Truth Table (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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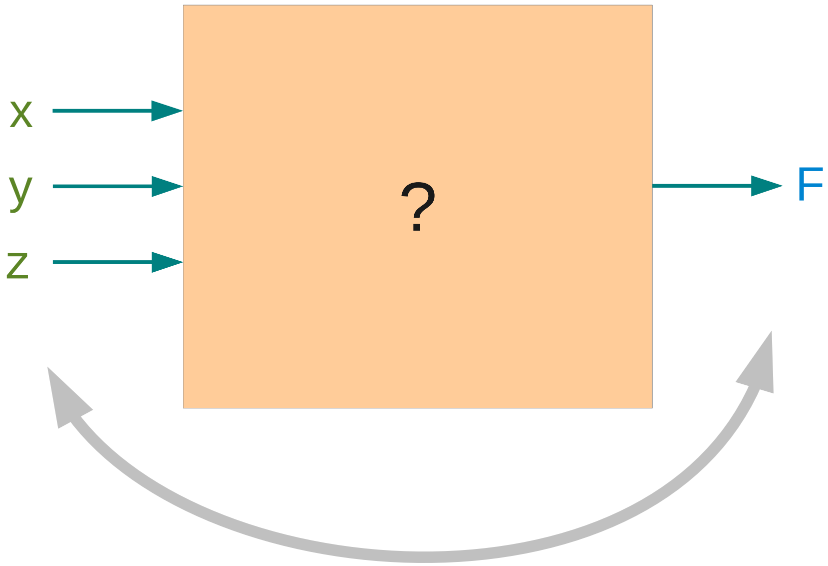
Truth Table



I/O relationship

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

inputs output

Truth Table and minterms (1)

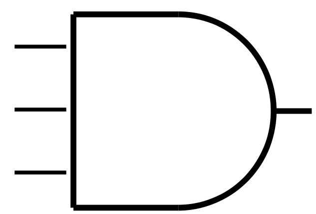
x	y	z		
0	0	0	the case when x=0 **and** y=0 **and** z=0	$\bar{x}\,\bar{y}\,\bar{z} = 1$
0	0	1	the case when x=0 **and** y=0 **and** z=1	$\bar{x}\,\bar{y}\,z = 1$
0	1	0	the case when x=0 **and** y=1 **and** z=0	$\bar{x}\,y\,\bar{z} = 1$
0	1	1	the case when x=0 **and** y=1 **and** z=1	$\bar{x}\,y\,z = 1$
1	0	0	the case when x=1 **and** y=0 **and** z=0	$x\,\bar{y}\,\bar{z} = 1$
1	0	1	the case when x=1 **and** y=0 **and** z=1	$x\,\bar{y}\,z = 1$
1	1	0	the case when x=1 **and** y=1 **and** z=0	$x\,y\,\bar{z} = 1$
1	1	1	the case when x=1 **and** y=1 **and** z=1	$x\,y\,z = 1$

inputs

All possible
combination of inputs

$x\,\bar{y}\,z = 1 \iff$

$x=1$, $\bar{y}=1$, $z=1$



$1 \iff$ $x=1$, $y=0$, $z=1$

For the output of an **and** gate to be 1,
all inputs must be 1

Truth Table and minterms (2)

index	x	y	z		minterm
0	0	0	0	the case when the minterm	$m_0 = \bar{x}\,\bar{y}\,\bar{z} = 1$
1	0	0	1	the case when the minterm	$m_1 = \bar{x}\,\bar{y}\,z = 1$
2	0	1	0	the case when the minterm	$m_2 = \bar{x}\,y\,\bar{z} = 1$
3	0	1	1	the case when the minterm	$m_3 = \bar{x}\,y\,z = 1$
4	1	0	0	the case when the minterm	$m_4 = x\,\bar{y}\,\bar{z} = 1$
5	1	0	1	the case when the minterm	$m_5 = x\,\bar{y}\,z = 1$
6	1	1	0	the case when the minterm	$m_6 = x\,y\,\bar{z} = 1$
7	1	1	1	the case when the minterm	$m_7 = x\,y\,z = 1$

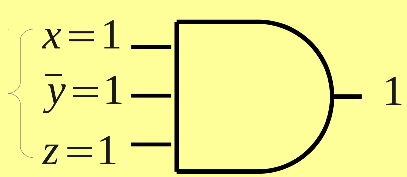
index

inputs

All possible
combination of inputs

minterm m_5 ➡ index 5

binary 101



$x\,\bar{y}\,z = 1$

0

bar

Truth Table and MAXterms (1)

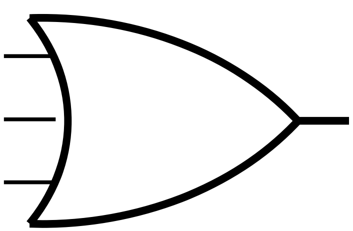
x	y	z		
0	0	0	the case when x=0 **and** y=0 **and** z=0	$x+y+z = 0$
0	0	1	the case when x=0 **and** y=0 **and** z=1	$x+y+\bar{z} = 0$
0	1	0	the case when x=0 **and** y=1 **and** z=0	$x+\bar{y}+z = 0$
0	1	1	the case when x=0 **and** y=1 **and** z=1	$x+\bar{y}+\bar{z} = 0$
1	0	0	the case when x=1 **and** y=0 **and** z=0	$\bar{x}+y+z = 0$
1	0	1	the case when x=1 **and** y=0 **and** z=1	$\bar{x}+y+\bar{z} = 0$
1	1	0	the case when x=1 **and** y=1 **and** z=0	$\bar{x}+\bar{y}+z = 0$
1	1	1	the case when x=1 **and** y=1 **and** z=1	$\bar{x}+\bar{y}+\bar{z} = 0$

inputs

All possible
combination of inputs

$\bar{x}+y+\bar{z} = 0 \Leftrightarrow$

$\bar{x}=0$, $y=0$, $\bar{z}=0$



$0 \Leftrightarrow$ $x=1$, $y=0$, $z=1$

For the output of an **or** gate to be 0,
all inputs must be 0

Truth Table and MAXterms (2)

index	x	y	z		
0	0	0	0	the case when the MAXterm	$M_0 = x+y+z = 0$
1	0	0	1	the case when the MAXterm	$M_1 = x+y+\overline{z} = 0$
2	0	1	0	the case when the MAXterm	$M_2 = x+\overline{y}+z = 0$
3	0	1	1	the case when the MAXterm	$M_3 = x+\overline{y}+\overline{z} = 0$
4	1	0	0	the case when the MAXterm	$M_4 = \overline{x}+y+z = 0$
5	1	0	1	the case when the MAXterm	$M_5 = \overline{x}+y+\overline{z} = 0$
6	1	1	0	the case when the MAXterm	$M_6 = \overline{x}+\overline{y}+z = 0$
7	1	1	1	the case when the MAXterm	$M_7 = \overline{x}+\overline{y}+\overline{z} = 0$

index

inputs

All possible
combination of inputs

minterm M_5 ➡ index 5

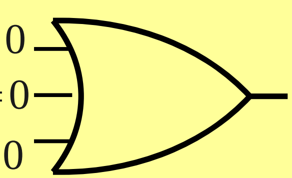
binary 101

$\overline{x}+y+\overline{z}$

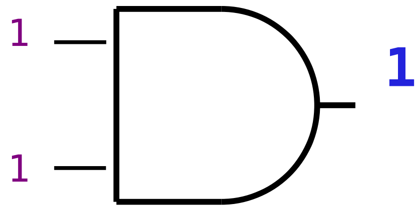
1

bar

$\overline{x}=0$
$y=0$
$\overline{z}=0$

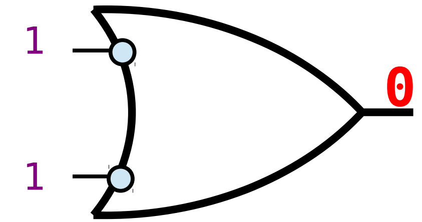


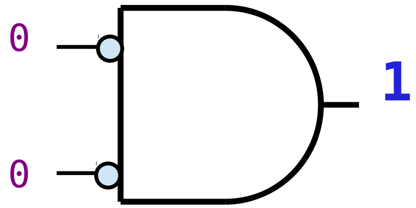
0



x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

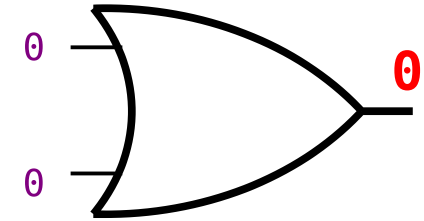
$\bar{x}$	$\bar{y}$	$\bar{x}+\bar{y}$
1	1	1
1	0	1
0	1	1
0	0	0





$\bar{x}$	$\bar{y}$	$\bar{x}\bar{y}$
1	1	1
1	0	0
0	1	0
0	0	0

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1



Boolean Function with minterms (1)

index	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

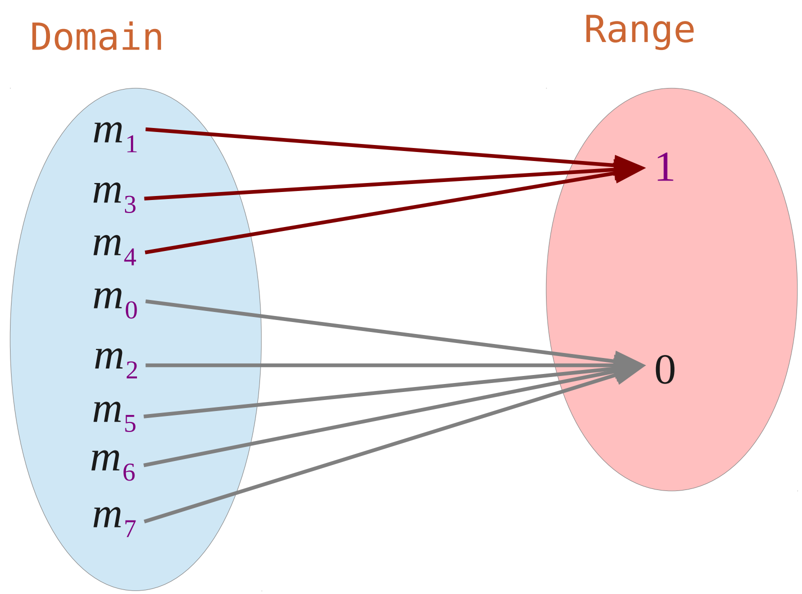
All possible
combination of inputs

The output F becomes 1,
for one of the <u>three</u> following cases

(the case when x=0 **and** y=0 **and** z=1) $\Longleftrightarrow$ $m_1 = \bar{x}\,\bar{y}\,z = 1$

or (the case when x=0 **and** y=1 **and** z=1) $\Longleftrightarrow$ $m_3 = \bar{x}\,y\,z = 1$

or (the case when x=1 **and** y=0 **and** z=0) $\Longleftrightarrow$ $m_4 = x\,\bar{y}\,\bar{z} = 1$



Boolean Function with minterms (2)

index	x	y	z	F
0	0	0	0	0
➡ 1	0	0	1	1
2	0	1	0	0
➡ 3	0	1	1	1
➡ 4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

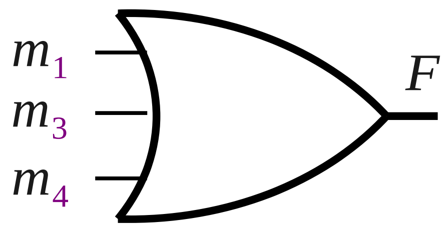
All possible
combination of inputs

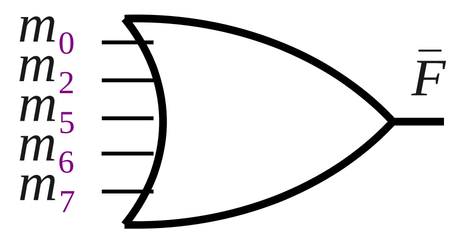
The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$

$\Longleftrightarrow \quad F = m_1 + m_3 + m_4$





For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with minterms (3)

index	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index inputs output

All possible
combination of inputs

The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$$m_1 + m_3 + m_4 = 1 \rightleftarrows F = 1$$

$$\Leftrightarrow \quad F = m_1 + m_3 + m_4$$

The output F becomes 0,

either $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$$m_0 + m_2 + m_5 + m_6 + m_7 = 1 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad \overline{F} = m_0 + m_2 + m_5 + m_6 + m_7$$

For the output of an **or** gate to be 1,
at least one must be 1

Boolean Function with Maxterms (1)

The output F becomes 0,
for one of the <u>five</u> following cases

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index — inputs — output

(the case when x=0 **and** y=0 **and** z=0) ⟺ $x + y + z = 0$

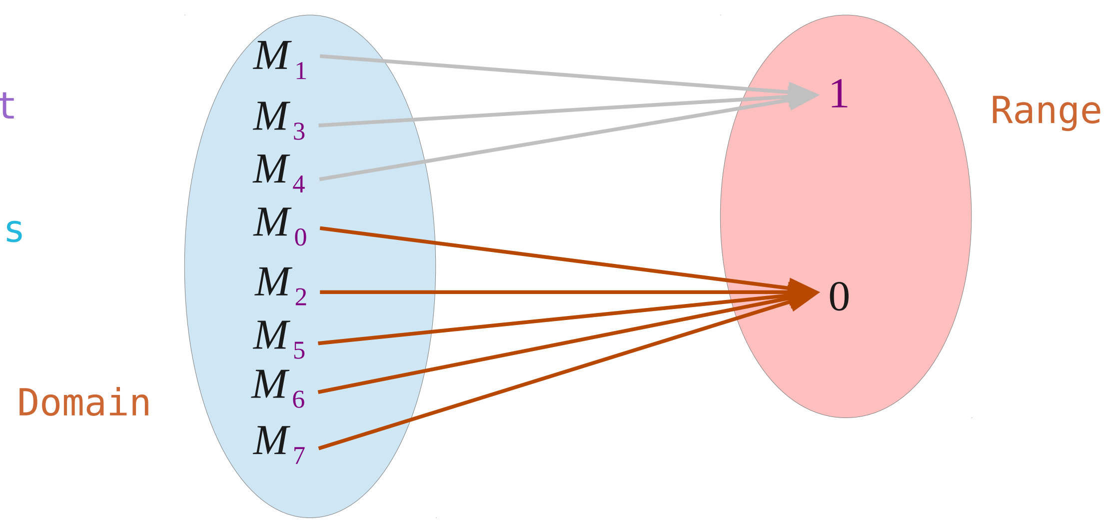
or (the case when x=0 **and** y=1 **and** z=0) ⟺ $x + \bar{y} + z = 0$

or (the case when x=1 **and** y=0 **and** z=1) ⟺ $\bar{x} + y + \bar{z} = 0$

or (the case when x=1 **and** y=1 **and** z=0) ⟺ $\bar{x} + \bar{y} + z = 0$

or (the case when x=1 **and** y=1 **and** z=1) ⟺ $\bar{x} + \bar{y} + \bar{z} = 0$

All possible
combination of inputs



Boolean Function with Maxterms (2)

index	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index inputs output

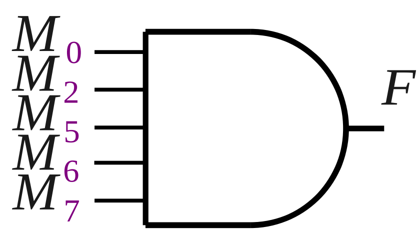
All possible
combination of inputs

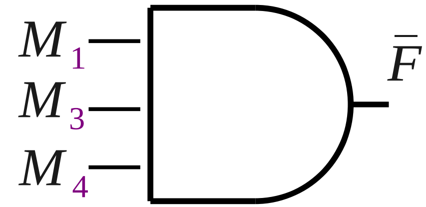
The output F becomes 0,

either $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \rightleftarrows F = 0$$

$$\Leftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$





For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function with Maxterms (2)

index	x	y	z	F
➡ 0	0	0	0	0
1	0	0	1	1
➡ 2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
➡ 5	1	0	1	0
➡ 6	1	1	0	0
➡ 7	1	1	1	0

index inputs output

All possible combination of inputs

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0 \rightleftarrows F = 0$$

$$\Longleftrightarrow \quad F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 1,

either $M_1=0$ or $M_3=0$ or $M_4=0$

$$M_1 \cdot M_3 \cdot M_4 = 0 \rightleftarrows F = 1$$

$$\Longleftrightarrow \quad \overline{F} = M_1 \cdot M_3 \cdot M_4$$

For the output of an **and** gate to be 0,
at least one input must be 0

Complimentary Relations

index	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index inputs output

All possible combination of inputs

$$m_i = \overline{M_i}$$

$$M_i = \overline{m_i}$$

$$F(x,y,z) = m_1 + m_3 + m_4$$

The output F becomes 1,

either $m_1=1$ or $m_3=1$ or $m_4=1$

For the output of an **or** gate to be 1,
at least one must be 1

$$\bar{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$$

$$\Leftrightarrow \quad F(x,y,z) = \overline{m_0 + m_2 + m_5 + m_6 + m_7}$$

$$= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_5} \cdot \overline{m_6} \cdot \overline{m_7}$$

$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

For the output of an **and** gate to be 0,
at least one input must be 0

Boolean Function Summary

	x	y	z	F
0	0	0	0	
1	0	0	1	1
2	0	1	0	
3	0	1	1	1
4	1	0	0	1
5	1	0	1	
6	1	1	0	
7	1	1	1	

The output F becomes 1,

for the cases

1) when $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$$F(x, y, z) = m_1 + m_3 + m_4 \Rightarrow \boxed{F = 1}$$

2) when $M_1 = 0$ or $M_3 = 0$ or $M_4 = 0$

$$\overline{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \Rightarrow F = 1 \; (\overline{F} = 0)$$

	x	y	z	F
0	0	0	0	0
1	0	0	1	
2	0	1	0	0
3	0	1	1	
4	1	0	0	
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 0,

for the cases

1) when $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$$\overline{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \Rightarrow F = 0 \; (\overline{F} = 1)$$

2) when $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \Rightarrow \boxed{F = 0}$$

Boolean Function Summary

	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$F(x,y,z) = m_1 + m_3 + m_4 \Rightarrow \boxed{F=1}$$

$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \Rightarrow \boxed{F=0}$$

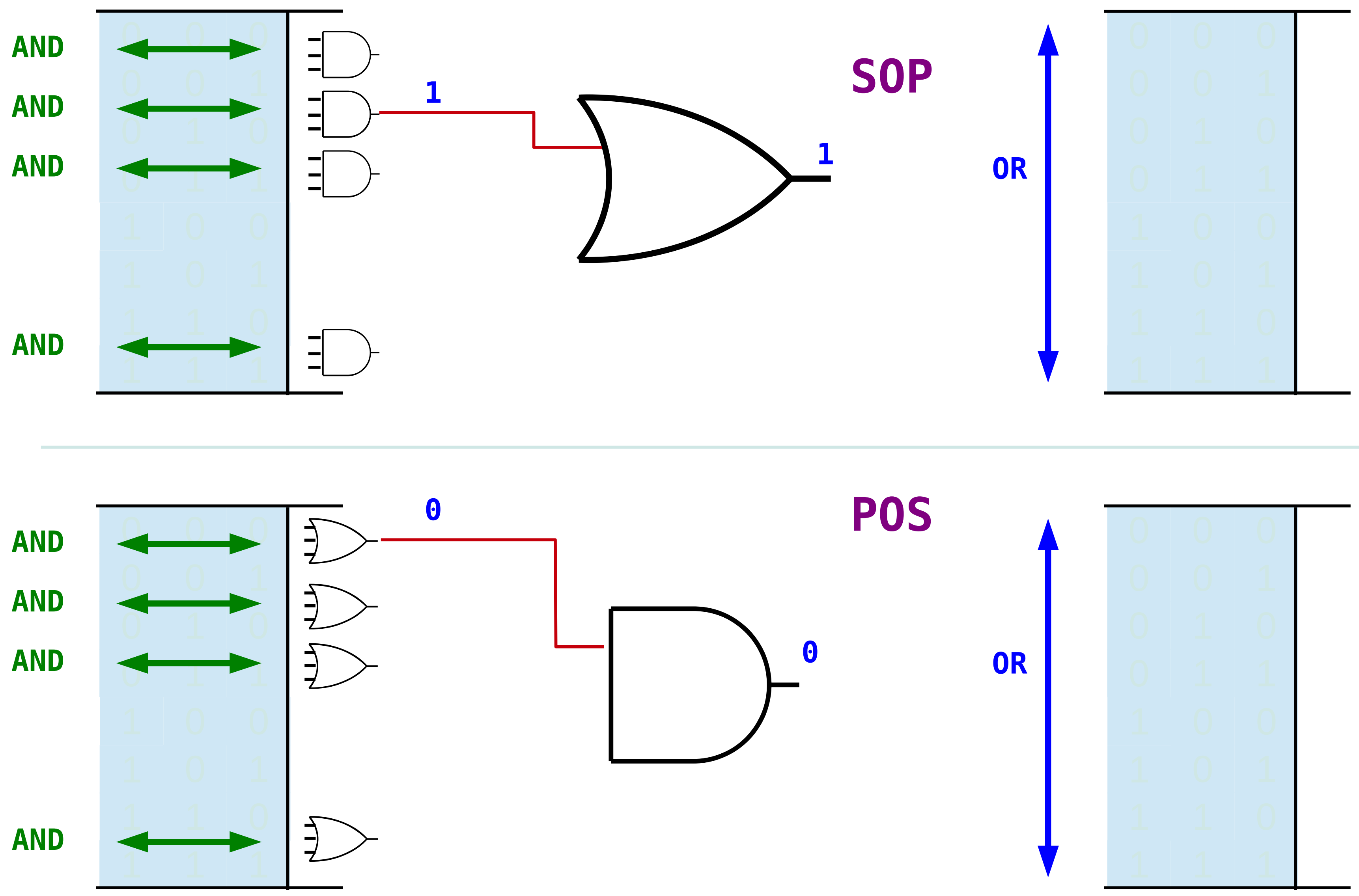
	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$\overline{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7 \Rightarrow F=0 \ (\overline{F}=1)$$

$$\overline{F}(x,y,z) = M_1 \cdot M_3 \cdot M_4 \Rightarrow F=1 \ (\overline{F}=0)$$

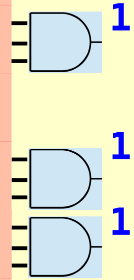
Young Won Lim
9/18/13

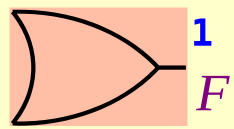
SOP and POS



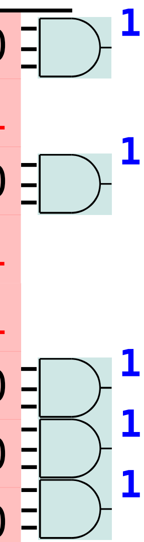
Boolean Function with minterms

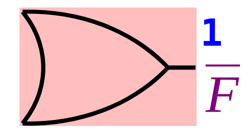
	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



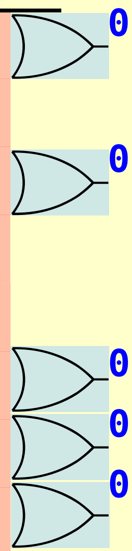


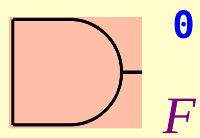
	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0



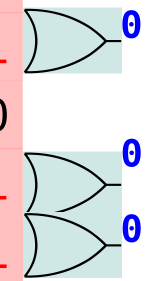


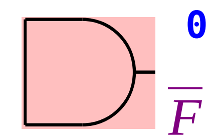
		x	y	z	F
⇨	0	0	0	0	0
	1	0	0	1	1
⇨	2	0	1	0	0
	3	0	1	1	1
	4	1	0	0	1
⇨	5	1	0	1	0
⇨	6	1	1	0	0
⇨	7	1	1	1	0





		x	y	z	F
	0	0	0	0	0
➡	1	0	0	1	1
	2	0	1	0	0
➡	3	0	1	1	1
➡	4	1	0	0	1
	5	1	0	1	0
	6	1	1	0	0
	7	1	1	1	0





Truth Table

References

[1] http://en.wikipedia.org/