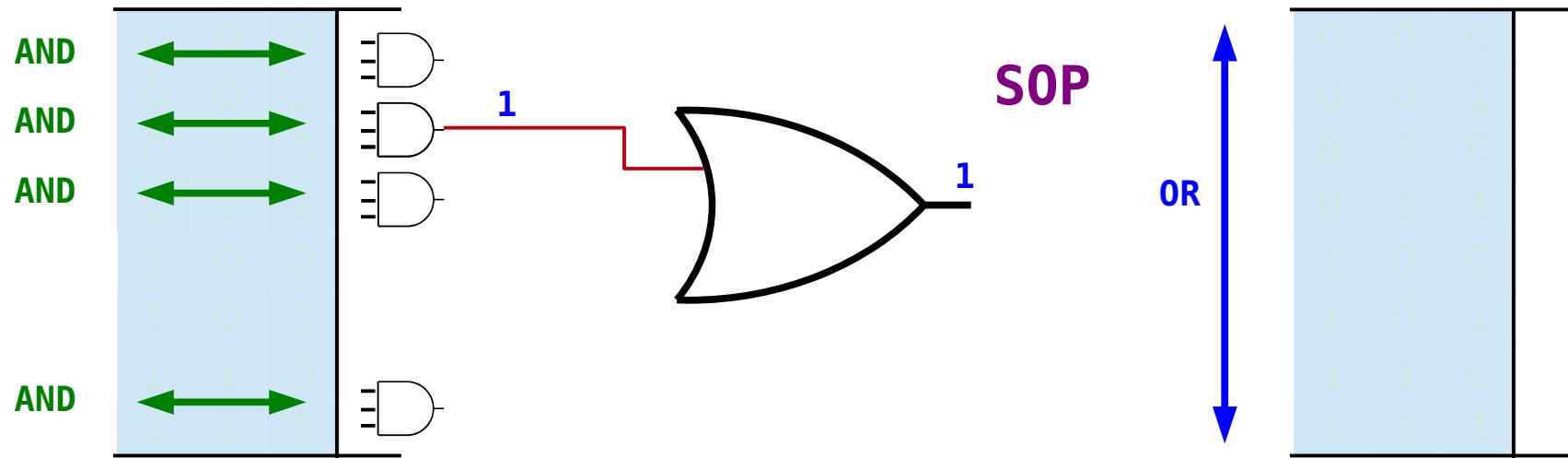
$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \rightarrow F=0$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\overline{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \rightarrow F=0 \quad (\overline{F}=1)$$

$$\overline{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \rightarrow F=1 \quad (\overline{F}=0)$$

SOP and POS



Boolean Function with minterms

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Diagram showing the function F as a 3-input AND gate followed by a NOT gate (inverter). The output of the AND gate is connected to the inverter.

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Diagram showing the function F as a 3-input OR gate followed by a NOT gate (inverter). The output of the OR gate is connected to the inverter.

Boolean Function with Maxterms

	x	y	z	F
⇒ 0	0	0	0	0
1	0	0	1	1
⇒ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
⇒ 5	1	0	1	0
⇒ 6	1	1	0	0
⇒ 7	1	1	1	0

Diagram illustrating a logic function F defined by the truth table above. The function is 1 for the minterm $(x=0, y=0, z=1)$ and 0 for all other minterms. The circuit consists of three inputs x, y, z and one output F . The output F is generated by a single AND gate (NAND gate with inverted inputs) whose inputs are the results of three AND gates (NAND gates with one inverted input). The first AND gate has inputs $\overline{x} \cdot y \cdot z$, the second has $x \cdot \overline{y} \cdot z$, and the third has $x \cdot y \cdot \overline{z}$.

	x	y	z	F
0	0	0	0	0
→ 1	0	0	1	1
2	0	1	0	0
→ 3	0	1	1	1
→ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

Diagram illustrating a logic function F defined by the truth table above. The function is 1 for the maxterm $(x=0, y=0, z=1)$ and 0 for all other maxterms. The circuit consists of three inputs x, y, z and one output F . The output F is generated by a single OR gate (NOR gate with inverted inputs) whose inputs are the results of three OR gates (NOR gates with one inverted input). The first OR gate has inputs $x \cdot \overline{y} \cdot \overline{z}$, the second has $\overline{x} \cdot y \cdot \overline{z}$, and the third has $\overline{x} \cdot \overline{y} \cdot z$.

Truth Table

References

[1] <http://en.wikipedia.org/>