Boolean Algebra (2E)

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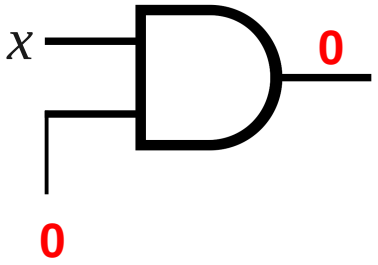
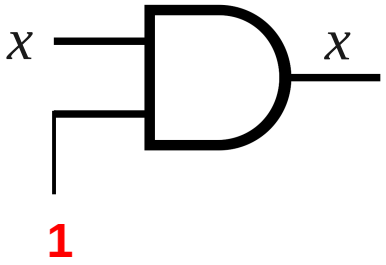
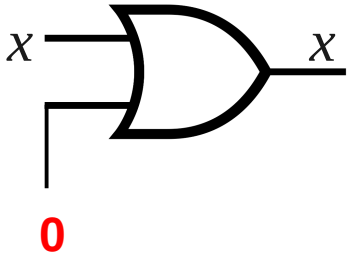
Identity and Null Element Theorem



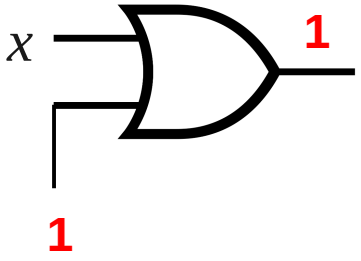
$$x \cdot 0 = 0$$



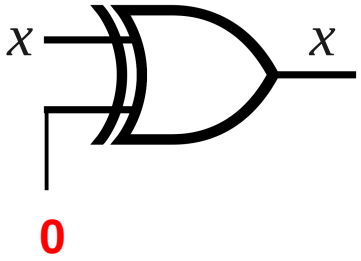
$$x \cdot 1 = x$$



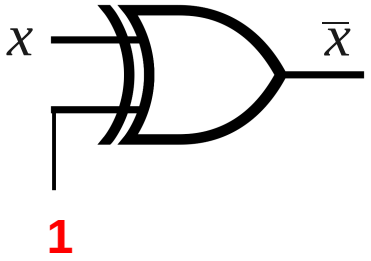
$$x + 0 = x$$



$$x + 1 = 1$$



$$x \oplus 0 = x$$



$$x \oplus 1 = \bar{x}$$

Distributive

$$x \cdot (y + z) = x \cdot y + x \cdot z \qquad \neq \quad x \cdot y + z$$

This parenthesis cannot be deleted

$$x + (y \cdot z) = (x + y) \cdot (x + z) \qquad = \quad x + y \cdot z$$

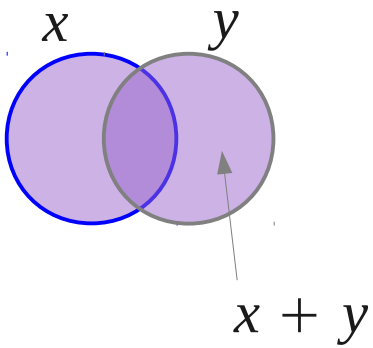
This parenthesis can be deleted

Operator precedence : $\cdot > +$

Inclusion

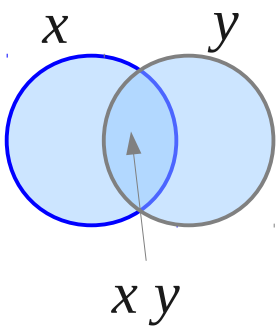
$$x \cdot (x + y) = x$$

$$\begin{aligned} x \cdot (x + y) &= x \cdot x + x \cdot y \\ &= x + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$



$$x + xy = x$$

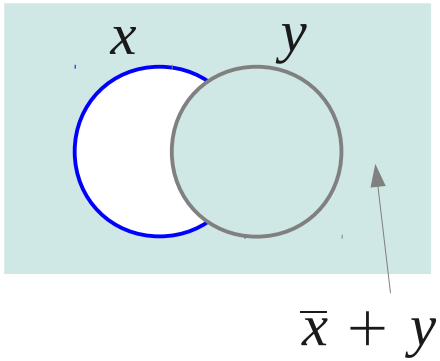
$$\begin{aligned} x + xy &= x \cdot 1 + x \cdot y \\ &= x \cdot (1 + y) \\ &= x \end{aligned}$$



Eliminate

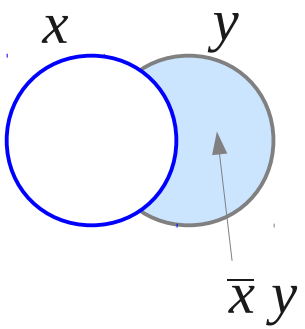
$$x \cdot (\bar{x} + y) = x\,y$$

$$\begin{aligned} x \cdot (\bar{x} + y) &= x \cdot \bar{x} + x \cdot y \\ &= 0 + x \cdot y \\ &= x \cdot y \end{aligned}$$



$$x + \bar{x}\,y = x + y$$

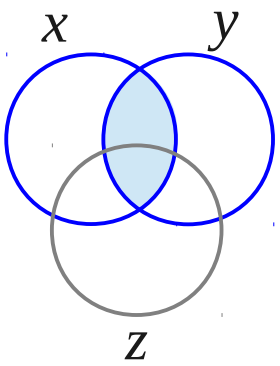
$$\begin{aligned} x + \bar{x}\,y &= (x + \bar{x}) \cdot (x + y) \\ &= 1 \cdot (x + y) \\ &= x + y \end{aligned}$$



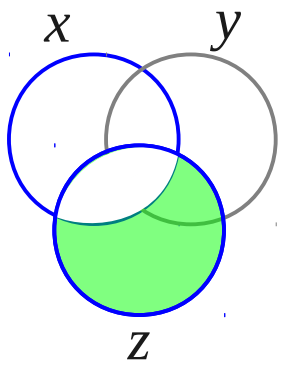
Consensus

$$x \cdot y + \bar{x} \cdot z + y \cdot z = x \cdot y + \bar{x} \cdot z$$

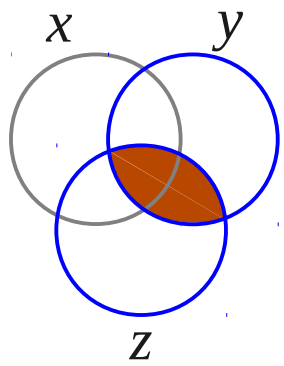
$x \cdot y$



$\bar{x} \cdot z$

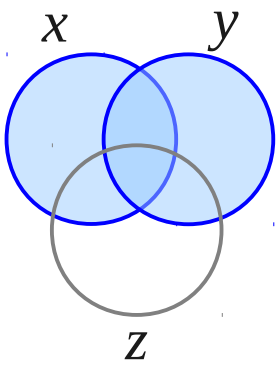


$y \cdot z$

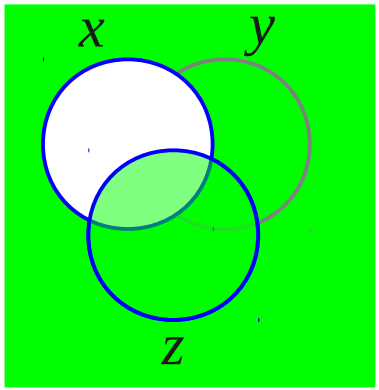


$$(x + y) \cdot (\bar{x} + z) \cdot (y + z) = (x + y) \cdot (\bar{x} + z)$$

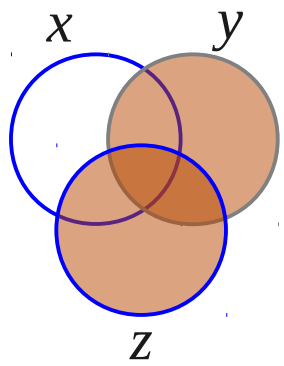
$x + y$



$\bar{x} + z$



$y \cdot z$



(x+y)(x+z) = x+yz

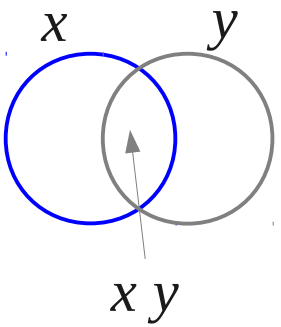
$$(x + y)(x + z) = x + yz$$

$$(x + y)(x + z) = xx + xz + xy + yz$$

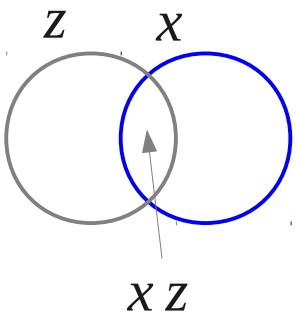
$$= \underline{x + xy} + xz + yz$$

$$= \underline{x + xz} + yz$$

$$= x + yz$$



$$x + xy = x$$



$$x + xz = x$$

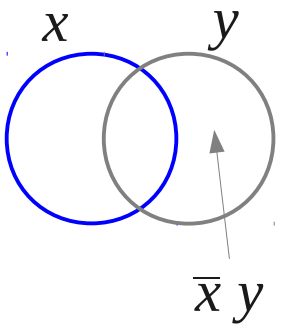
x + x'y = x + y

$$x + \bar{x}\,y = x + y$$

$$x + \bar{x}\,y = (x + \bar{x})\cdot(x + y)$$

$$= 1 \cdot (x + y)$$

$$= x + y$$



References

References

[1] http://en.wikipedia.org/
[2] M. M. Mano, C. R. Kime, “Logic and Computer Design Fundamentals”, 4th ed.
[3] D.M. Harris, S. L. Harris, “Digital Design and Computer Architecture”