Cylindrical Beamformer

- Octave Special Functions

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Based on the paper:

Daren J. Zywicki and Glenn J. Rix

Mitigation of Near-Field Effects for Seismic Surface Wave Velocity Estimation with Cylindrical Beamformers

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Steering Vector (1)

$$\boldsymbol{e}(\boldsymbol{k}) = [\exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_1) \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_2) \quad \cdots \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_s)]^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = \exp\{\, -j[\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1)) \quad \Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2)) \quad \cdots \quad \Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s))]\,\}^T$$

$$\boldsymbol{e}(\boldsymbol{k}) = [\exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_1) \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_2) \quad \cdots \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_s)]^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = [\exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1))) \quad \exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2))) \quad \cdots \quad \exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s)))]^T$$

$$\boldsymbol{e}(\boldsymbol{k}) = [e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_1} \quad e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_2} \quad \cdots \quad e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_s}]^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = [e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1))} \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2))} \quad \cdots \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s))}]^T$$

Steering Vector (2)

$$\boldsymbol{e}(\boldsymbol{k}) = [\exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_1) \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_2) \quad \cdots \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_s)]^T$$

$$\boldsymbol{e}(\boldsymbol{k}) = [\exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_1) \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_2) \quad \cdots \quad \exp(-j\,\boldsymbol{k}\cdot\boldsymbol{x}_s)]^T$$

$$\boldsymbol{e}(\boldsymbol{k}) = [e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_1} \quad e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_2} \quad \cdots \quad e^{-j\boldsymbol{k}\cdot\boldsymbol{x}_s}]^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = \exp\{\, -j[\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1)) \quad \Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2)) \quad \cdots \quad \Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s))]\,\}^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = [\exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1))) \quad \exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2))) \quad \cdots \quad \exp(-j\,\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s)))]^T$$

$$\boldsymbol{h}(\boldsymbol{k}) = [e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1))} \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2))} \quad \cdots \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s))}]^T$$

Steering Vector (3)

$$\boldsymbol{h}(\boldsymbol{k}) = [e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_1))} \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_2))} \quad \cdots \quad e^{-j\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x}_s))}]^T$$

$$H_0(\boldsymbol{k}\cdot\boldsymbol{x}) = J_0(\boldsymbol{k}\cdot\boldsymbol{x}) + jY_0(\boldsymbol{k}\cdot\boldsymbol{x})$$

Bessel functions of the 1st kind : J_α

Bessel functions of the 2nd kind : Y_α

Φ : phase angle of the argument

$$\Phi(H_0(\boldsymbol{k}\cdot\boldsymbol{x})) = \tan^{-1}\left(\frac{Y_0(\boldsymbol{k}\cdot\boldsymbol{x})}{J_0(\boldsymbol{k}\cdot\boldsymbol{x})}\right)$$

$$\tan^{-1}\left(\frac{Y_0(\boldsymbol{k}\cdot\boldsymbol{x})}{J_0(\boldsymbol{k}\cdot\boldsymbol{x})}\right)$$

Bessel Functions

Bessel functions of the 1st kind : J_α

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+\alpha+1)} \left(\frac{1}{2}x\right)^{2m}$$

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+1)} \left(\frac{1}{2}x\right)^{2m}$$

Bessel functions of the 2nd kind : Y_α

$$Y_\alpha(x) = \frac{J_\alpha(x)\cos(\alpha\pi) - J_{-\alpha}(x)}{\sin(\alpha\pi)}$$

$$Y_n(x) = \lim_{\alpha \to n} Y_\alpha(x)$$

$$Y_n(x) = \frac{1}{\pi}\int_0^\pi \sin(x\sin\theta - n\theta)\,d\theta - \frac{1}{\pi}\int_0^\infty [e^{nt} + (-1)^n e^{-nt}]e^{-x\sinh t}\,dt$$

$$Y_0(x) = \frac{1}{\pi}\int_0^\pi \sin(x\sin\theta)\,d\theta - \frac{1}{\pi}\int_0^\infty 2e^{-x\sinh t}\,dt$$

Octave Special Functions

Loadable Function: [j, ierr] = **besselj** (alpha, x, opt)
Loadable Function: [y, ierr] = **bessely** (alpha, x, opt)
Loadable Function: [i, ierr] = **besseli** (alpha, x, opt)
Loadable Function: [k, ierr] = **besselk** (alpha, x, opt)
Loadable Function: [h, ierr] = **besselh** (alpha, k, x, opt)

Compute Bessel or Hankel functions of various kinds:

besselj Bessel functions of the first kind.
If the argument opt is supplied, the result is multiplied by exp(-abs(imag(x))).

bessely Bessel functions of the second kind.
If the argument opt is supplied, the result is multiplied by exp(-abs(imag(x))).

besseli Modified Bessel functions of the first kind.
If the argument opt is supplied, the result is multiplied by exp(-abs(real(x))).

besselk Modified Bessel functions of the second kind.
If the argument opt is supplied, the result is multiplied by exp(x).

besselh Compute Hankel functions of the first (k = 1) or second (k = 2) kind.
If the argument opt is supplied, the result is multiplied by exp (-I*x) for k = 1 or exp (I*x) for k = 2.

Steering Vector in Octave

$$h(k) = [e^{-j\Phi(H_0(k\cdot x_1))} \quad e^{-j\Phi(H_0(k\cdot x_2))} \quad \cdots \quad e^{-j\Phi(H_0(k\cdot x_s))}]^T$$

$$H_0^{(1)}(k\cdot x) = J_0(k\cdot x) + jY_0(k\cdot x)$$ Hankel functions of the first (k = 1)

$$H_0^{(2)}(k\cdot x) = J_0(k\cdot x) - jY_0(k\cdot x)$$ Hankel functions of the second (k = 2)

Bessel functions of the 1st kind : J_α

Bessel functions of the 2nd kind : Y_α

besselh (alpha, k, x, opt)

besselh (0, 1, x)

Φ : phase angle of the argument

$$\Phi(H_0(k\cdot x)) = \tan^{-1}\left(\frac{Y_0(k\cdot x)}{J_0(k\cdot x)}\right)$$

Steering Vector (3)

Sdfsdf
Mapping Function: besseli (alpha, x)
Mapping Function: besselj (alpha, x)
Mapping Function: besselk (alpha, x)
Mapping Function: bessely (alpha, x)
Compute Bessel functions of the following types:

besselj
Bessel functions of the first kind.

bessely
Bessel functions of the second kind.

besseli
Modified Bessel functions of the first kind.

besselk
Modified Bessel functions of the second kind.
The second argument, x, must be a real matrix, vector, or scalar.

The first argument, alpha, must be greater than or equal to zero. If alpha is a range, it must have an increment equal to one.

If alpha is a scalar, the result is the same size as x.

If alpha is a range, x must be a vector or scalar, and the result is a matrix with length(x) rows and length(alpha) columns.

References

[1] http://en.wikipedia.org/
[2] D. J. Zywicki and G. J. Rix, "Mitigation of Near-Field Effects for Seismic Surface Wave Velocity Estimation with Cylidrical Beamformers"
[3] http://www.network-theory.co.uk/docs/octave3/octave_193.html