Binary Numbers (1A)

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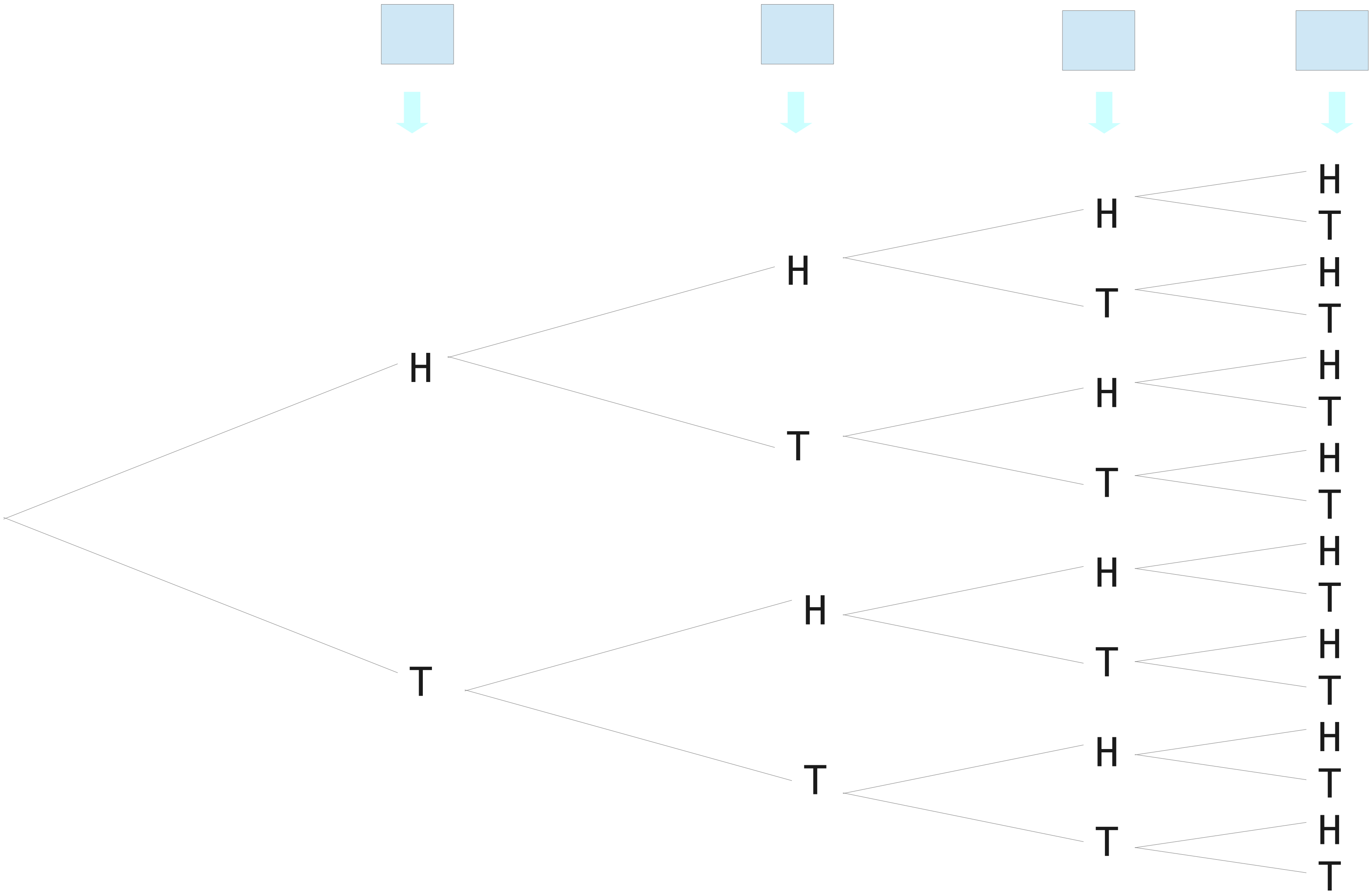
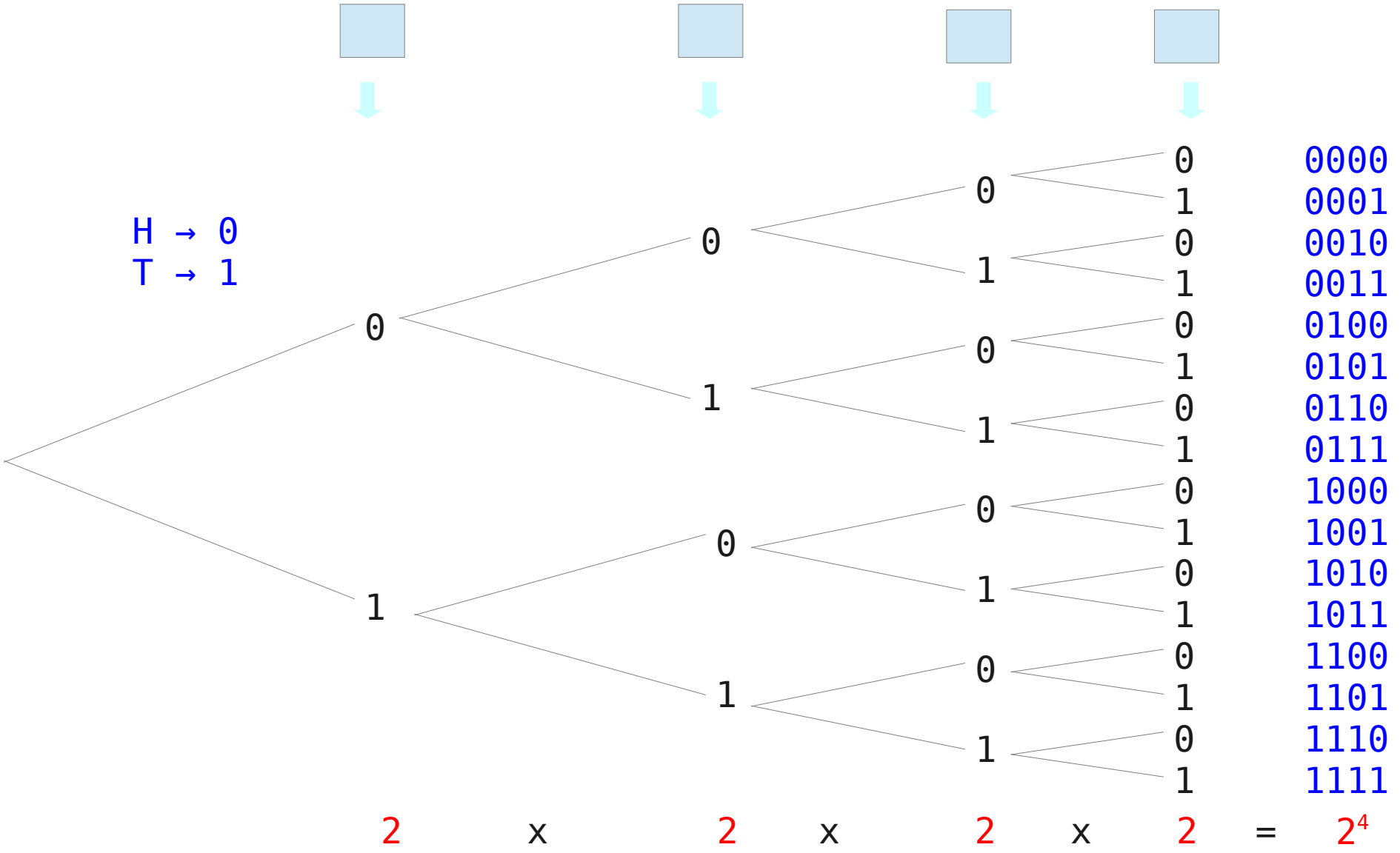
Coin Tossing

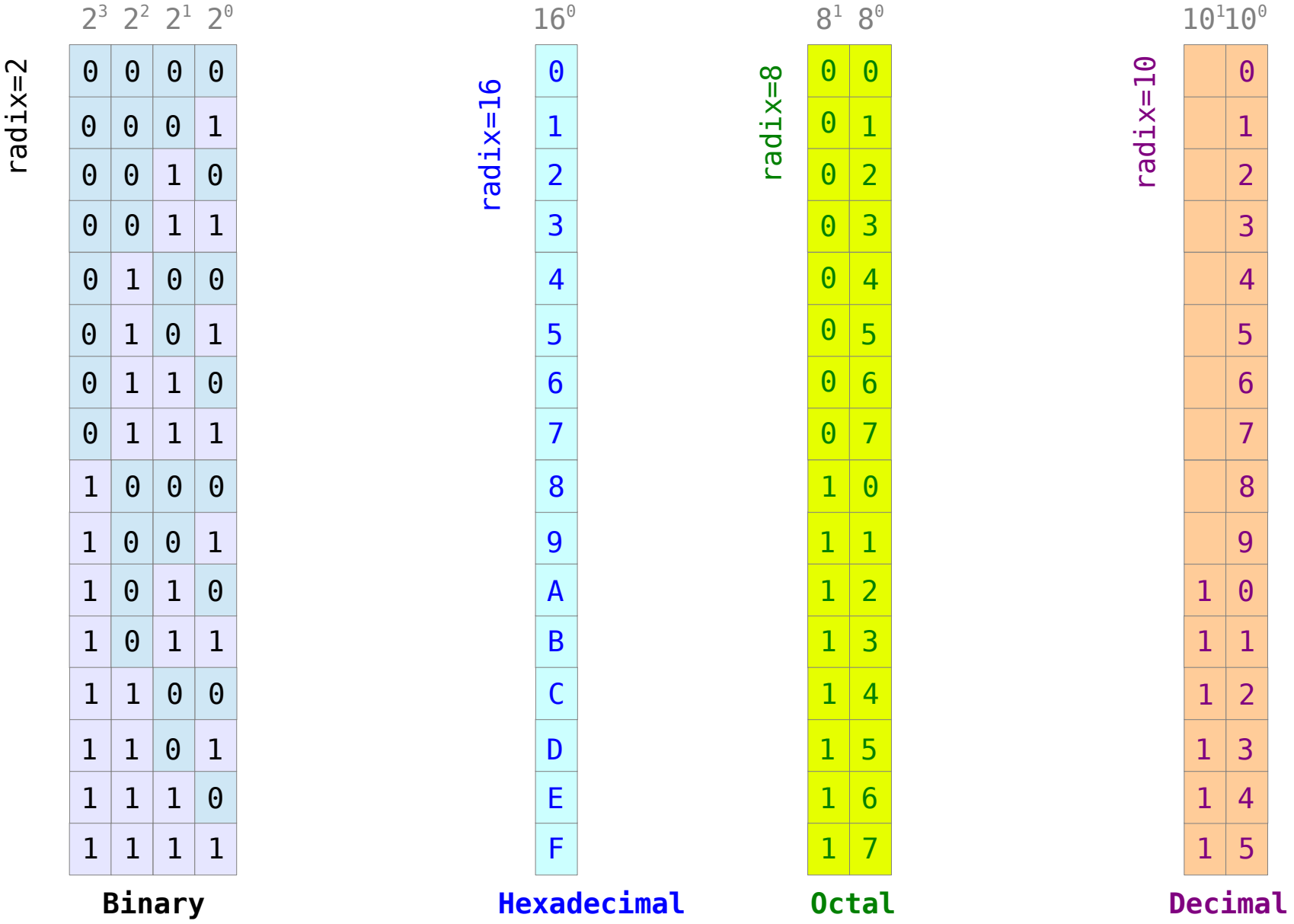


$2 \times 2 \times 2 \times 2 = 2^4$

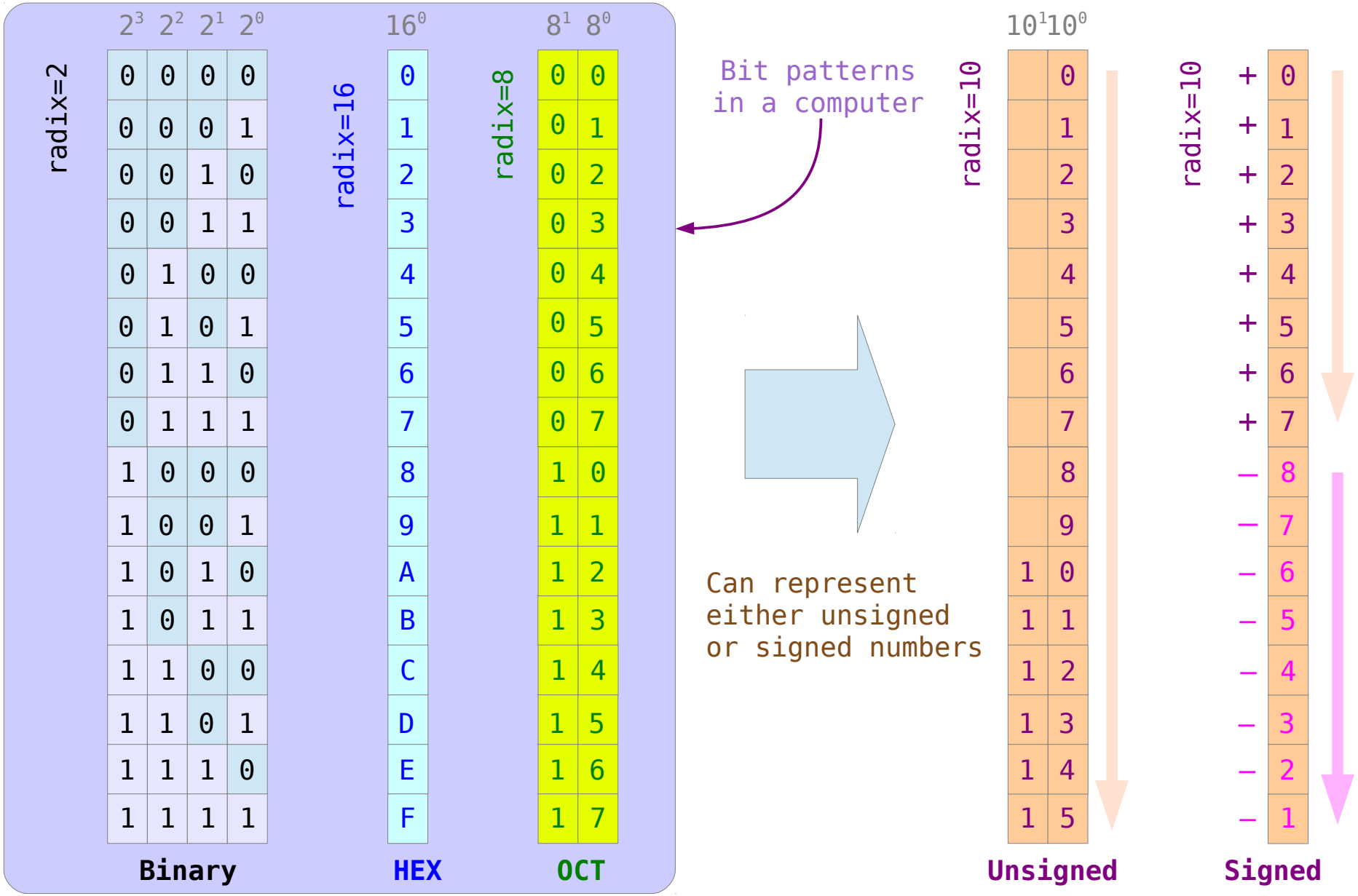
Coin Tossing & Binary Numbers



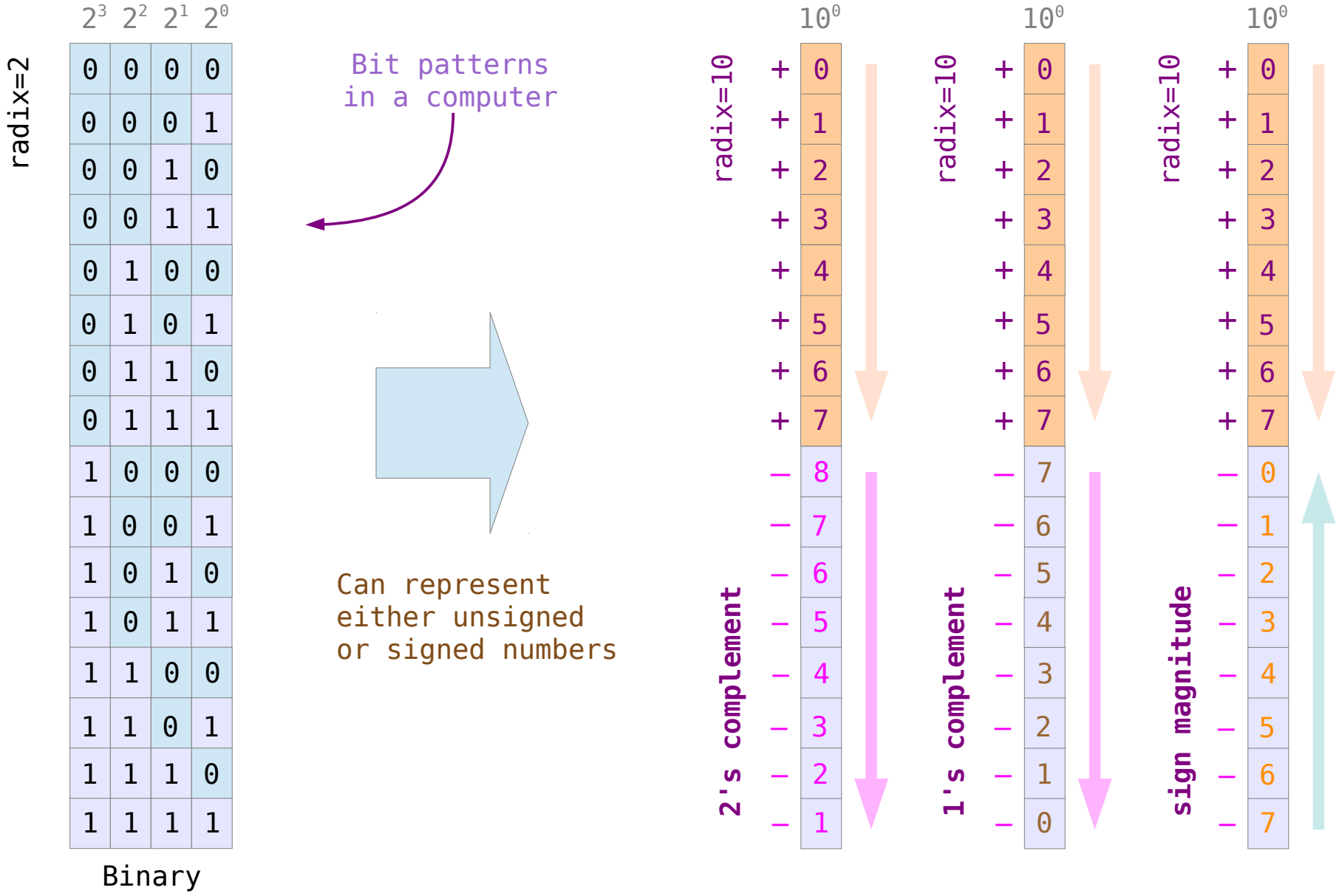
Number Systems



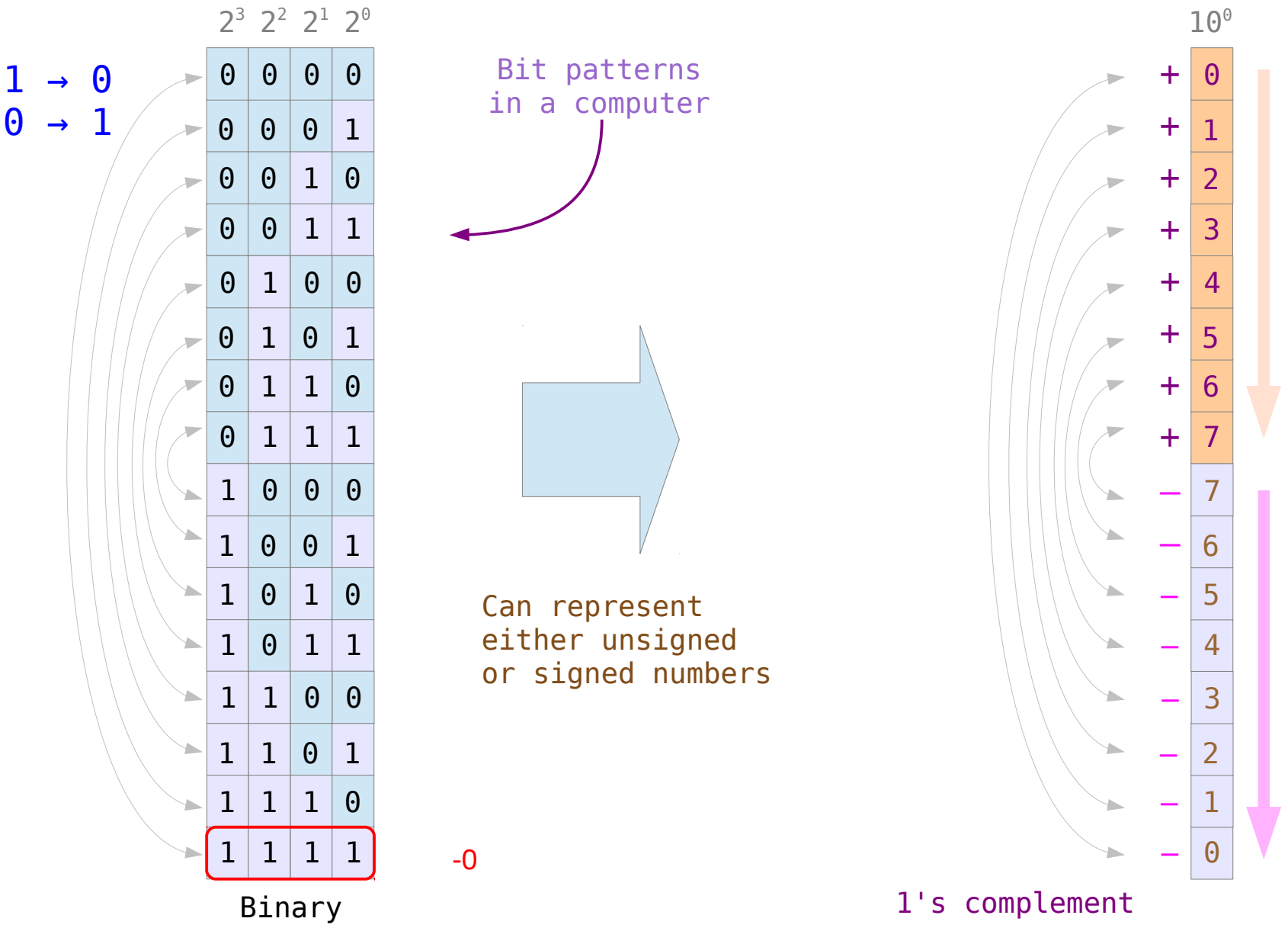
Bit Patterns



4-Bit Signed Numbers



1's Complement Bit Pattern



The Sum 1's Complement Numbers

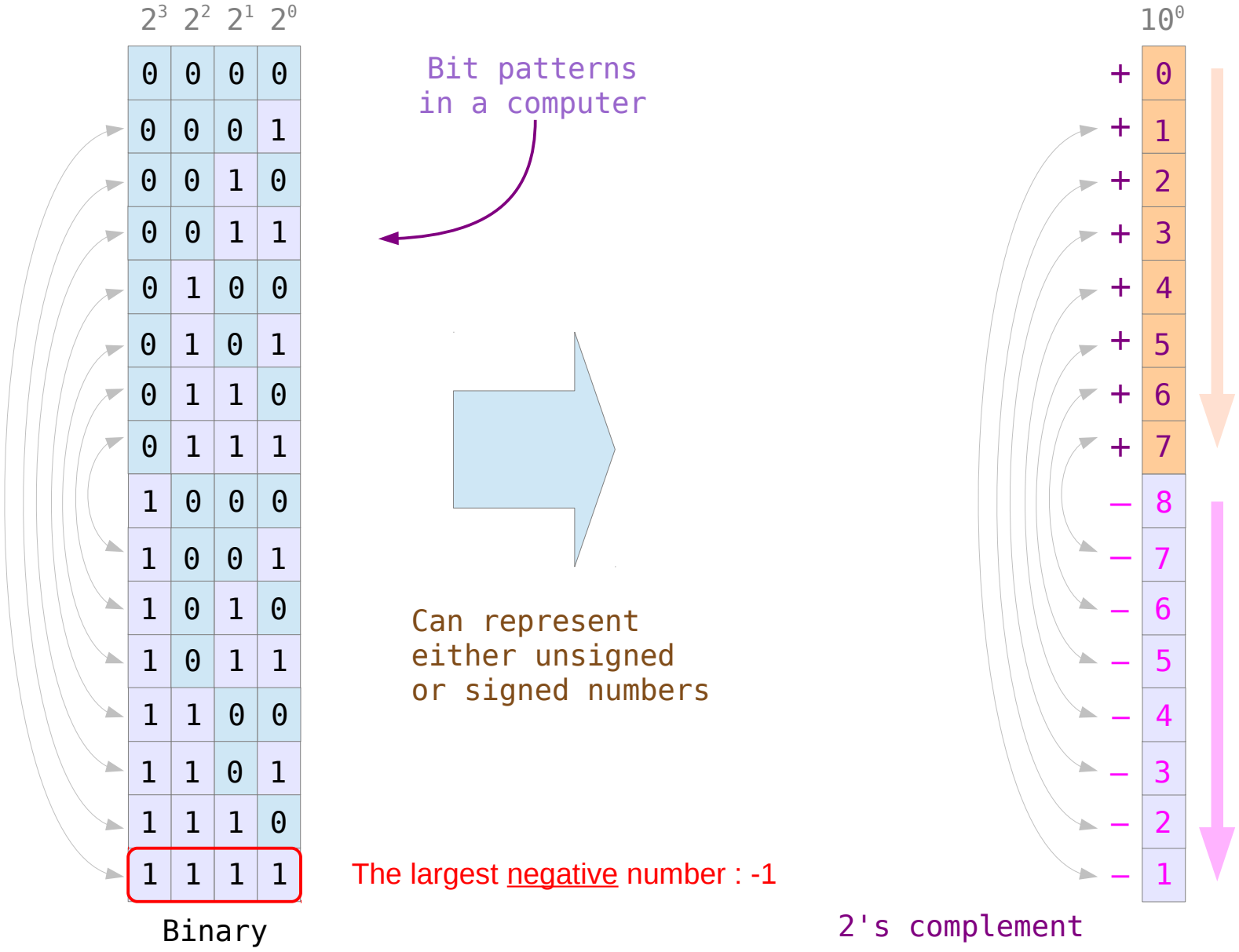
0	0	0	0	+	0	0	1	0	0	+	4
1	1	1	1	−	0	1	0	1	1	−	4
1	1	1	1	−	0	1	1	1	1	−	0
0	0	0	1	+	1	0	1	0	1	+	5
1	1	1	0	−	1	1	0	1	0	−	5
1	1	1	1	−	0	1	1	1	1	−	0
0	0	1	0	+	2	0	1	1	0	+	6
1	1	0	1	−	2	1	0	0	1	−	6
1	1	1	1	−	0	1	1	1	1	−	0
0	0	1	1	+	3	0	1	1	1	+	7
1	1	0	0	−	3	1	0	0	0	−	7
1	1	1	1	−	0	1	1	1	1	−	0

$$\begin{array}{r} +X \\ -X \\ \hline (2^n - 1) \end{array}$$

n-bit

No carry

2's Complement Bit Pattern



The Sum of 2's Complement Numbers

```
  0 0 0 0     0
  1 0 0 0   − 8
-------------------
1 1 0 0 0     8
```

```
1 1
  0 1 0 0   + 4
  1 1 0 0   − 4
-------------------
1 0 0 0 0   − 0
```

$$\begin{array}{r} +X \\ -X \\ \hline 2^n \end{array}$$

(n+1)-bit

```
1 1 1 1
  0 0 0 1   + 1
  1 1 1 1   − 1
-------------------
1 0 0 0 0     0
```

```
1 1 1 1
  0 1 0 1   + 5
  1 0 1 1   − 5
-------------------
1 0 0 0 0     0
```

```
1 1 1
  0 0 1 0   + 2
  1 1 1 0   − 2
-------------------
1 0 0 0 0     0
```

```
1 1 1 1
  0 1 1 0   + 6
  1 0 1 0   − 6
-------------------
1 0 0 0 0     0
```

```
1 1 1 1
  0 0 1 1   + 3
  1 1 0 1   − 3
-------------------
1 0 0 0 0     0
```

```
1 1 1 1
  0 1 1 1   + 7
  1 0 0 1   − 7
-------------------
1 0 0 0 0     0
```

Decimal to Binary (1)

$2 \;)\; 14$

$2 \;)\; 7 \quad \ldots \quad 0 \qquad 14 = 2 \cdot 7 + 0$

$2 \;)\; 3 \quad \ldots \quad 1 \qquad 7 = 2 \cdot 3 + 1$

$1 \quad \ldots \quad 1 \qquad 3 = 2 \cdot 1 + 1$

$$14 = 2 \cdot 7 + 0$$

$$14 = 2 \cdot (2 \cdot 3 + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \qquad 1 \cdot 2^3$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \qquad 1 \cdot 2^2$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \qquad 1 \cdot 2^1$$

Decimal to Binary (2)

$$14 = 2 \cdot 7 + 0$$

$$14 = 2 \cdot (2 \cdot 3 + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0$$

$$14 = 7 \cdot 2 + 0$$

$$14 = (3 \cdot 2 + 1) \cdot 2 + 0$$

$$14 = ((1 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0$$

$$(\cdots((A_{n-1}r + A_{n-2})r + A_{n-3})r + \cdots + A_1)r + A_0$$

A_{n-1}	A_{n-2}	A_{n-3}		$\cdots$		A_1	A_0

References

[1] http://en.wikipedia.org/
[2] M. M. Mano, C. R. Kime, “Logic and Computer Design Fundamentals”, 4th ed.
[3] M. M. Mano, M. D. Ciletti, “Digital Design”, 5th ed.
[4] D. M. Harris, S. L. Harris, “Digital Design and Computer Architecture”