CMOS Combi-1 (H.1)

20151118

References

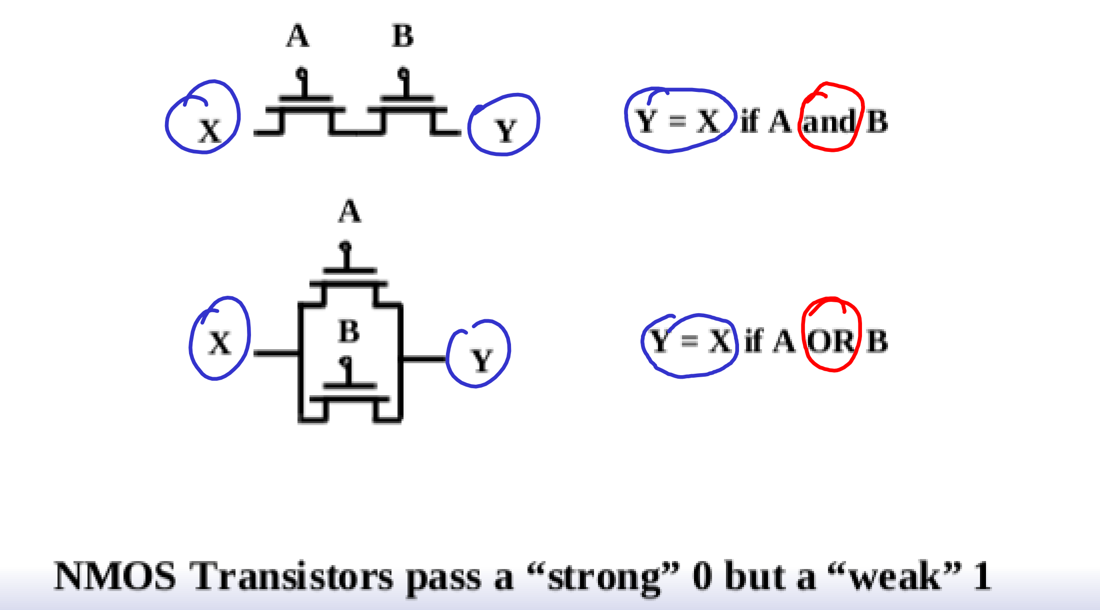
Some Figures from the following sites

[1] http://pages.hmc.edu/harris/cmosvlsi/4e/index.html
Weste & Harris Book Site

[2] en.wikipedia.org

[3] Digital Integrated Circuits : A Design Perspective,
Jan M. Rabaey,
(http://bwrcs.eecs.berkeley.edu/Classes/IcBook/)

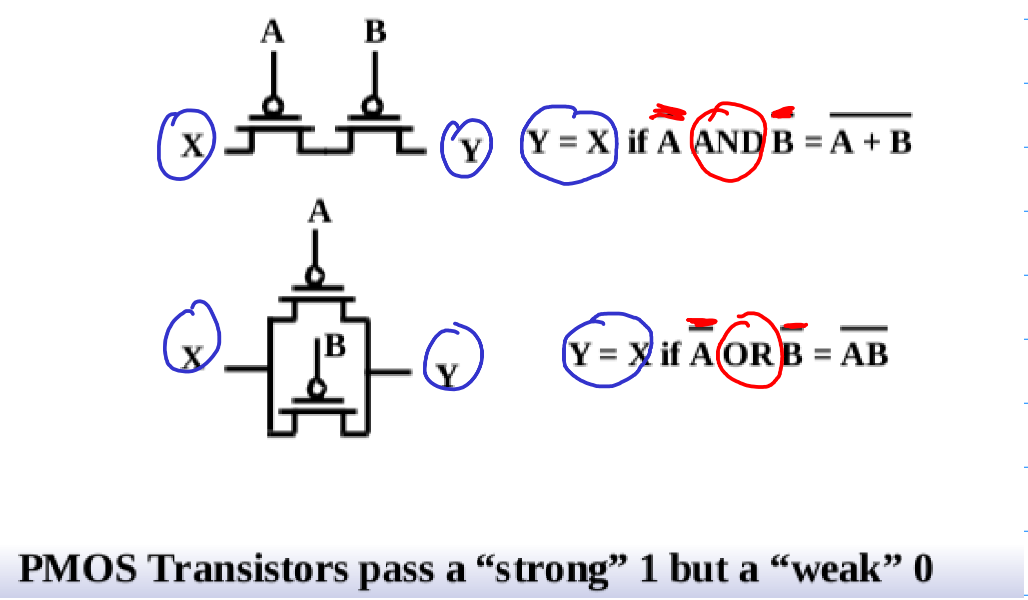
[4] Digital Electronics and Design with VHDL
Pedroni



Y = X if A and B

Y = X if A OR B

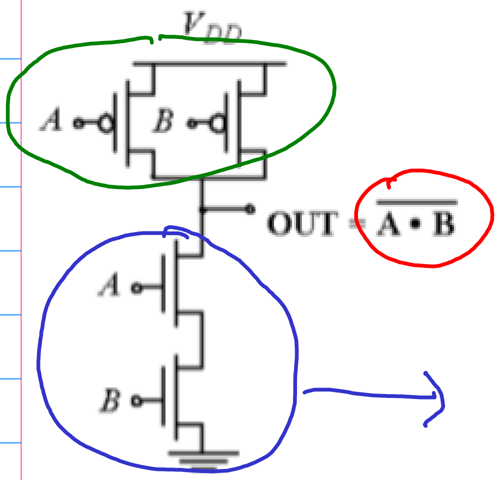
NMOS Transistors pass a "strong" 0 but a "weak" 1



Y = X if $\bar{A}$ AND $\bar{B}$ = $\overline{A + B}$

Y = X if $\bar{A}$ OR $\bar{B}$ = $\overline{AB}$

PMOS Transistors pass a "strong" 1 but a "weak" 0



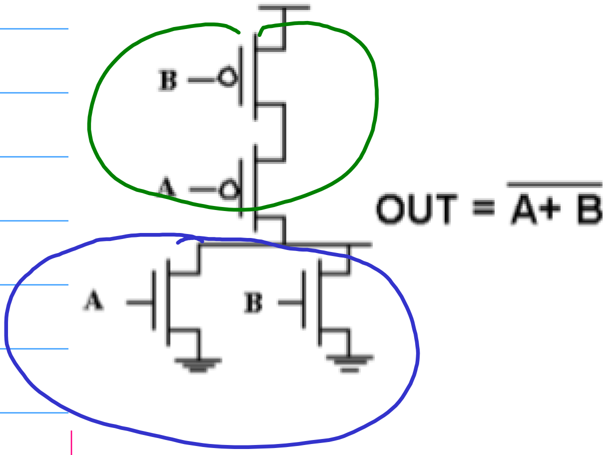
if $\bar{A} + \bar{B}$, then OUT ⇒ 1

$\bar{A} + \bar{B}$

if $A \cdot B$, then OUT ⇒ 0

$\overline{A \cdot B}$ NAND

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$



if $\bar{A} \cdot \bar{B}$, then OUT ⇒ 1

$\bar{A} \cdot \bar{B}$

if $A + B$, then OUT ⇒ 0

$\overline{A+B}$ NOR

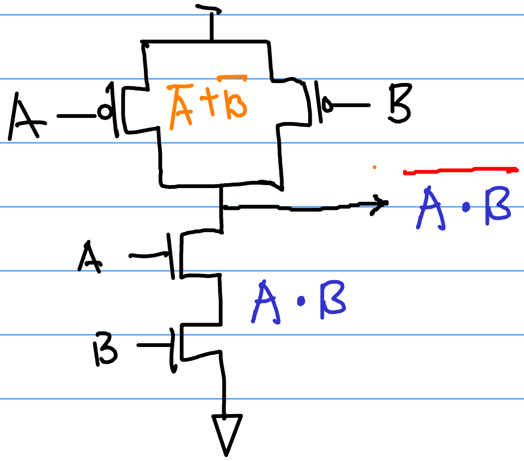
$$\bar{A} \cdot \bar{B} = \overline{A+B}$$

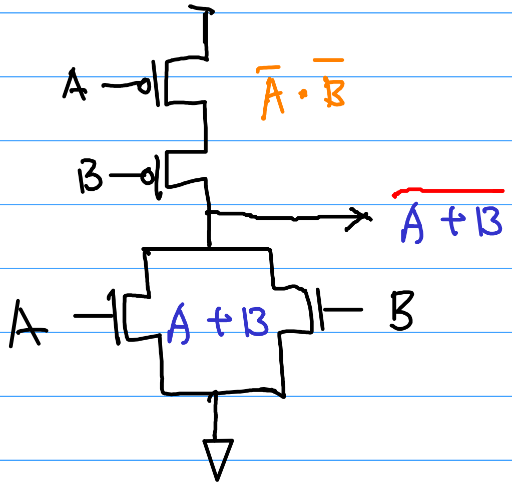
NAND $\overline{A \cdot B}$

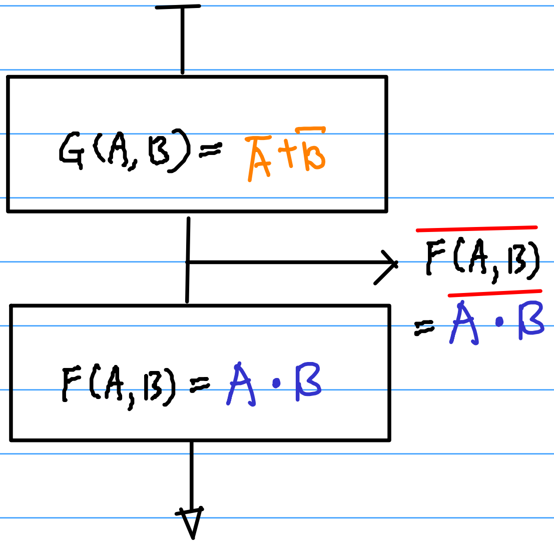
$= \bar{A} + \bar{B}$

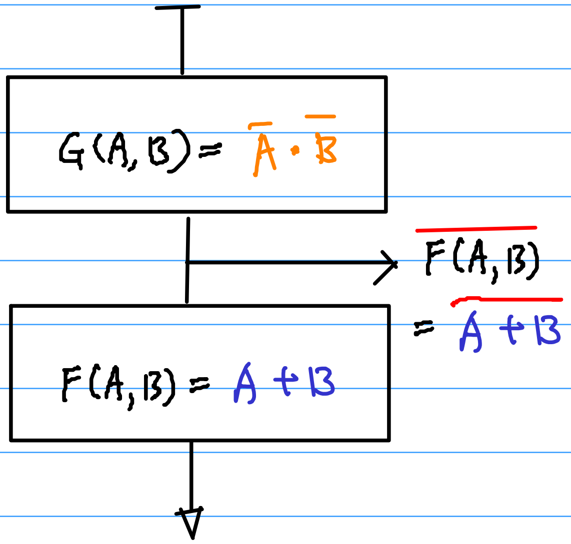
NOR $\overline{A + B}$

$= \bar{A} \cdot \bar{B}$

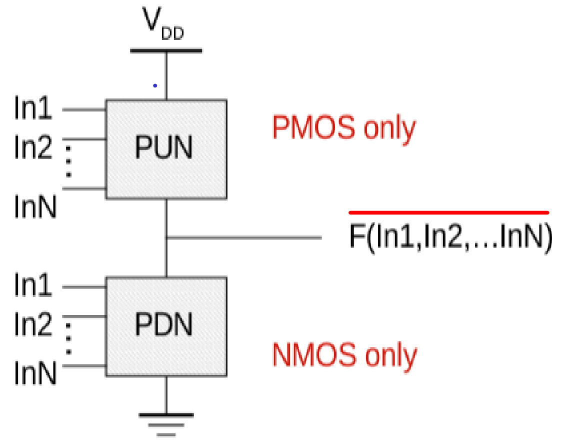








Logic Function



PUN and PDN are dual logic networks

Pull up Network — pMos

implements G

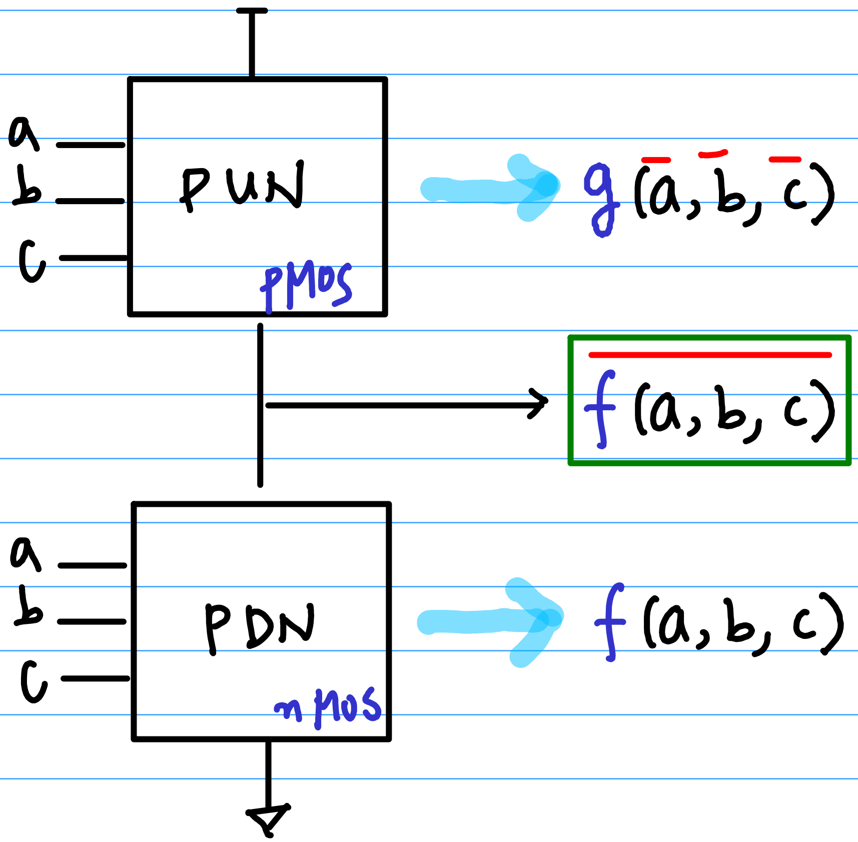
Pull down Network — nMos

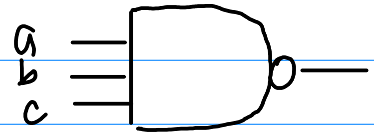
implements $\overline{F}$

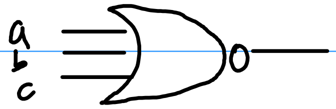
$$\overline{F(in1, in2, \ldots, inN)} = G(\overline{in1}, \overline{in2}, \ldots, \overline{inN})$$

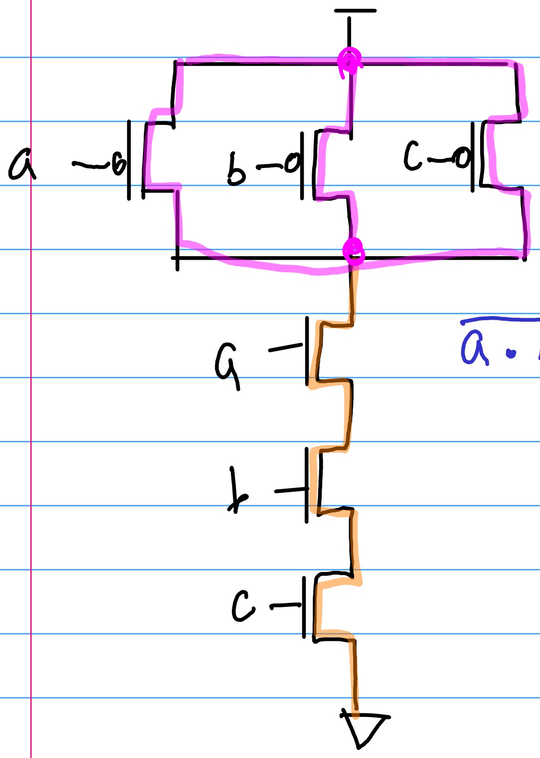
pMos: $G(\overline{in1}, \overline{in2}, \ldots, \overline{inN})$

nMos: $\overline{F(in1, in2, \ldots, inN)}$

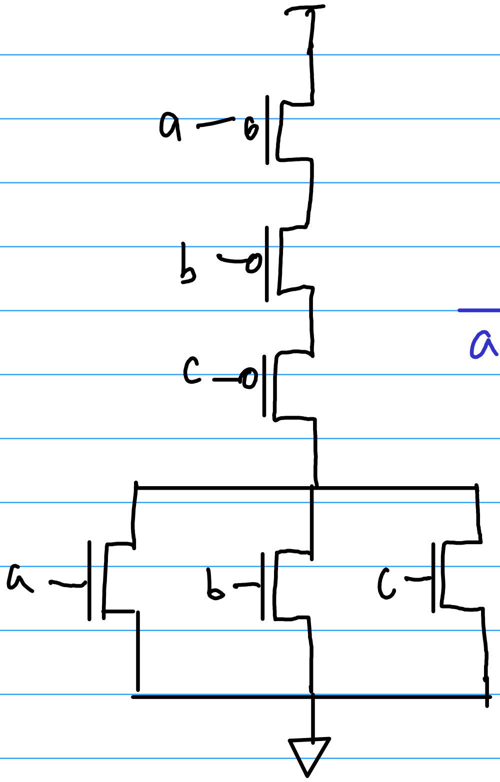

a
b
c
PUN
pMOS
$g(\bar{a},\bar{b},\bar{c})$
$\overline{f(a,b,c)}$
a
b
c
PDN
nMOS
$f(a,b,c)$



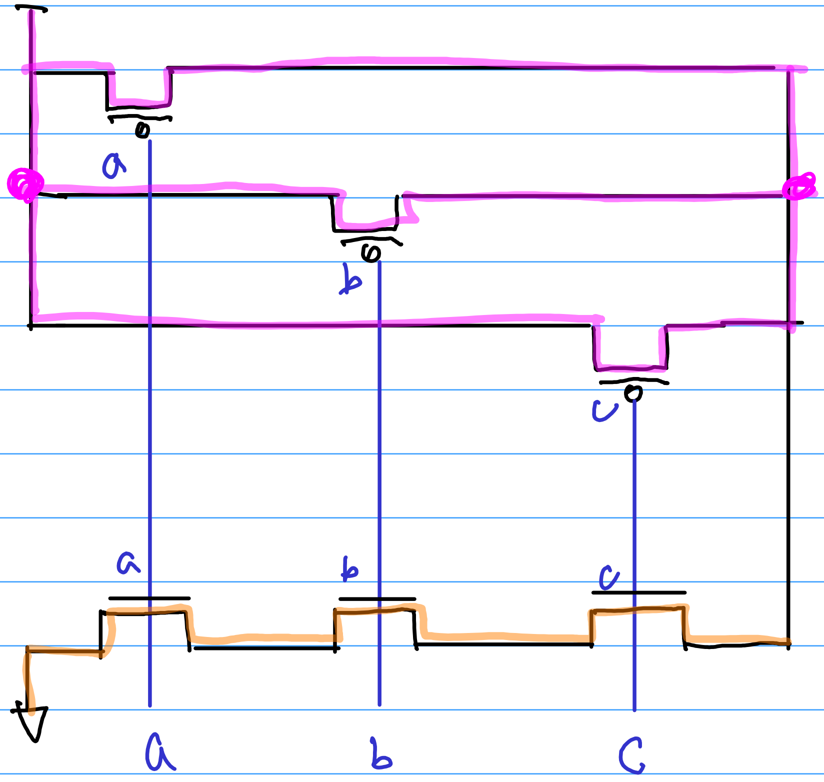




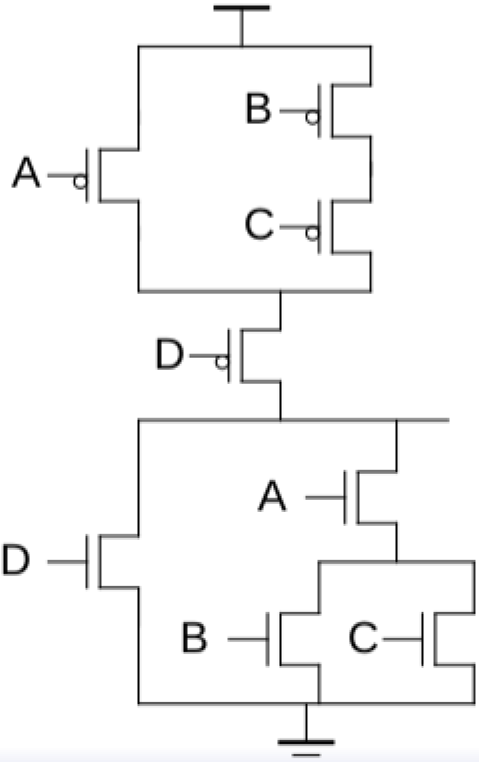
$\overline{a \cdot b \cdot c}$



$\overline{a + b + c}$



look like a physical layout



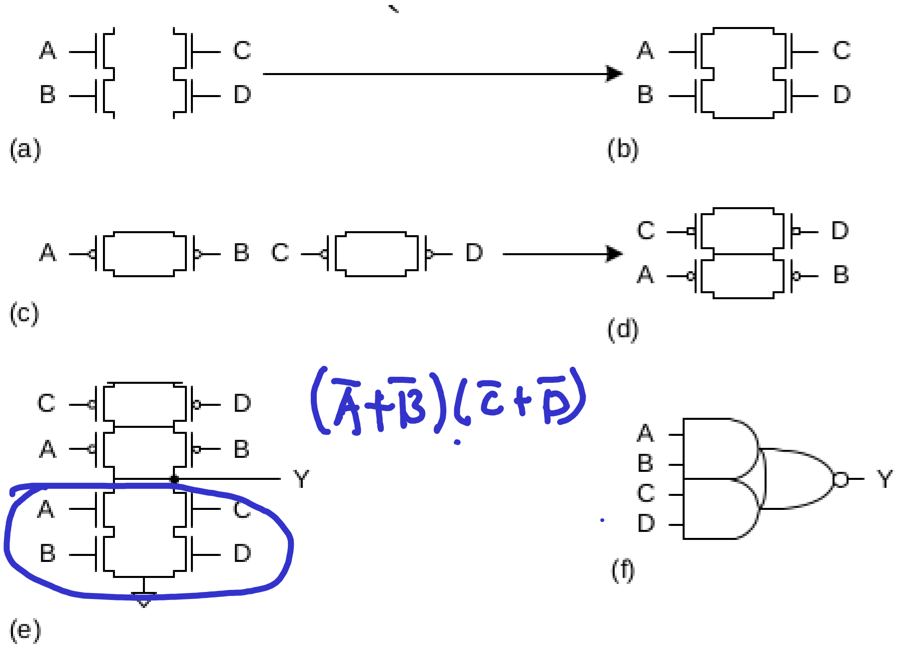
$$OUT = \overline{D + A \cdot (B + C)}$$

if $(\bar{A}+(\bar{B}\cdot\bar{C}))\cdot\bar{D}$, OUT ⇒ 1

$(\bar{A}+(\bar{B}\cdot\bar{C}))\cdot\bar{D}$

if $D + A\cdot(B+C)$, OUT ⇒ 0

$\overline{D + A\cdot(B+C)}$



$(A\cdot B)+(C\cdot D)$

$(\bar{C}+\bar{D})\cdot(\bar{A}+\bar{B})$

$(\bar{A}+\bar{B})(\bar{C}+\bar{D})$

$\underline{\overline{AB+CD}}$

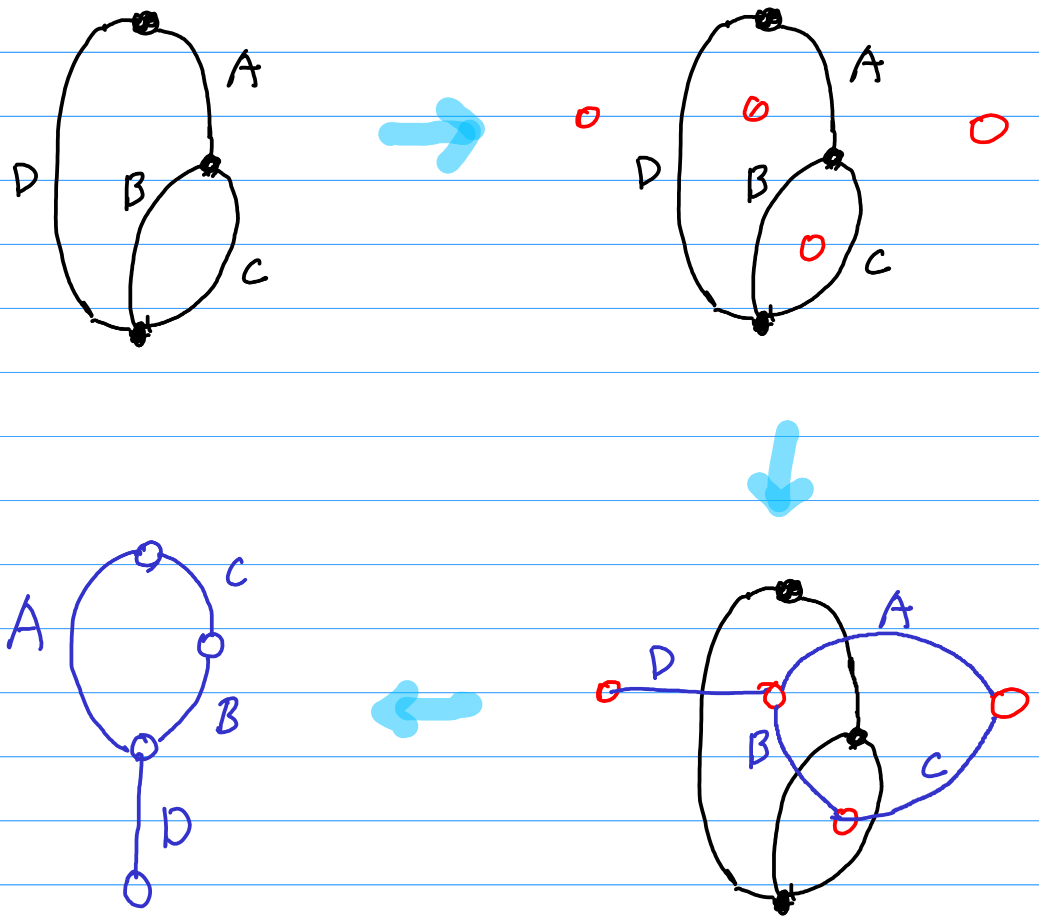
$\overline{(A\cdot B)+(C\cdot D)}$

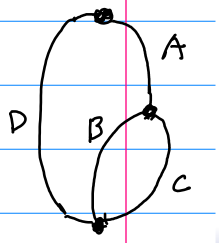
$= \overline{A\cdot B}\cdot\overline{C\cdot D}$

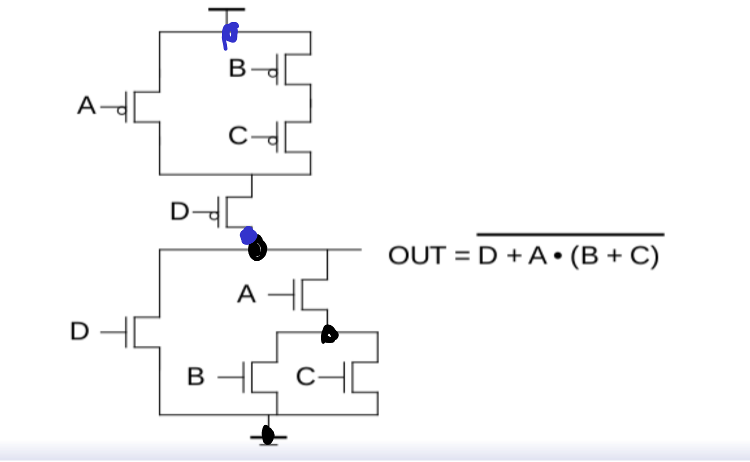
$= (\bar{A}+\bar{B})\cdot(\bar{C}+\bar{D})$

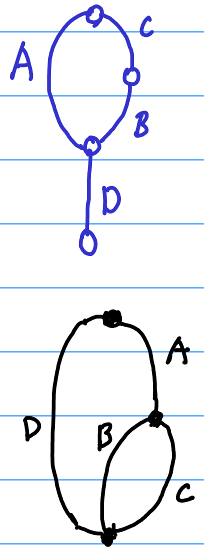
Finding (PUN) from (PDN)

$OUT = \overline{D + A \cdot (B+C)}$





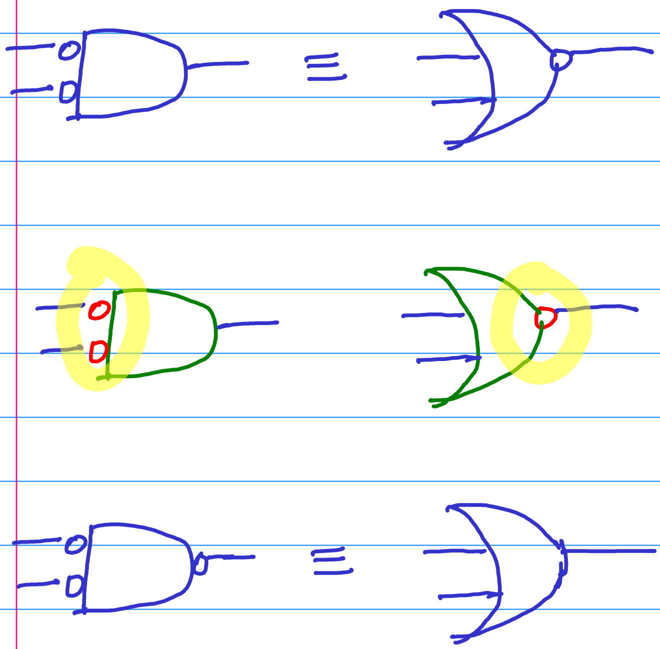


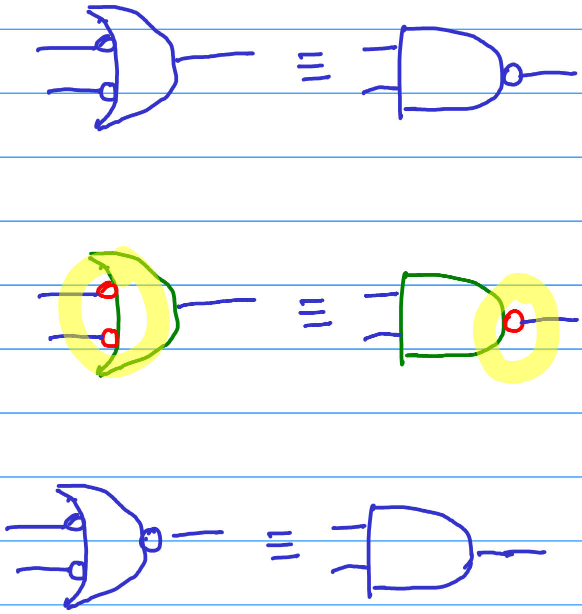


Bubble Pushing and DeMorgan's Law

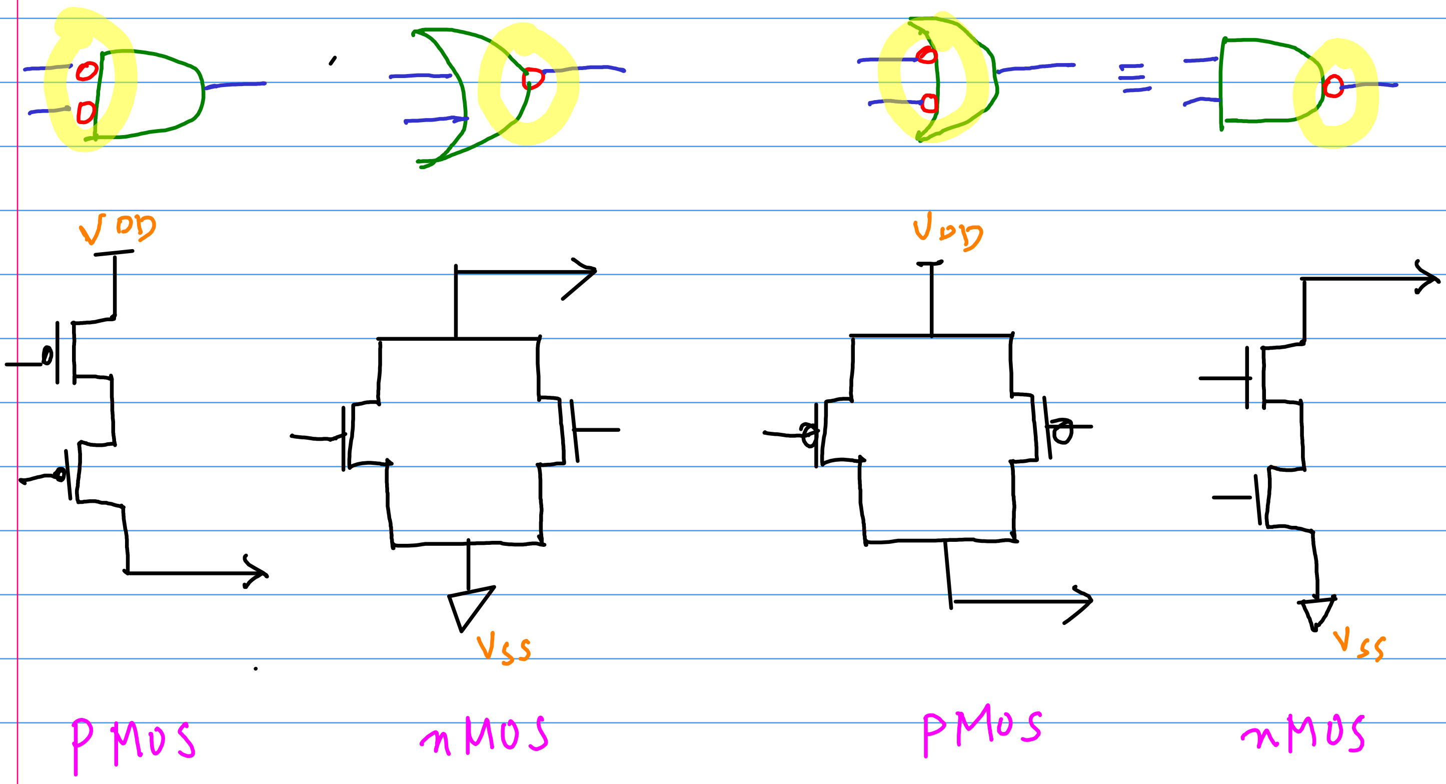
$\overline{A} \cdot \overline{B} = \overline{(A + B)}$

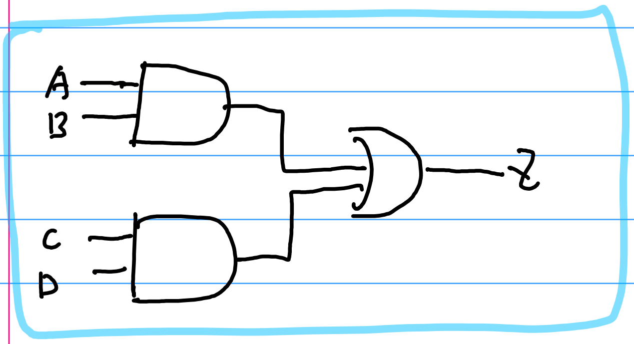
$(\overline{A} + \overline{B}) = \overline{A \cdot B}$



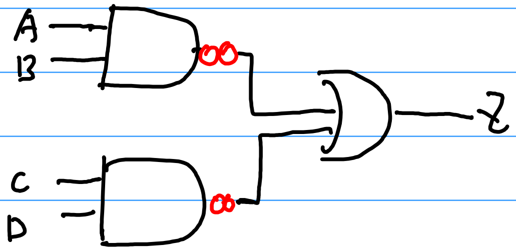


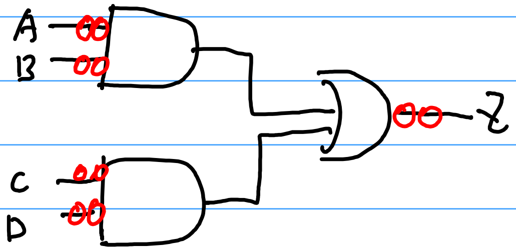
Basic nMOS, pMOS Configurations

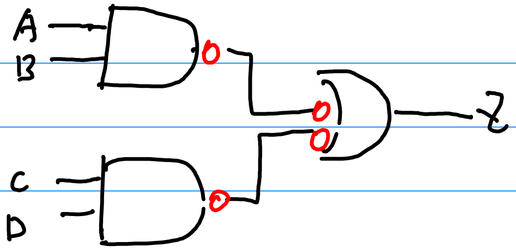


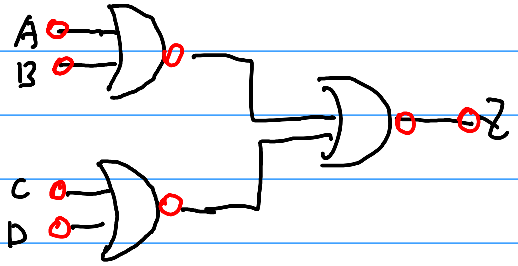


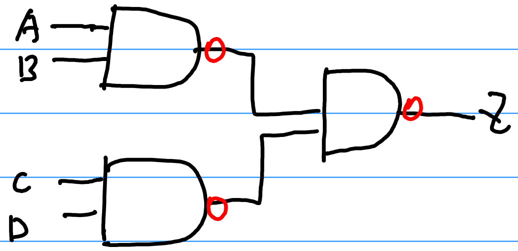
Sum of Product (SOP)

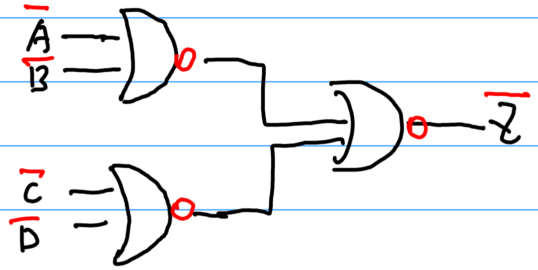




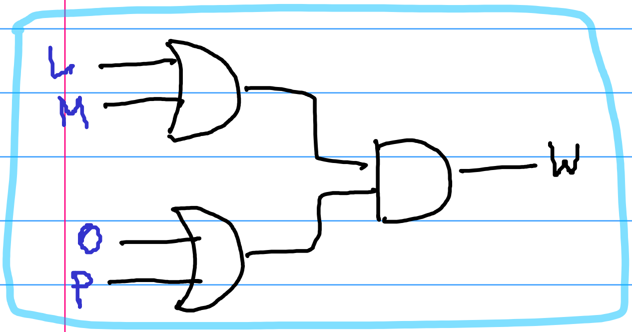


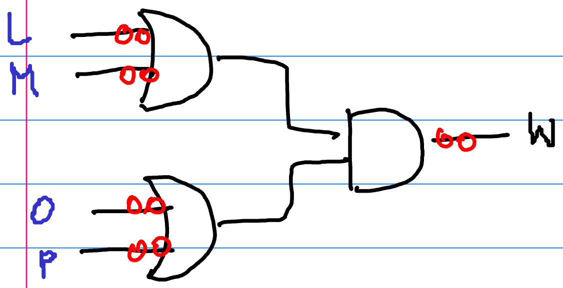


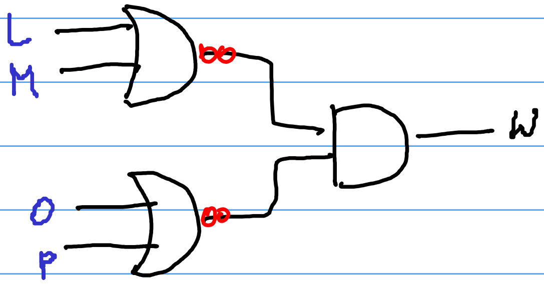


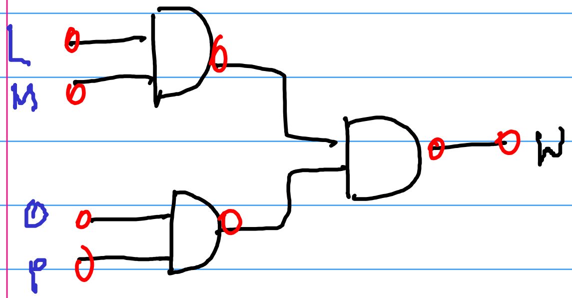


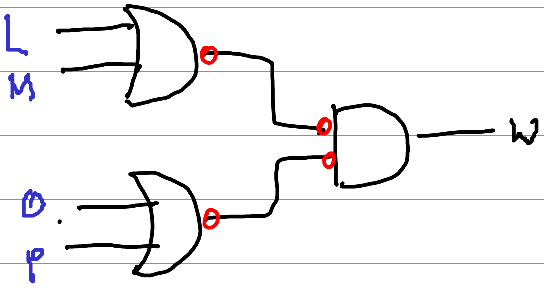
Product of Sum (POS)

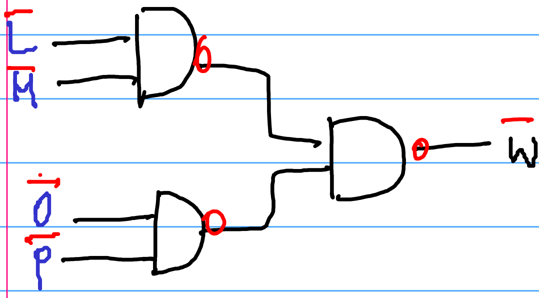


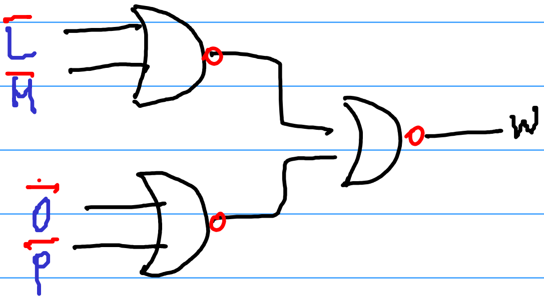


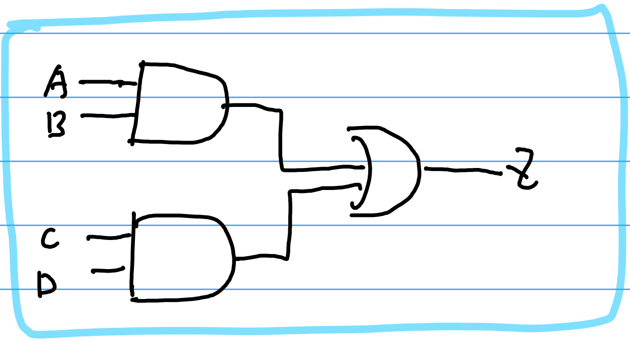




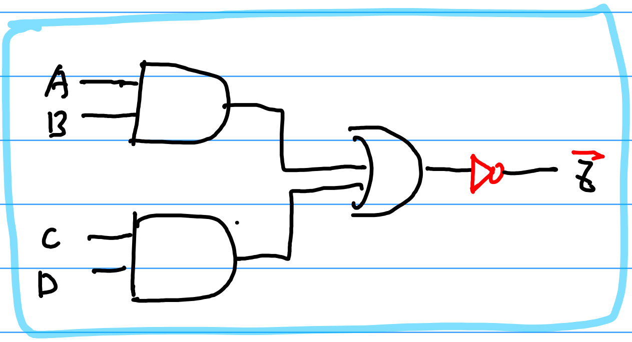




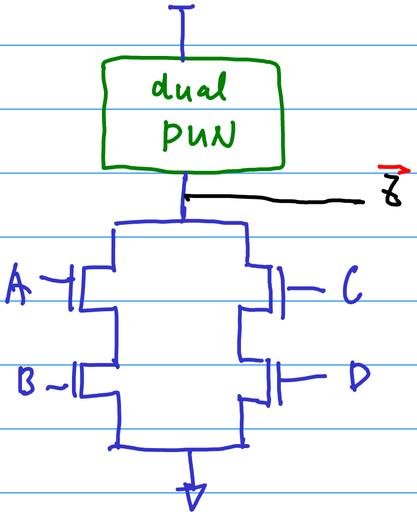




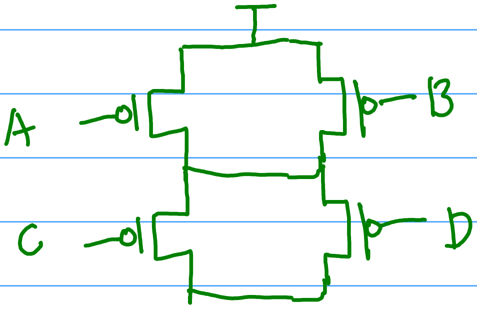
Sum of Product (SOP)



AND - OR - INV (AOI)

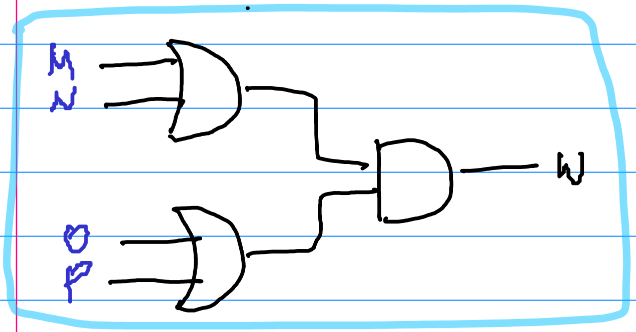


$$\overline{(A \cdot B + C \cdot D)}$$

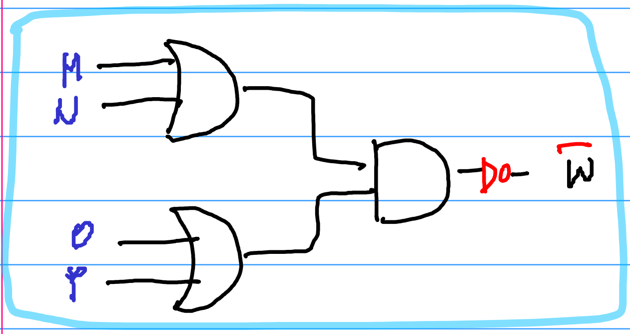


$$\overline{A \cdot B} \cdot \overline{C \cdot D}$$

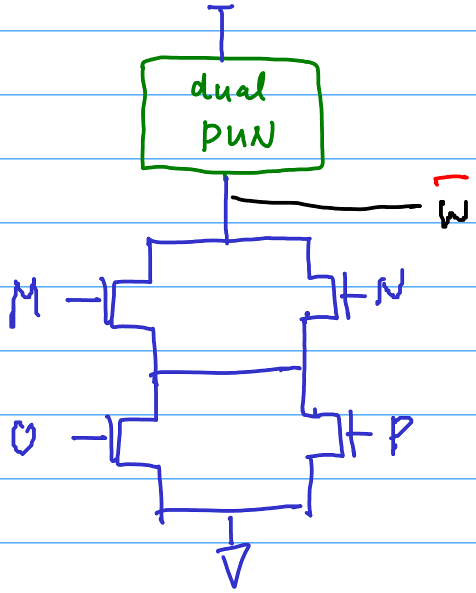
$$= (\bar{A} + \bar{B}) \cdot (\bar{C} + \bar{D})$$



Product of Sum (POS)



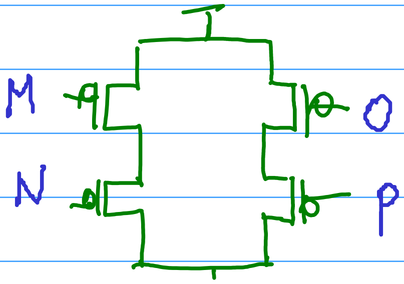
OR - AND - INV (OAI)



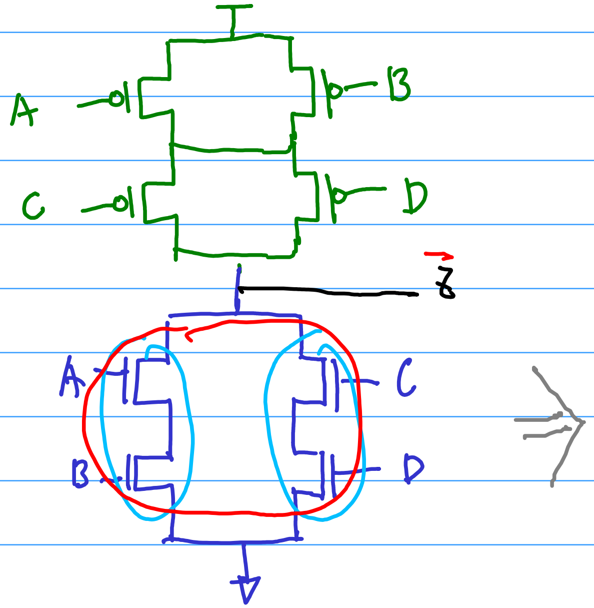
$$\overline{(M+N)(O+P)}$$

$$\overline{(M+N)}+\overline{(O+P)}$$

$$=(\overline{M}\cdot\overline{N})+(\overline{O}\cdot\overline{P})$$

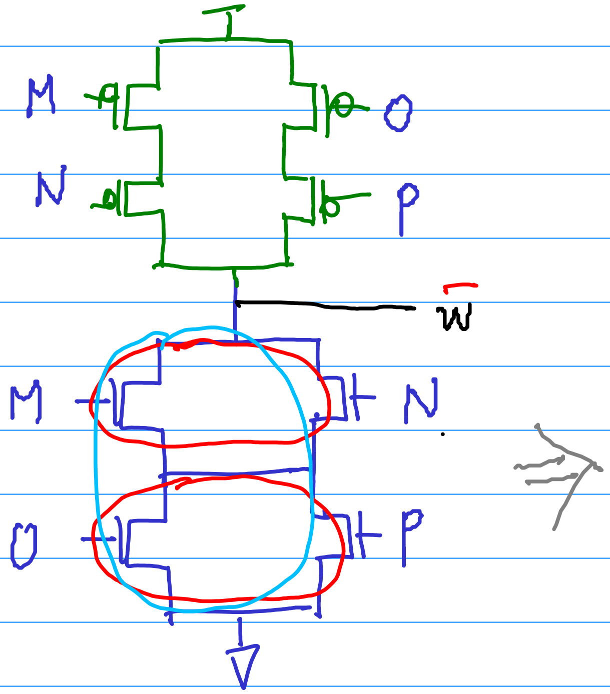


AOI , OAI



$$\overline{(A \cdot B + C \cdot D)}$$

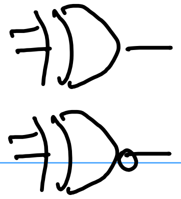
AND - OR - INV



$$\overline{(M + N) \cdot (O + P)}$$

OR - AND - INV

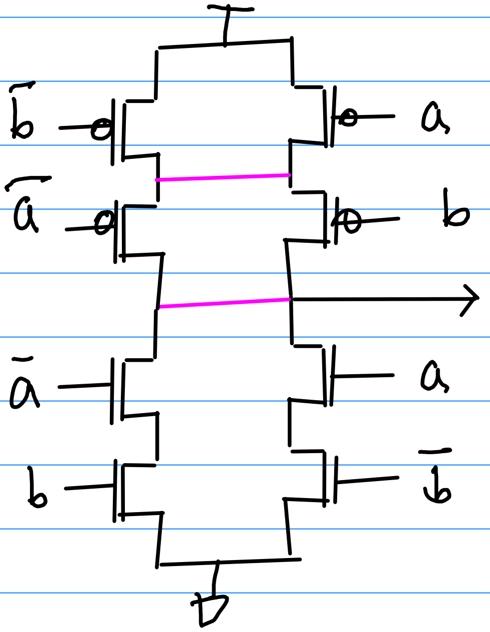
XNOR, XOR using AOI



$a \oplus b = \bar{a} \cdot b + a\bar{b}$

$\overline{a \oplus b} = ab + \bar{a}\bar{b}$

XNOR
(AOI)



$\overline{a \oplus b}$

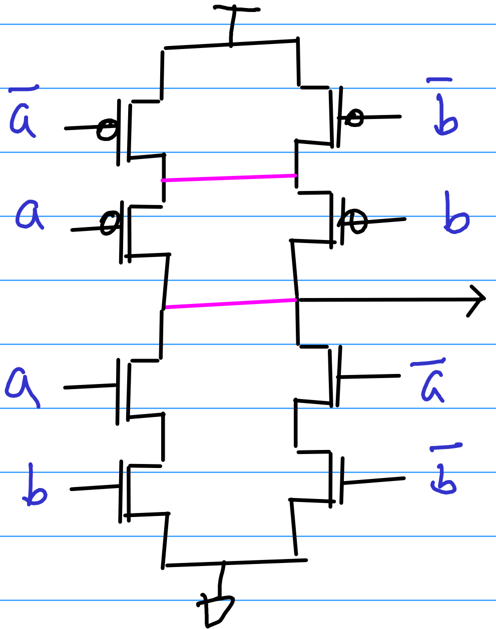
$(a+\bar{b}) \cdot (\bar{a}+b)$

$= 0 + \bar{a}\bar{b} + ab + 0$

$= ab + \bar{a}\bar{b} = \overline{a \oplus b}$

$\bar{a}b + a\bar{b} = a \oplus b$

XOR
(AOI)



$(\bar{a}+\bar{b}) \cdot (a+b) = a\bar{b} + \bar{a}b$

$= a \oplus b$

$\overline{\overline{a \oplus b}}$

$ab + \bar{a}\bar{b} = \overline{a \oplus b}$

(AOI)

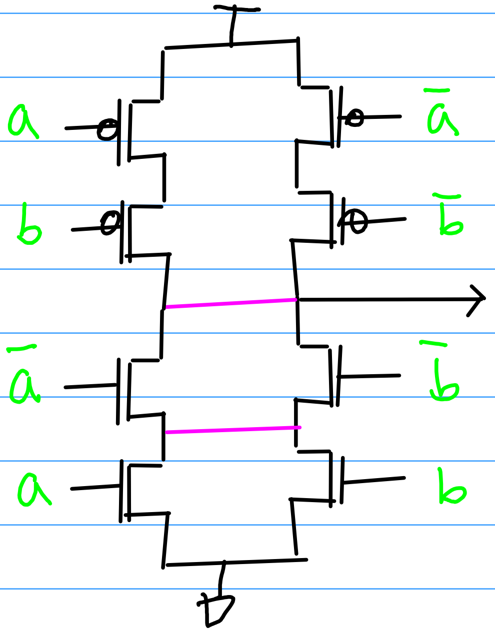
XNOR, XOR using OAI

$$a \oplus b = \bar{a}\cdot b + a\bar{b}$$

$$\overline{a \oplus b} = ab + \bar{a}\bar{b}$$

XNOR

(OAI)



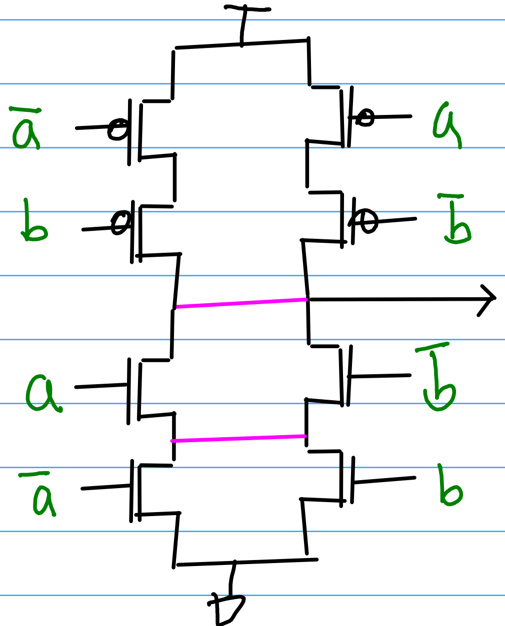
$$ab + \bar{a}\bar{b} = \overline{a \oplus b}$$

$$(\bar{a}+\bar{b})\cdot(a+b)$$

$$= a\bar{b} + \bar{a}b = a \oplus b$$

XOR

(OAI)



$$\bar{a}b + a\bar{b} = a \oplus b$$

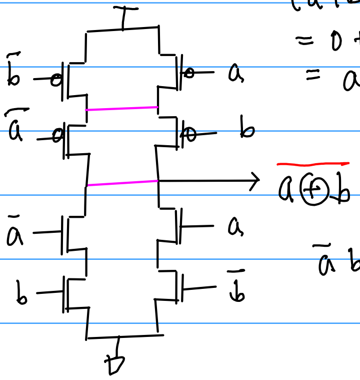
$$(a+\bar{b})\cdot(\bar{a}+b)$$

$$= ab + \bar{a}\bar{b} = \overline{a \oplus b}$$

(AOI)

* Which one is better?

XNOR (AOI)



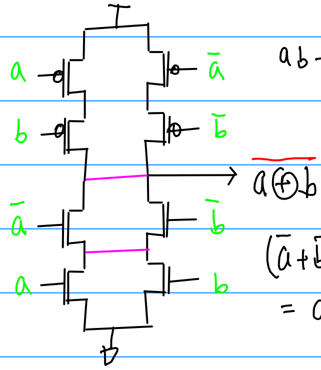
$(a+\bar b)\cdot(\bar a+b)$

$= 0 + \bar a \bar b + ab + 0$

$= ab + \bar a \bar b = \overline{a \oplus b}$

$\bar a b + a \bar b = a \oplus b$

XNOR (OAI)

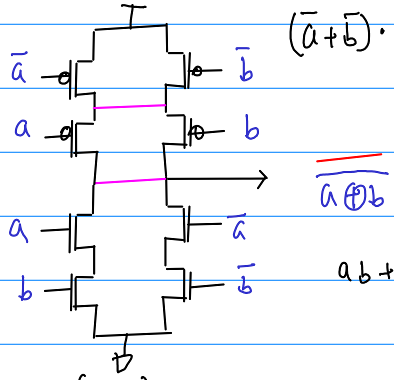


$ab + \bar a \bar b = \overline{a \oplus b}$

$(\bar a+\bar b)\cdot(a+b)$

$= a\bar b + \bar a b = a \oplus b$

XOR (AOI)



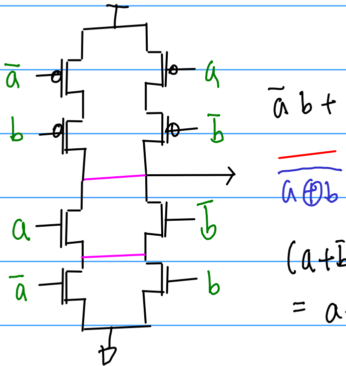
(AOI)

$(\bar a+\bar b)\cdot(a+b) = a\bar b + \bar a b$

$= a \oplus b$

$ab + \bar a \bar b = \overline{a \oplus b}$

XOR (OAI)



(AOI)

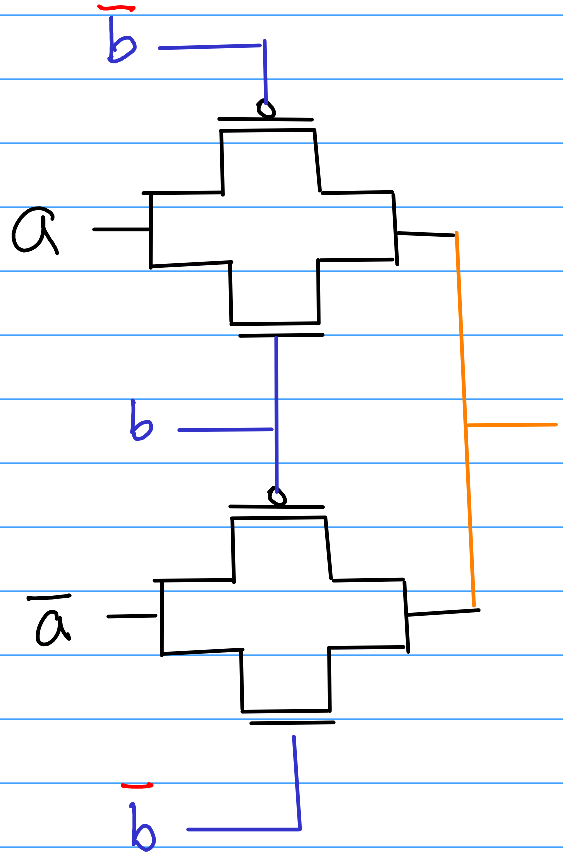
$\bar a b + a \bar b = a \oplus b$

$(a+\bar b)\cdot(\bar a+b)$

$= ab + \bar a \bar b = \overline{a \oplus b}$

XNOR, XOR using PG

XNOR
(PG)

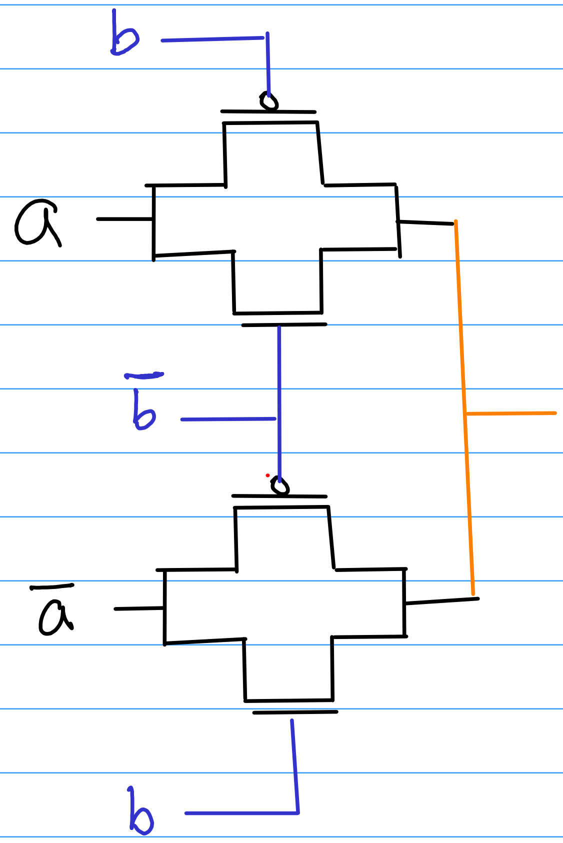


pass a if b

$ab + \bar{a}\bar{b} = \overline{a \oplus b}$

pass $\bar{a}$ if $\bar{b}$

XOR
(PG)



pass a if $\bar{b}$

$a\bar{b} + \bar{a}b = a \oplus b$

pass $\bar{a}$ if b

minterm, Maxterm

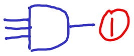
Boolean Function with minterms (1)

		x	y	z	F
	0	0	0	0	0
➡	1	0	0	1	1
	2	0	1	0	0
➡	3	0	1	1	1
➡	4	1	0	0	1
	5	1	0	1	0
	6	1	1	0	0
	7	1	1	1	0

index

inputs output

All possible combination of inputs



The output F becomes 1,
for one of the three following cases

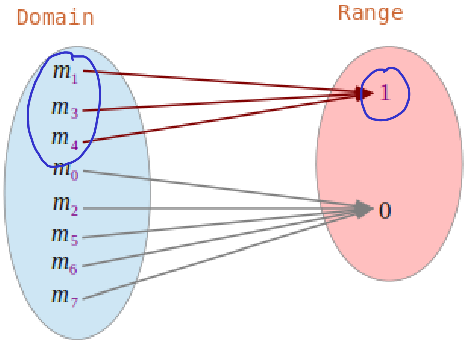
(the case when x=0 and y=0 and z=1) ⟺ $m_1 = \bar{x}\bar{y}z = 1$

or (the case when x=0 and y=1 and z=1) ⟺ $m_3 = \bar{x}yz = 1$

or (the case when x=1 and y=0 and z=0) ⟺ $m_4 = x\bar{y}\bar{z} = 1$

input conditions that make Fi = 1

function value



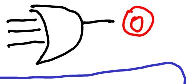
Boolean Function with Maxterms (1)

		x	y	z	F
➡	0	0	0	0	0
	1	0	0	1	1
➡	2	0	1	0	0
	3	0	1	1	1
	4	1	0	0	1
➡	5	1	0	1	0
➡	6	1	1	0	0
➡	7	1	1	1	0

index

inputs output

All possible combination of inputs



The output F becomes 0,
for one of the five following cases

(the case when x=0 and y=0 and z=0) ⟺ $M_0 = x+y+z = 0$

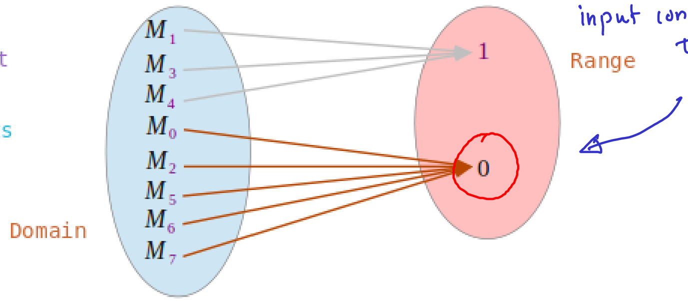
or (the case when x=0 and y=1 and z=0) ⟺ $M_2 = x+\bar{y}+z = 0$

or (the case when x=1 and y=0 and z=1) ⟺ $M_5 = \bar{x}+y+\bar{z} = 0$

or (the case when x=1 and y=1 and z=0) ⟺ $M_6 = \bar{x}+\bar{y}+z = 0$

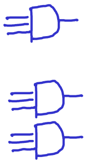
or (the case when x=1 and y=1 and z=1) ⟺ $M_7 = \bar{x}+\bar{y}+\bar{z} = 0$

input conditions that make F = ⓪



AOI, OAI using minterm, Maxterm

Complimentary Relations



	x	y	z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

index

inputs output

All possible combination of inputs

$$m_i = \overline{M_i}$$

$$M_i = \overline{m_i}$$

$$F(x,y,z) = m_1 + m_3 + m_4$$

The output F becomes 1,

either $m_1=1$ or $m_3=1$ or $m_4=1$

For the output of an **or** gate to be 1, at least one must be 1

$$\bar F(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$$

→ nMOS

AOI

$$\Leftrightarrow F(x,y,z) = \overline{m_0 + m_2 + m_5 + m_6 + m_7}$$

$$= \overline{m_0}\cdot\overline{m_2}\cdot\overline{m_5}\cdot\overline{m_6}\cdot\overline{m_7}$$

→ pMOS

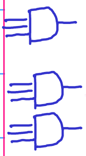
$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

For the output of an **and** gate to be 0, at least one input must be 0

Boolean Function Summary



	x	y	z	F
0	0	0	0	
1	0	0	1	1
2	0	1	0	
3	0	1	1	1
4	1	0	0	1
5	1	0	1	
6	1	1	0	
7	1	1	1	

The output F becomes 1,

for the cases

1) when $m_1=1$ or $m_3=1$ or $m_4=1$

$F(x,y,z) = m_1 + m_3 + m_4$ ➡ $F=1$

pMOS

2) when $M_1=0$ or $M_3=0$ or $M_4=0$

$\bar F(x,y,z) = M_1 \cdot M_3 \cdot M_4$ ➡ $F=1$ $(\bar F=0)$

nMOS OAI

	x	y	z	F
0	0	0	0	0
1	0	0	1	
2	0	1	0	0
3	0	1	1	
4	1	0	0	
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

The output F becomes 0,

for the cases

1) when $m_0=1$ or $m_2=1$ or $m_5=1$ or $m_6=1$ or $m_7=1$

$\bar F(x,y,z) = m_0+m_2+m_5+m_6+m_7$ ➡ $F=0$ $(\bar F=1)$

nMOS AOI

2) when $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$ ➡ $F=0$

pMOS

$$\boxed{\begin{tikzcd} m_1 + m_3 + m_4 = F \arrow[r] & M_1 \cdot M_3 \cdot M_4 = \overline{F} \\ m_0 + m_2 + m_5 + m_6 + m_7 = \overline{F} \arrow[ru] & M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = F \end{tikzcd}}$$

F

$\boxed{m_1 + m_3 + m_4 = 1} = F$

$(m_1=1) \vee (m_3=1) \vee (m_4=1) \Leftrightarrow F=1$

$\boxed{M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0} = F$

$(M_0=0) \wedge (M_2=0) \wedge (M_5=0) \wedge$
$(M_6=0) \wedge (M_7=0) \Leftrightarrow F=0$

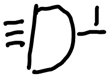
$\overline{F}$

$\boxed{m_0 + m_2 + m_5 + m_6 + m_7 = 1} = \overline{F}$

$(m_0=1) \vee (m_2=1) \vee (m_5=1) \vee$
$(m_6=1) \vee (m_7=1) \Leftrightarrow F=0$

$\boxed{M_1 \cdot M_3 \cdot M_4 = 0} = \overline{F}$

$(M_1=0) \wedge (M_3=0) \wedge (M_4=0)$
$\Leftrightarrow F=1$



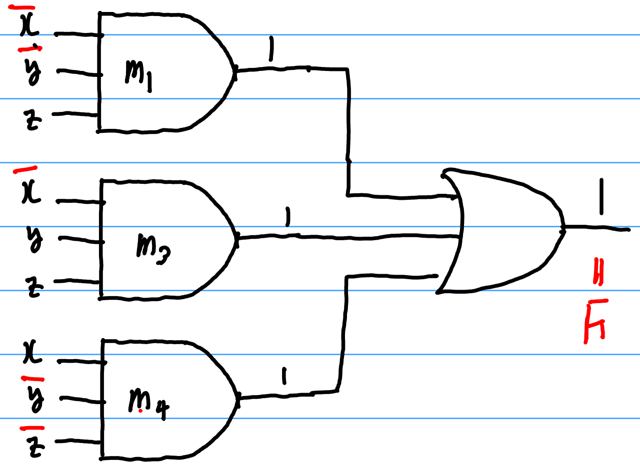
→ $m_1 + m_3 + m_4 = F$ $\qquad$ $M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = F$

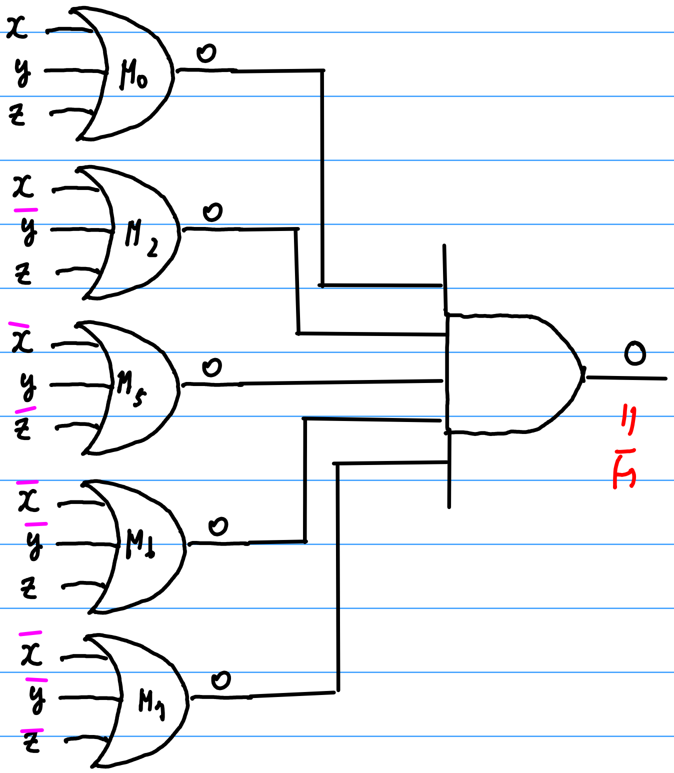
$m_0 + m_2 + m_5 + m_6 + m_7 = \overline{F}$ $\qquad$ $M_1 \cdot M_3 \cdot M_4 = \overline{F}$

Ⓕ $\boxed{m_1 + m_3 + m_4 = 1} = F$ $\qquad$ $\boxed{M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0} = F$

$(m_1 = 1) \vee (m_3 = 1) \vee (m_4 = 1) \Leftrightarrow F = 1$

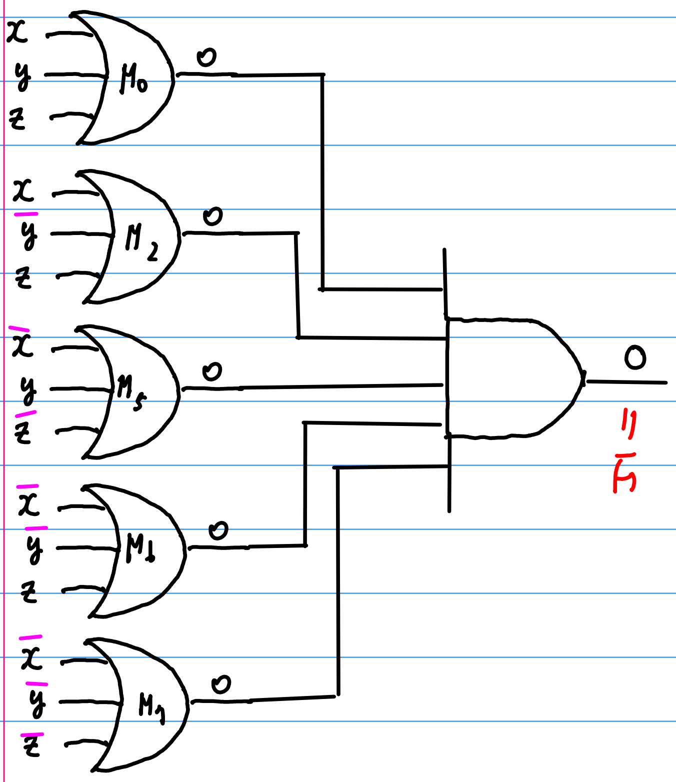
$(M_0 = 0) \wedge (M_2 = 0) \wedge (M_5 = 0) \wedge$
$(M_6 = 0) \wedge (M_7 = 0) \Leftrightarrow F = 0$

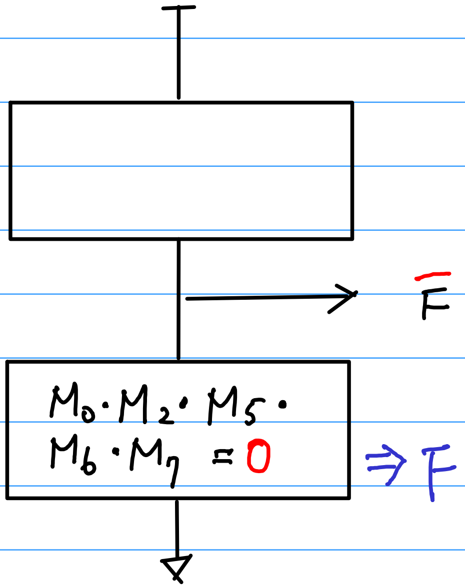




Ⓕ

	x	y	z	$x+y+z$	$x+\bar{y}+z$	$\bar{x}+y+\bar{z}$	$\bar{x}+\bar{y}+z$	$\bar{x}+\bar{y}+\bar{z}$
⓪	0	0	0	0	1	1	1	1
1	0	0	1	1	1	1	1	1
②	0	1	0	1	0	1	1	1
3	0	1	1	1	1	1	1	1
4	1	0	0	1	1	1	1	1
⑤	1	0	1	1	1	0	1	1
⑥	1	1	0	1	1	1	0	1
⑦	1	1	1	1	1	1	1	0





$m_1 + m_3 + m_4 = F$ $\quad$ $M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = F$

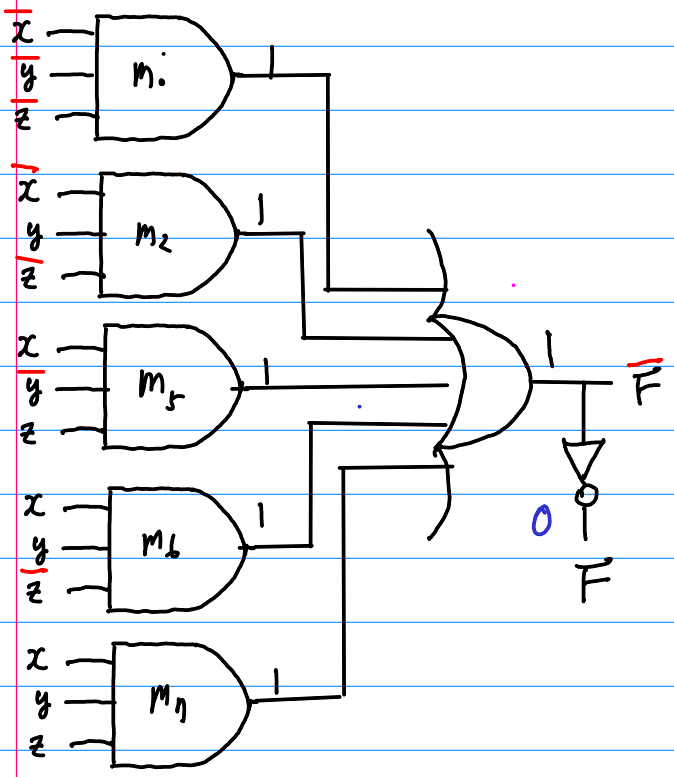
→ $m_0 + m_2 + m_5 + m_6 + m_7 = \overline{F}$ $\quad$ $M_1 \cdot M_3 \cdot M_4 = \overline{F}$

$\overline{F}$

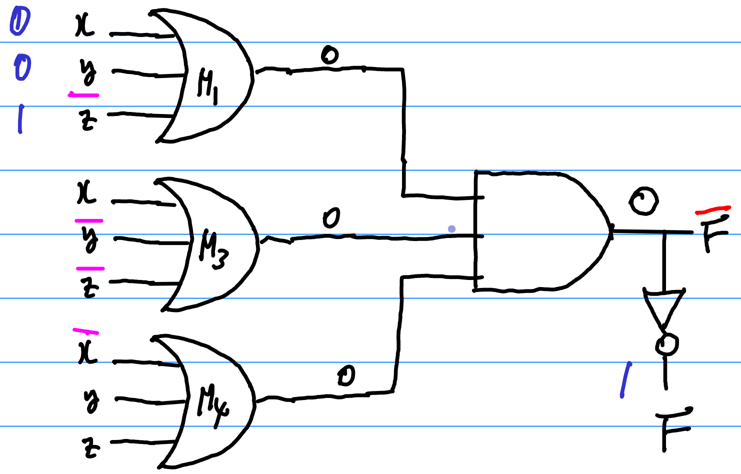
$\boxed{m_0 + m_2 + m_5 + m_6 + m_7 = 1} = \overline{F}$ $\quad$ $\boxed{M_1 \cdot M_3 \cdot M_4 = 0} = \overline{F}$

$(m_0=1) \vee (m_2=1) \vee (m_5=1) \vee (m_6=1) \vee (m_7=1) \Leftrightarrow F=0$

$(M_1=0) \wedge (M_3=0) \wedge (M_4=0) \Leftrightarrow F=1$



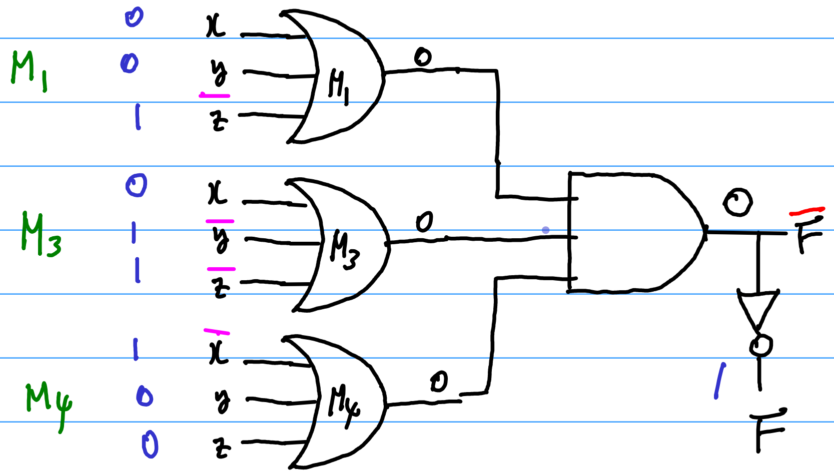
AOI



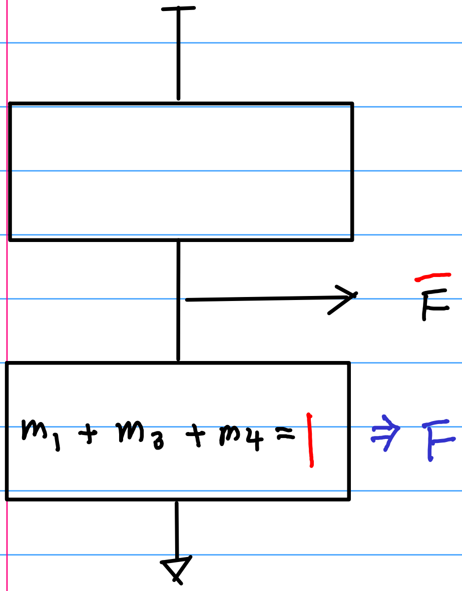
OAI

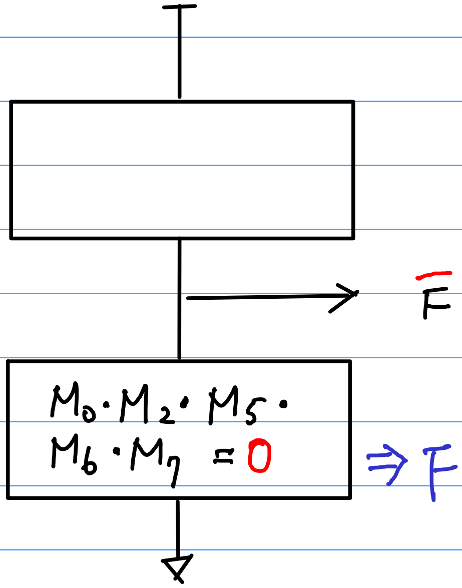
$\overline{F}$

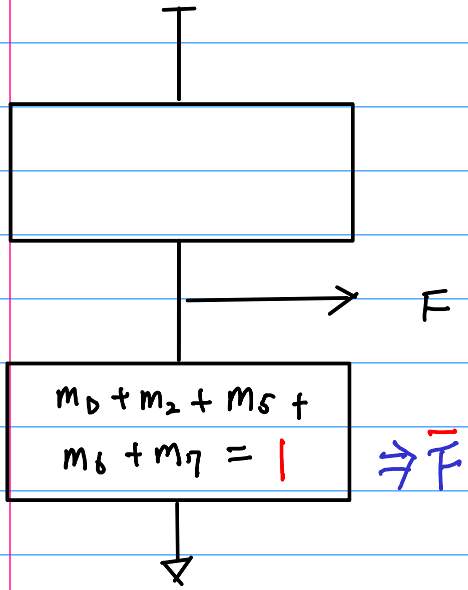
	x	y	z	$x+y+\bar{z}$	$x+\bar{y}+\bar{z}$	$\bar{x}+y+z$
0	0	0	0	1	1	1
①	0	0	1	0	1	1
2	0	1	0	1	1	1
③	0	1	1	1	0	1
④	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	1	1
7	1	1	1	1	1	1

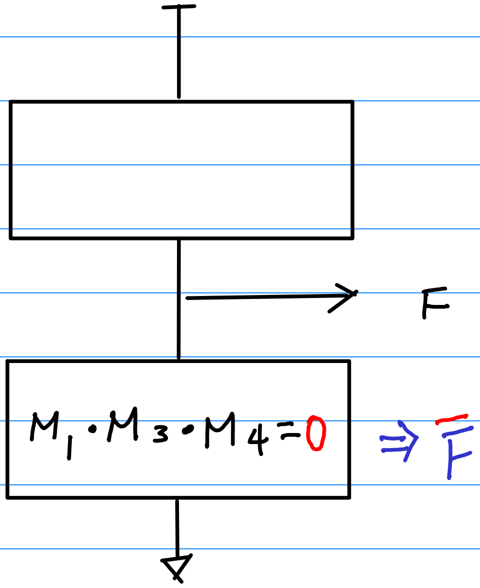


$$\begin{tikzcd}
m_1 + m_3 + m_4 = F \arrow[dr] & M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = F \\
m_0 + m_2 + m_5 + m_6 + m_7 = \overline{F} \arrow[ur] & M_1 \cdot M_3 \cdot M_4 = \overline{F}
\end{tikzcd}$$



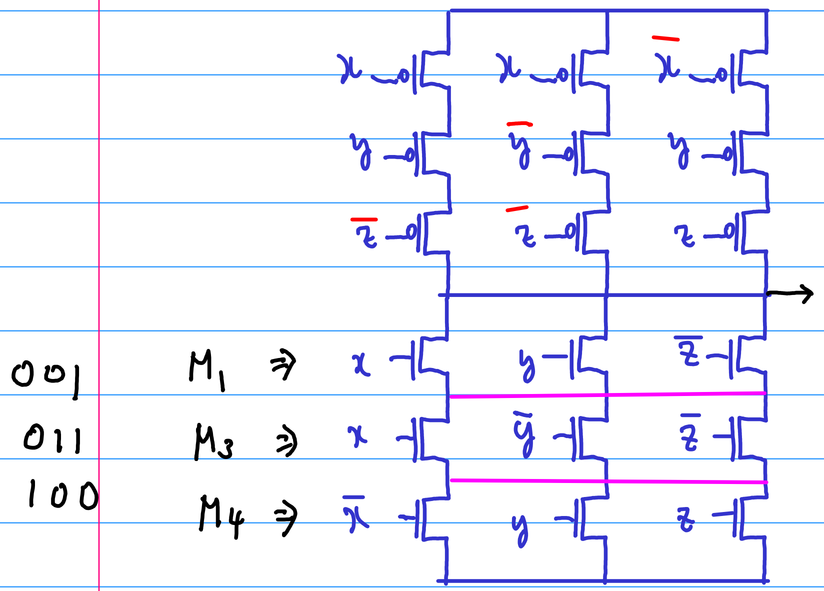


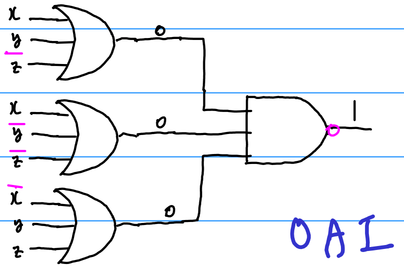




$F(x,y,z) = m_1 + m_3 + m_4$

$= \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z}$





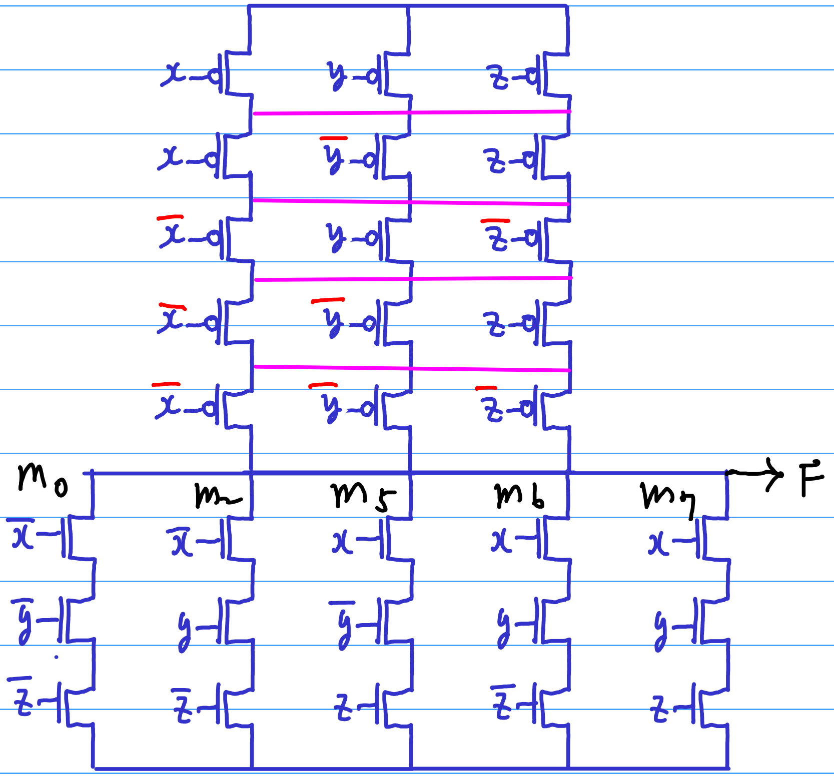
$\bar{F}(x,y,z) = M_1 \cdot M_3 \cdot M_4$

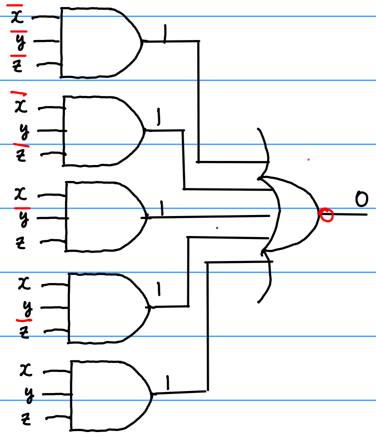
$(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + y + z)$

smaller

$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$

$= (x+y+z)(x+\bar{y}+z)(\bar{x}+y+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$



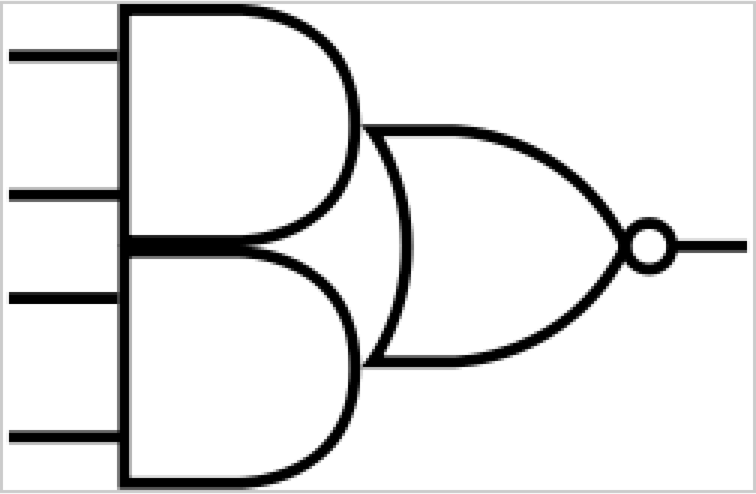


AOI

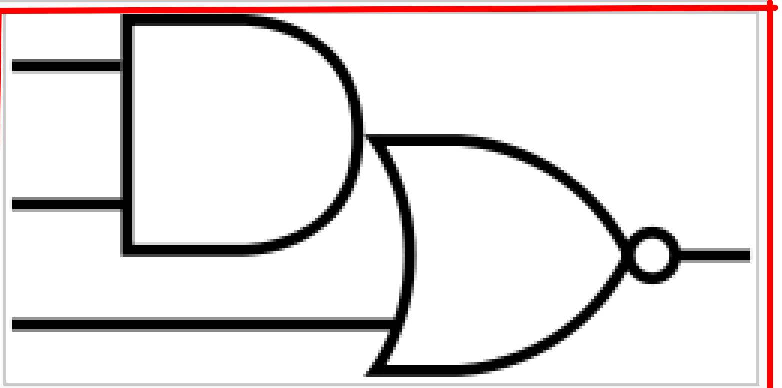
$\bar{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$

$= \bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}z + xy\bar{z} + xyz$

* depend on the number of '1's in the truth table.

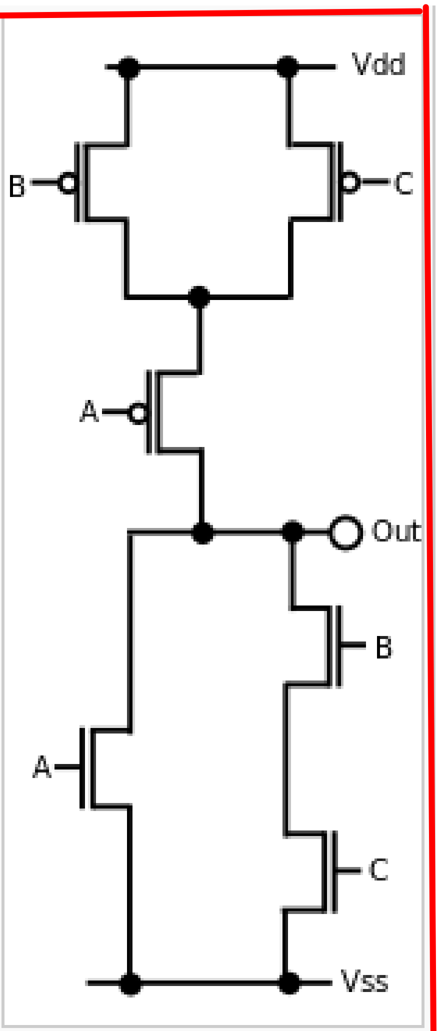


2-2 AOI Symbol



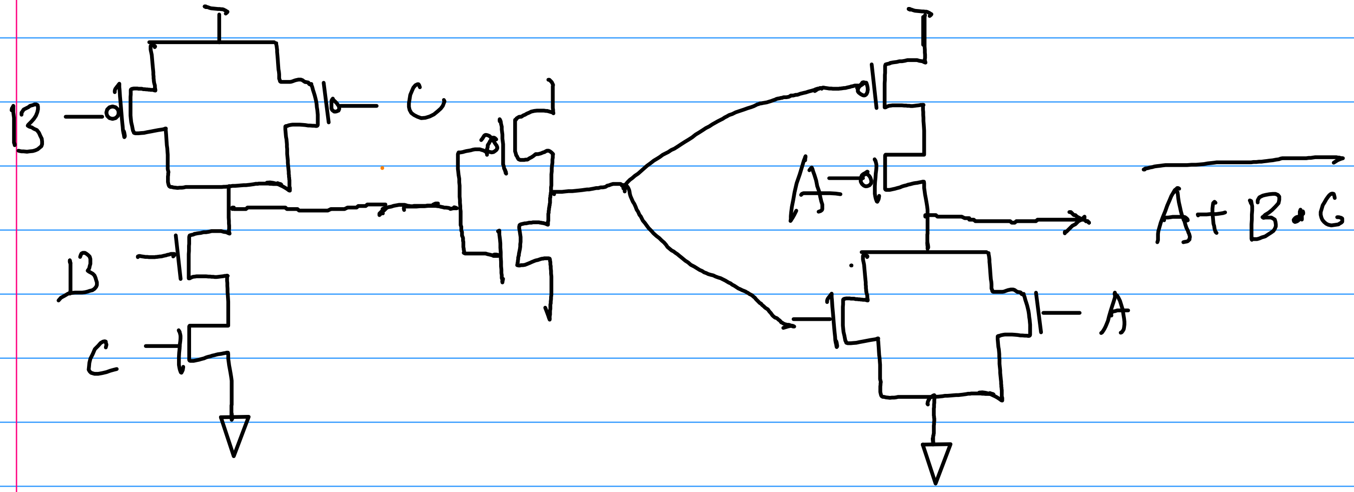
2-1 AOI Symbol

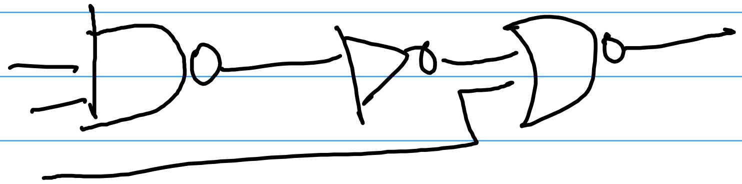
⇒



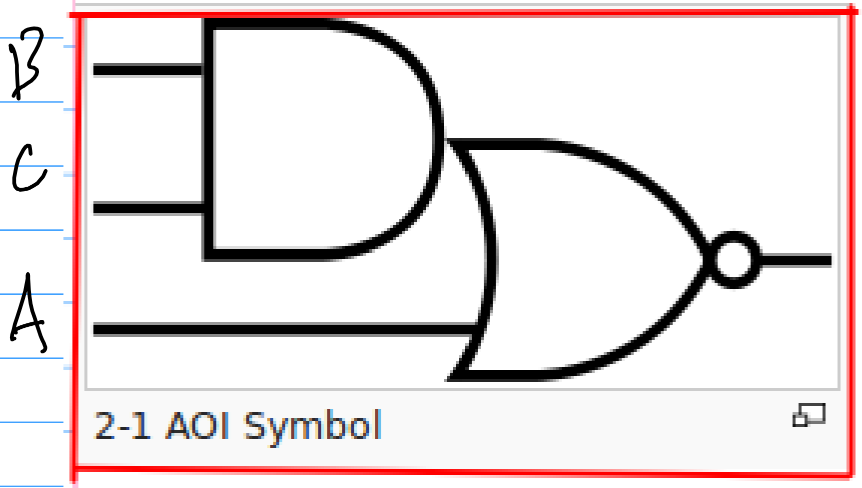
CMOS implementation of a 2-1 AOI gate

$4 + 2 + 4 = \boxed{8}$





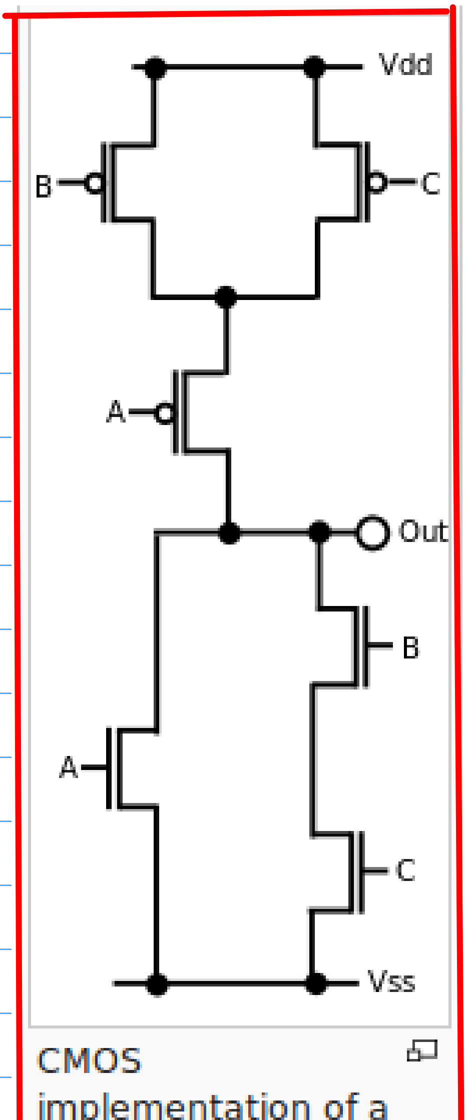
$\boxed{6}$



2-1 AOI Symbol

↑

also included in a standard cell library



CMOS implementation of a