CORDIC Background (4A)

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CORDIC Background

J. S. Walther, A Unified Algorithm for Elementary Functions

Unified CORDIC Iteration Eq

$$x_{i+1} = x_i - \boldsymbol{m} \cdot y_i \cdot d_i \cdot 2^{-i}$$
$$y_{i+1} = y_i + x_i \cdot d_i \cdot 2^{-i}$$
$$z_{i+1} = z_i - d_i \cdot \boldsymbol{e}_i$$

$$\boldsymbol{m} = 1 \Rightarrow \boldsymbol{e}_i = \tan^{-1}(2^{-i})$$
$$\boldsymbol{m} = 0 \Rightarrow \boldsymbol{e}_i = (2^{-i})$$
$$\boldsymbol{m} = -1 \Rightarrow \boldsymbol{e}_i = \mathbf{tanh}^{-1}(2^{-i})$$

$$\mathbf{tan}\,\alpha = \frac{\mathbf{sin}\,\alpha}{\mathbf{cos}\,\alpha}$$

$$\mathbf{tanh}\,\alpha = \frac{\mathbf{sinh}\,\alpha}{\mathbf{cosh}\,\alpha}$$

$$\mathbf{exp}\,\alpha = \mathbf{sinh}\,\alpha + \mathbf{cosh}\,\alpha$$

$$\mathbf{ln}\,\alpha = 2\,\mathbf{tanh}^{-1}(y/x)$$
$$x = \alpha + 1$$
$$y = \alpha - 1$$

$$(\alpha)^{1/2} = (x^2 - y^2)^{1/2}$$
$$x = \alpha + 1/4$$
$$y = \alpha - 1/4$$

Unified CORDIC Iteration Eq

$$x_{i+1} = x_i - \boldsymbol{m} \cdot y_i \cdot d_i \cdot 2^{-i}$$
$$y_{i+1} = y_i + x_i \cdot d_i \cdot 2^{-i}$$
$$z_{i+1} = z_i - d_i \cdot \boldsymbol{e}_i$$

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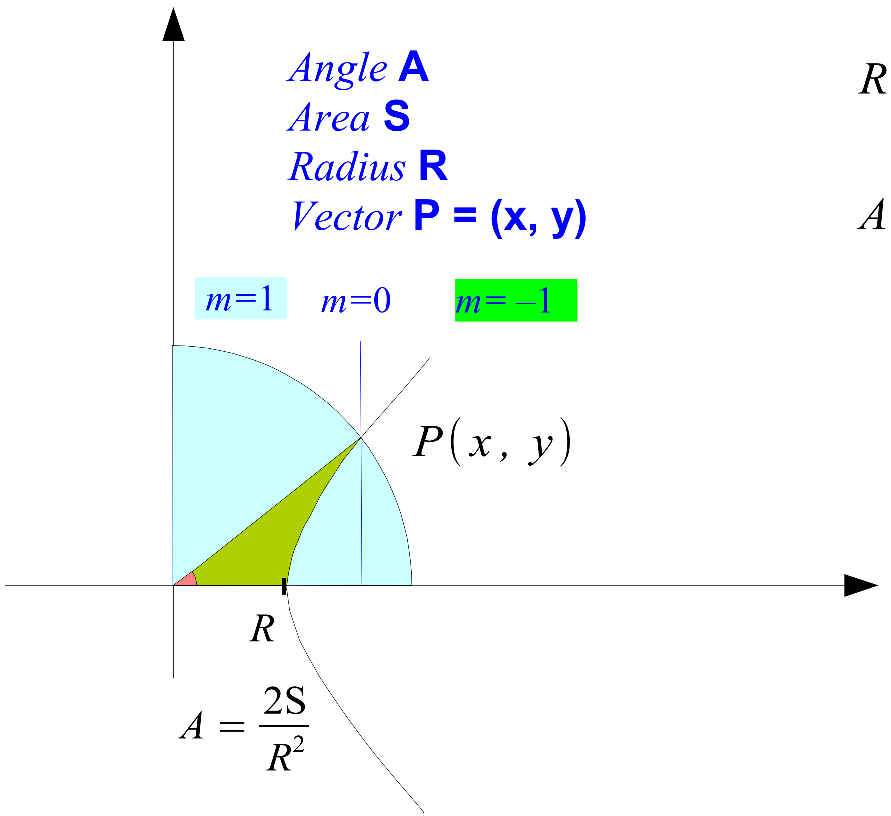
$$\boldsymbol{m} = -1 \Rightarrow \boldsymbol{e}_i = \mathbf{tanh}^{-1}(2^{-i})$$

$$\mathbf{cosh}\, i\, x = \frac{1}{2}(e^{ix} + e^{-ix}) = \cos x$$
$$\mathbf{sinh}\, i\, x = \frac{1}{2}(e^{ix} - e^{-ix}) = i\, \sin x$$
$$\mathbf{tanh}\, i\, x = \frac{(e^{ix} + e^{-ix})}{(e^{ix} - e^{-ix})} = i\, \tan x$$

$$\mathbf{cosh}\, x = \frac{1}{2}(e^{x} + e^{-x}) = \cos i\, x$$
$$\mathbf{sinh}\, x = \frac{1}{2}(e^{x} - e^{-x}) = -i\, \sin ix$$
$$\mathbf{tanh}\, x = \frac{(e^{x} + e^{-x})}{(e^{x} - e^{-x})} = -i\, \tan i\, x$$

2S = A (Area:S Angle:A)



$$R = (x^2 + y^2)^{1/2}$$

$$A = \tan^{-1}\left(\frac{y}{x}\right)$$

$$R = (x^2 - y^2)^{1/2}$$

$$A = -i \tan^{-1}\left(i \frac{y}{x}\right)$$

$$= \mathbf{tanh}^{-1}\left(\frac{y}{x}\right)$$

$$R = x$$

$$A =$$

Unified CORDIC Iteration Eq

$P = (x, y)$

$$R = \sqrt{x^2 + m\ y^2}$$

$$A = \frac{1}{\sqrt{m}} \tan^{-1}\left(\sqrt{m}\ \frac{y}{x}\right)$$

$P_{i+1} = (x_{i+1}, y_{i+1})$ ➡ $P_i = (x_i, y_i)$

$$x_{i+1} = x_i + m\ y_i\ \delta_i$$

$$y_{i+1} = y_i - x_i\ \delta_i$$

$$A_{i+1} = A_i - \alpha_i$$

$$R_{i+1} = R_i \cdot K_i$$

$$\alpha_i = \frac{1}{\sqrt{m}} \tan^{-1}\left(\sqrt{m}\ \delta_i\right)$$

$$K_i = \sqrt{1 + m\ \delta_i^2}$$

$$A_n = A_0 - \alpha$$

$$R_n = R_0 \cdot K$$

$$\alpha = \sum_{i=0}^{n-1} \alpha_i$$

$$K = \prod_{i=0}^{n-1} K_i$$

Unified CORDIC Iteration Eq

$$A_{i+1} = A_i - \alpha_i \qquad \alpha_i = \frac{1}{\sqrt{m}} \tan^{-1}\left(\sqrt{m}\,\delta_i\right)$$

$$R_{i+1} = R_i \cdot K_i \qquad K_i = \sqrt{1 + m\,\delta_i^2}$$

$$A_{i+1} = A_i - \frac{1}{\sqrt{m}} \tan^{-1}\left(\sqrt{m}\,\delta_i\right)$$

$$R_{i+1} = R_i \cdot \sqrt{1 + m\,\delta_i^2}$$

$$\boldsymbol{m} = +1$$

$$x_{i+1} = x_i + y_i\,\delta_i$$
$$y_{i+1} = y_i - x_i\,\delta_i$$

$$A_{i+1} = A_i - \tan^{-1}\left(\delta_i\right)$$

$$R_{i+1} = R_i \cdot \sqrt{1 + \delta_i^2}$$

$$\boldsymbol{m} = -1$$

$$x_{i+1} = x_i - y_i\,\delta_i$$
$$y_{i+1} = y_i - x_i\,\delta_i$$

$$A_{i+1} = A_i + \boldsymbol{i}\tan^{-1}\left(\boldsymbol{i}\,\delta_i\right)$$

$$R_{i+1} = R_i \cdot \sqrt{1 - \delta_i^2}$$

$$\boldsymbol{m} = 0$$

$$x_{i+1} = x_i$$
$$y_{i+1} = y_i - x_i\,\delta_i$$

$$A_{i+1} = A_i - \delta_i$$

$$R_{i+1} = R_i$$

Unified CORDIC Iteration Eq

$$\boldsymbol{m} = -1 \quad \Rightarrow \quad \boldsymbol{e}_i = \mathbf{tanh}^{-1}\left(2^{-i}\right)$$

$$\mathbf{cosh}\, i\, x = \frac{1}{2}\left(e^{ix} + e^{-ix}\right) = \cos x \qquad \mathbf{cosh}\, x = \frac{1}{2}\left(e^{x} + e^{-x}\right) = \cos i\, x$$

$$\mathbf{sinh}\, i\, x = \frac{1}{2}\left(e^{ix} - e^{-ix}\right) = i\ \sin x \qquad \mathbf{sinh}\, x = \frac{1}{2}\left(e^{x} - e^{-x}\right) = -i\ \sin ix$$

$$\mathbf{tanh}\, i\, x = \frac{\left(e^{ix} + e^{-ix}\right)}{\left(e^{ix} - e^{-ix}\right)} = i\ \tan x \qquad \mathbf{tanh}\, x = \frac{\left(e^{x} + e^{-x}\right)}{\left(e^{x} - e^{-x}\right)} = -i\ \tan i\, x$$

Unified CORDIC Iteration Eq

References

[1] http://en.wikipedia.org/
[2] CORDIC FAQ, www.dspguru.com
[3] R. Andraka, A survey of CORDIC algorithms for FPGA based computers
[4] J. S. Walther, A Unified Algorithm for Elementary Functions