General CORDIC Description (1A)

-
-

Young Won Lim
04/09/2012

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

CORDIC Background

1. CORDIC FAQ, G. R. Griffin, www.dspguru.com/info/faqs/cordic2.htm

Complex Multiplication

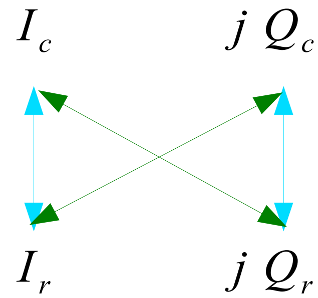
Given Complex Value $\quad C = I_c + j\,Q_c$

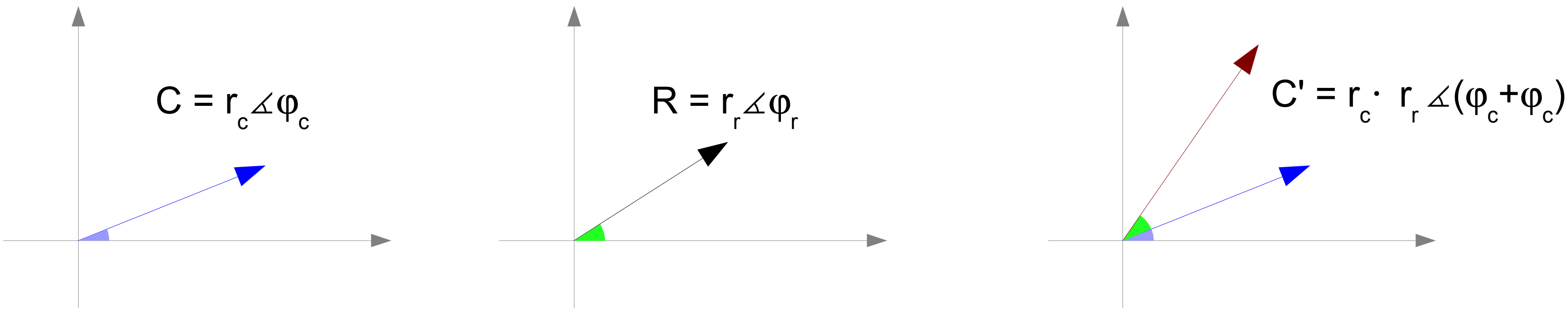
Rotation Value $\quad R = I_r + j\,Q_r$

Rotated Complex Value $\quad C' = I_c' + j\,Q_c'$

$$C' = C \cdot R$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c)\cdot(I_r + j\,Q_r) \\ &= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r) \end{aligned}$$

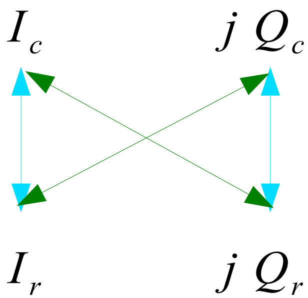




Adding / Subtracting Phase

To **add** R' phase to C

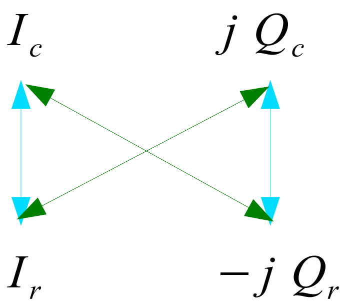
$$C' = C \cdot R$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (I_r + j\,Q_r)$$
$$= (I_c I_r - Q_c Q_r) + j\,(Q_c I_r + I_c Q_r)$$

To **sub** R' phase to C

$$C' = C \cdot R^*$$

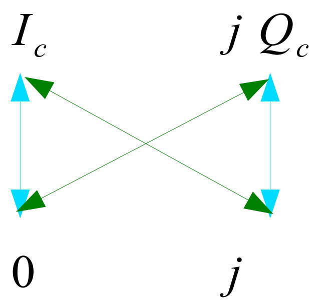


$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (I_r - j\,Q_r)$$
$$= (I_c I_r + Q_c Q_r) + j\,(Q_c I_r - I_c Q_r)$$

Adding / Subtracting 90 Degrees

To add R' phase to C

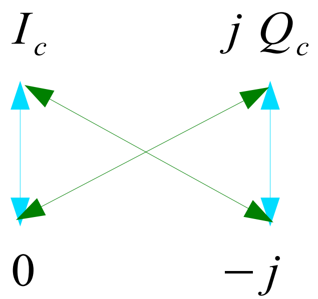
$$C' = C \cdot (+j)$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 + j)$$
$$= (-Q_c) + j\,(I_c)$$

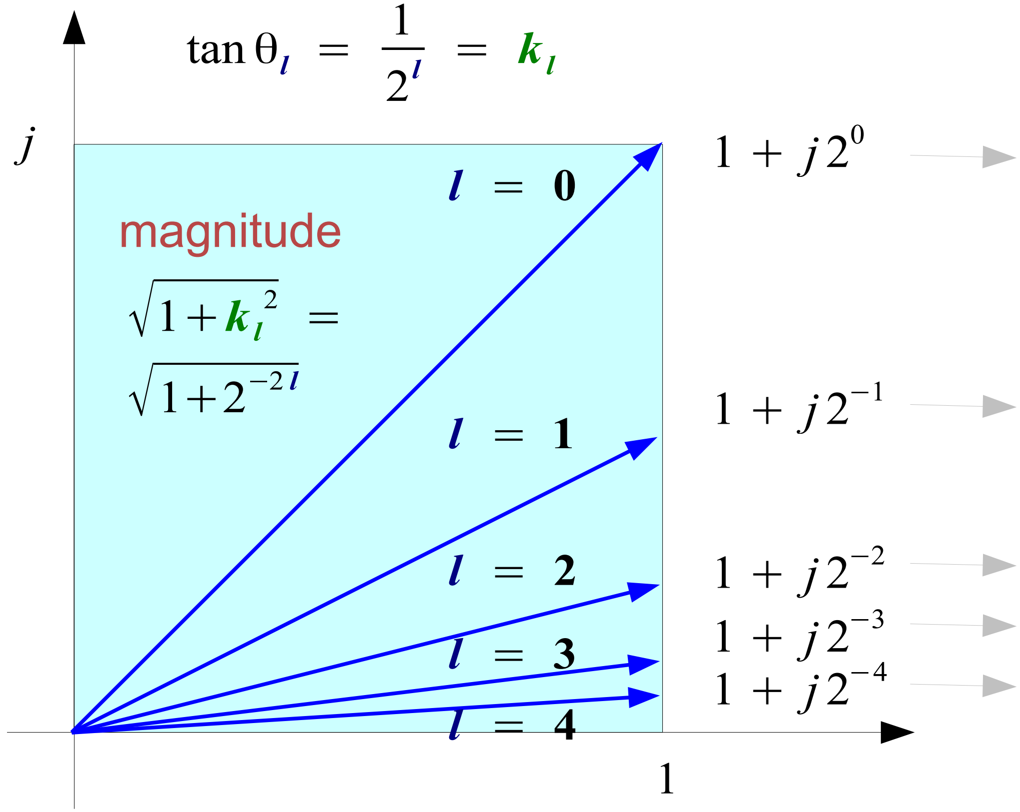
To sub R' phase to C

$$C' = C \cdot (-j)$$



$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (0 - j)$$
$$= (Q_c) + j\,(-I_c)$$

Elementary Angle: $\tan^{-1}(k_l)$



$$\theta_l = \tan^{-1}(2^{-l}) = \tan^{-1}(k_l)$$

$$\theta_0 = \tan^{-1}(2^0) = 45.00000^\circ$$

$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505^\circ$$

$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624^\circ$$

$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502^\circ$$

$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633^\circ$$

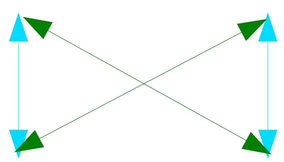
Represent arbitrary angle θ

in terms of $\pm\theta_0, \pm\theta_1, \pm\theta_2, \pm\theta_3, \cdots, \pm\theta_l, \cdots$

$$\left(k_l = \tan\theta_l = \frac{1}{2^l}, \quad l = 0, 1, 2, \cdots\right)$$

Adding / Subtracting $\tan^{-1}(k_l)$

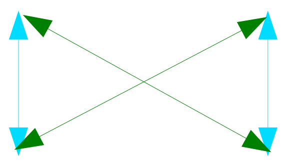
To add R' phase to C



$$C' = C \cdot (1 + j\,k_l)$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot \boxed{(1 + j\,k_l)} \\ &= (I_c - k_l Q_c) + j\,(Q_c + k_l I_c) \\ &= (I_c - 2^{-l} Q_c) + j\,(Q_c + 2^{-l} I_c) \end{aligned}$$

To sub R' phase to C



$$C' = C \cdot (1 - j\,k_l)$$

$$\begin{aligned} I_c' + j\,Q_c' &= (I_c + j\,Q_c) \cdot \boxed{(1 - j\,k_l)} \\ &= (I_c + k_l Q_c) + j\,(Q_c - k_l I_c) \\ &= (I_c + 2^{-l} Q_c) + j\,(Q_c - 2^{-l} I_c) \end{aligned}$$

$$\theta_l = \tan^{-1}(k_l) = \tan^{-1}(2^{-l})$$

$$k_l = \frac{1}{2^l}, \quad l = 0, 1, 2, \cdots$$

Phase and Magnitude of $1 + jk_l$ (1)

Cumulative Magnitude

L	$k = \frac{1}{2^l}$	$R = 1 + jk$	*Phase of R*	*Magnitude of R*	***CORDIC Gain***
0	1.0	1 + j1.0	45°	1.41421356	1.414213562
1	0.5	1 + j0.5	26.56505°	1.11803399	1.581138830
2	0.25	1 + j0.25	14.03624°	1.03077641	1.629800601
3	0.125	1 + j0.125	7.12502°	1.00778222	1.642484066
4	0.0625	1 + j0.0625	3.57633°	1.00195122	1.645688916
5	0.03125	1 + j0.03125	1.78991°	1.00048816	1.646492279
6	0.015625	1 + j0.015625	0.89517°	1.00012206	1.646693254
7	0.007813	1 + j0.007813	0.44761°	1.00003052	1.646743507
•••	•••	•••	•••	•••	•••
					1.647 ←

$R = 1 + jk_l \xrightarrow{k_l = 1/2^l \quad l = 0, 1, 2, \cdots}$

$$|R| = \sqrt{1^2 + k_l^2} = \sqrt{1 + 2^{-2l}} > 1.0$$

Phase and Magnitude of $1 + jk_l$ (2)

Represent arbitrary angle θ

in terms of $\pm\theta_0, \pm\theta_1, \pm\theta_2, \pm\theta_3, \cdots, \pm\theta_l, \cdots$ $\left(k_l = \tan\theta_l = \frac{1}{2^l}, \quad l = 0,1,2,\cdots\right)$

Binary Search ➡ Shift and Add ➡ No multiplier

	Phase of R
$\theta_0 = \tan^{-1}(2^0) =$	45°
$\theta_1 = \tan^{-1}(2^{-1}) =$	26.56505°
$\theta_2 = \tan^{-1}(2^{-2}) =$	14.03624°
$\theta_3 = \tan^{-1}(2^{-3}) =$	7.12502°
$\theta_4 = \tan^{-1}(2^{-4}) =$	3.57633°
$\theta_5 = \tan^{-1}(2^{-5}) =$	1.78991°
$\theta_6 = \tan^{-1}(2^{-6}) =$	0.89517°
$\theta_7 = \tan^{-1}(2^{-7}) =$	0.44761°
	•••

Each successive angle $\approx \frac{1}{2}$ of the previous.

First four angles: > 92°

$\theta \in$ [-180, +180]

$R = 0 \pm j$

$\theta \in$ [-90, +90]

$R = 1 \pm j\,k_l$

4 or more rotations

45° + 26.56505° + 14.03624° + 7.12502°
= 92.726° > 92°

Phase and Magnitude of 1 + jk_l (3)

Cumulative Magnitude

$R = 1 \pm j\,k_l$	*Magnitude of R* $\sqrt{1^2 + k_l^2}$ > 1.0	*CORDIC Gain*
$R_0 = 1 \pm j\,(1/2^0)$	$\sqrt{1^2 + 1^2}$ = 1.41421356	1.414213562
$R_1 = 1 \pm j\,(1/2^1)$	$\sqrt{1^2 + (1/2)^2}$ = 1.11803399	1.581138830
$R_2 = 1 \pm j\,(1/2^2)$	$\sqrt{1^2 + (1/2^2)^2}$ = 1.03077641	1.629800601
$R_3 = 1 \pm j\,(1/2^3)$	$\sqrt{1^2 + (1/2^3)^2}$ = 1.00778222	1.642484066
$R_4 = 1 \pm j\,(1/2^4)$	$\sqrt{1^2 + (1/2^4)^2}$ = 1.00195122	1.645688916
$R_5 = 1 \pm j\,(1/2^5)$	$\sqrt{1^2 + (1/2^5)^2}$ = 1.00048816	1.646492279
$R_6 = 1 \pm j\,(1/2^6)$	$\sqrt{1^2 + (1/2^6)^2}$ = 1.00012206	1.646693254
$R_7 = 1 \pm j\,(1/2^7)$	$\sqrt{1^2 + (1/2^7)^2}$ = 1.00003052	1.646743507
	...	... → 1.647

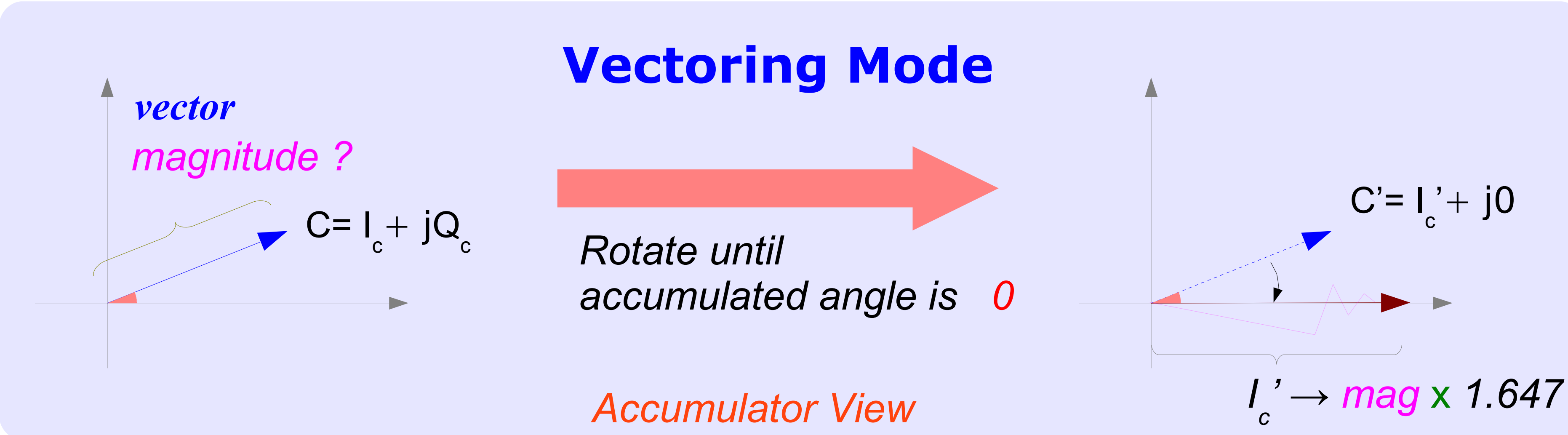
Iteration

The actual CORDIC Gain depends on the number of iterations

For each iteration, the corresponding θ_i is always used (either $+\theta_i$ or $-\theta_i$)

The magnitude is growing → for correction, multiply by 0.6073 = 1 / 1.647

Vector Mode – Calculating Magnitude



Each iteration, the magnitude is increased by $\sqrt{1^2 + k_i^2}$

CORDIC Gain (cumulative gain)

$\simeq 1.647 = 0.607^{-1}$

$I_c' \rightarrow$ *magnitude* x *1.647*

magnitude $\rightarrow I_c'$ x *0.607*

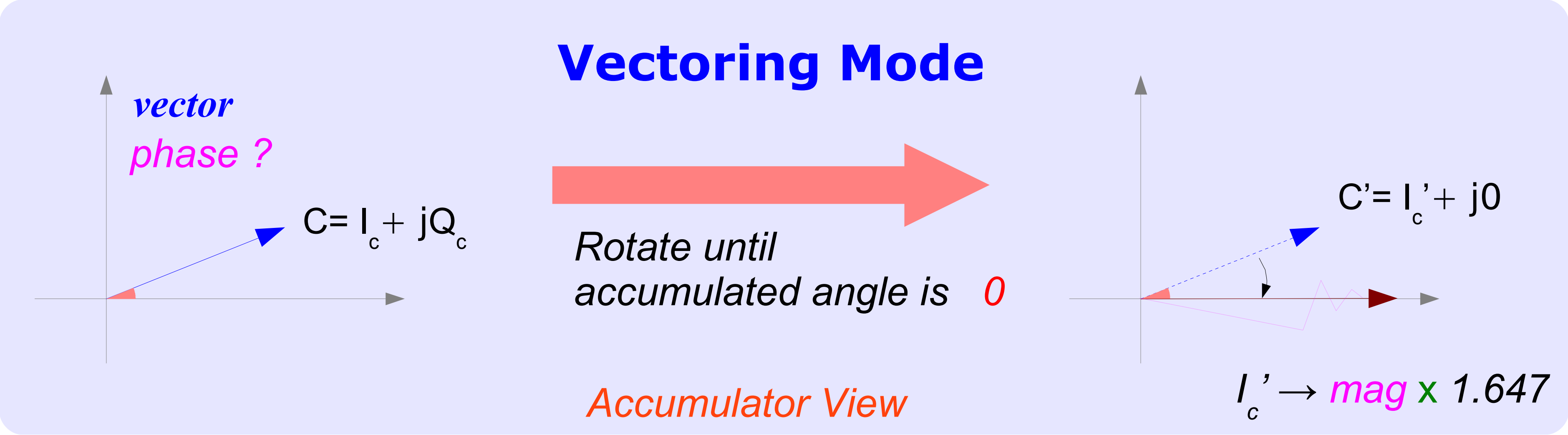
To compensate, multiply by 0.607

Can't perform this gain adjustment by simple shift and add

- *This adjustment can be done in other part of a system when relative magnitude is enough*

Vector Mode – Calculating Phase

Vectoring Mode



Phase of R

$$\theta_0 = \tan^{-1}(2^0) = 45^\circ$$
$$\theta_1 = \tan^{-1}(2^{-1}) = 26.56505^\circ$$
$$\theta_2 = \tan^{-1}(2^{-2}) = 14.03624^\circ$$
$$\theta_3 = \tan^{-1}(2^{-3}) = 7.12502^\circ$$
$$\theta_4 = \tan^{-1}(2^{-4}) = 3.57633^\circ$$
$$\theta_5 = \tan^{-1}(2^{-5}) = 1.78991^\circ$$

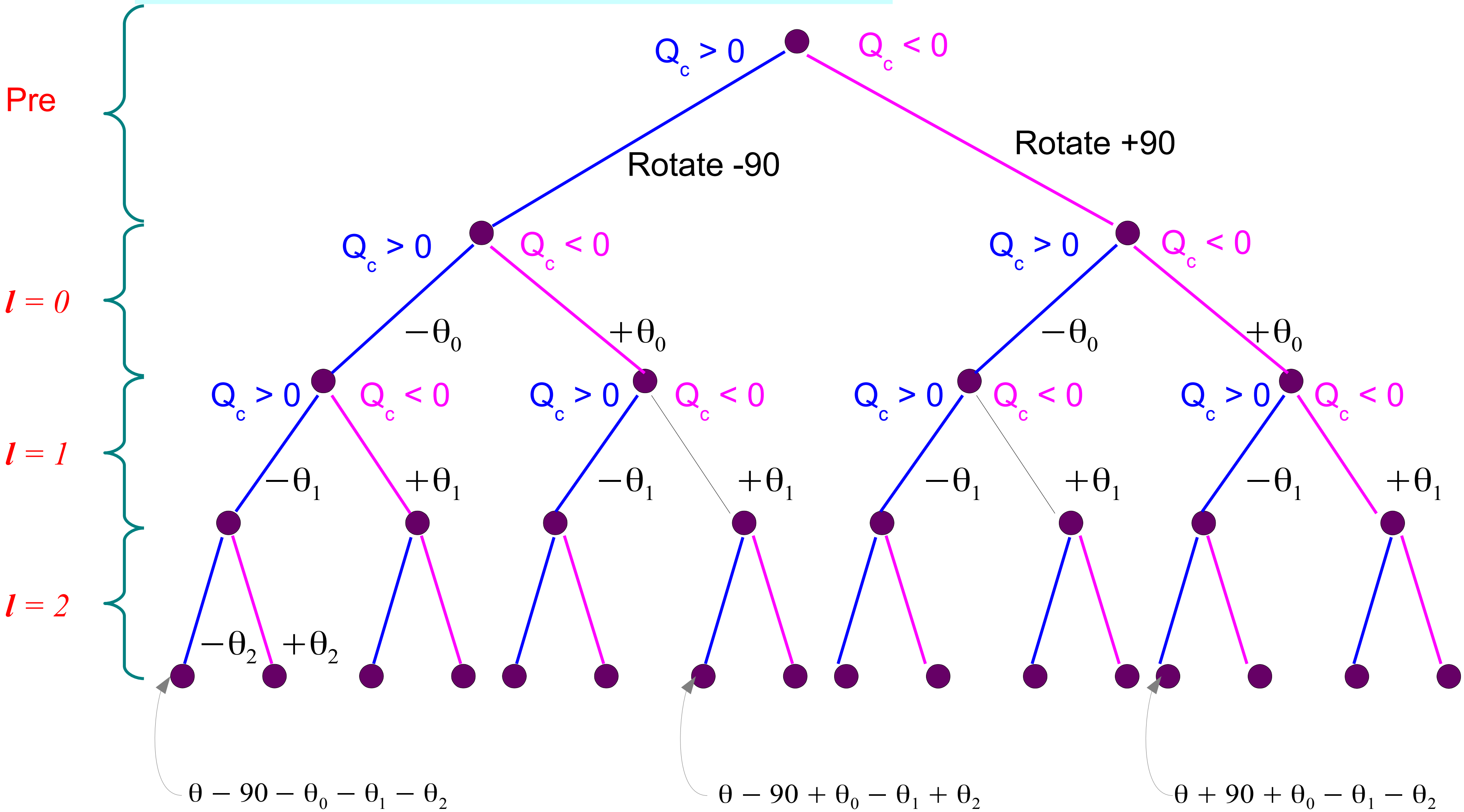
...

$$\theta \quad \pm\theta_0 \pm\theta_1 \pm\theta_2 \pm\theta_3 \cdots = 0$$

Accumulate each rotating angles then negate the result

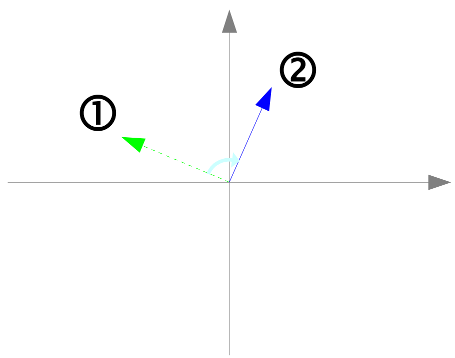
Angle Update in Vector Mode

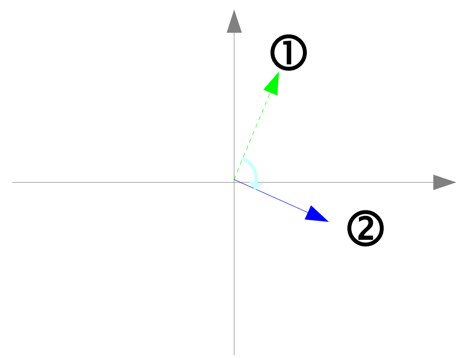
$$I_c' + j\,Q_c' = (I_c + j\,Q_c) \cdot (1 + j\,k_l)$$

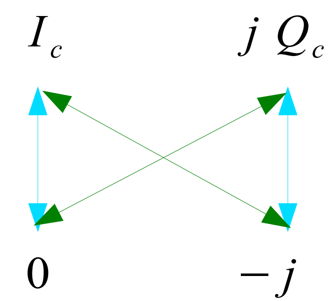


Multiplication of ±j

Positive Phase ($Q_c > 0$) ➡ *Rotate by –90 degrees*

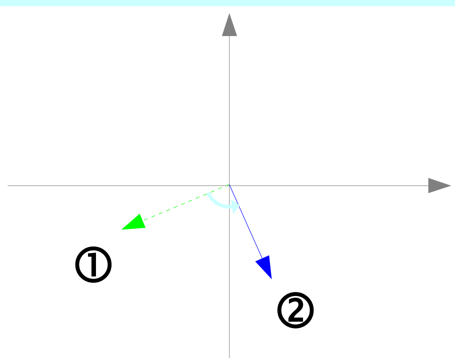


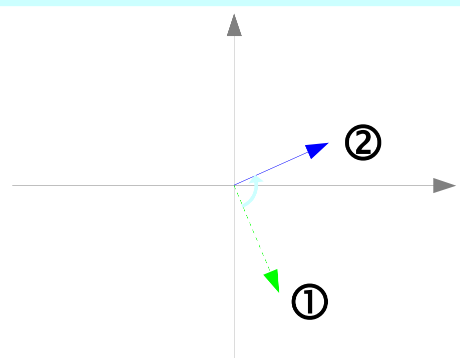


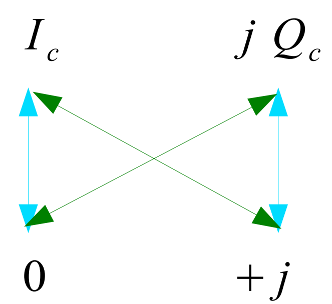


$$(Q_c) + j\,(-I_c)$$

Negative Phase ($Q_c < 0$) ➡ *Rotate by +90 degrees*





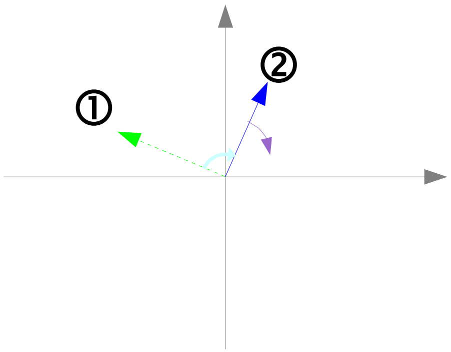


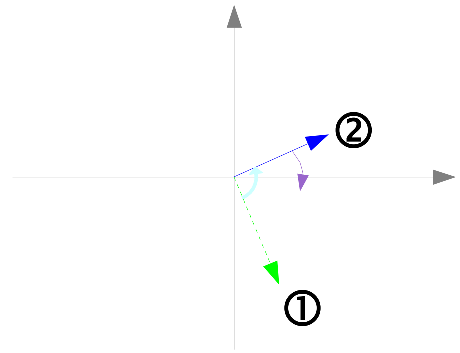
$$(-Q_c) + j\,(I_c)$$

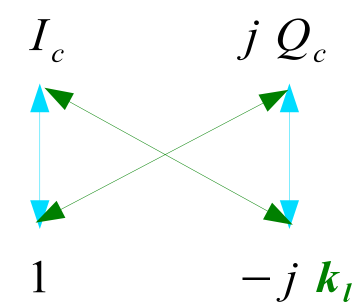
Resulting Phase ➡ [-90, +90]

Multiplication of $1 \pm jk_l$

Positive Phase ($Q_c > 0$) ➡ *Rotate by* $1 - j\, k_l$

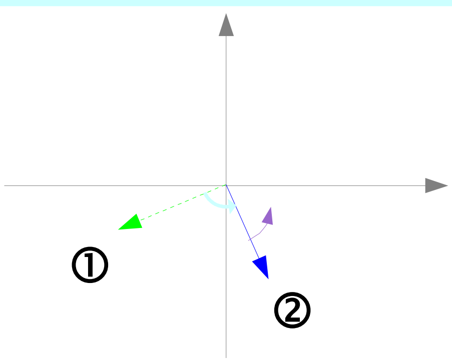


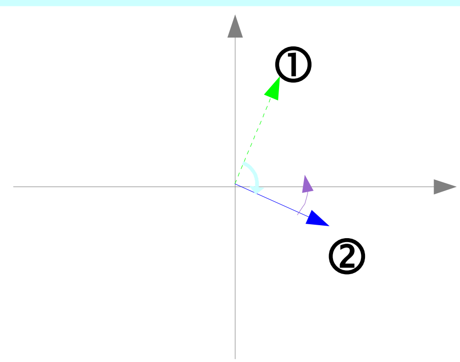


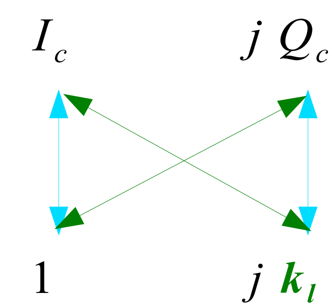


$$(I_c + 2^{-l} Q_c) + j\,(Q_c - 2^{-l} I_c)$$

Negative Phase ($Q_c < 0$) ➡ *Rotate by* $1 + j\, k_l$



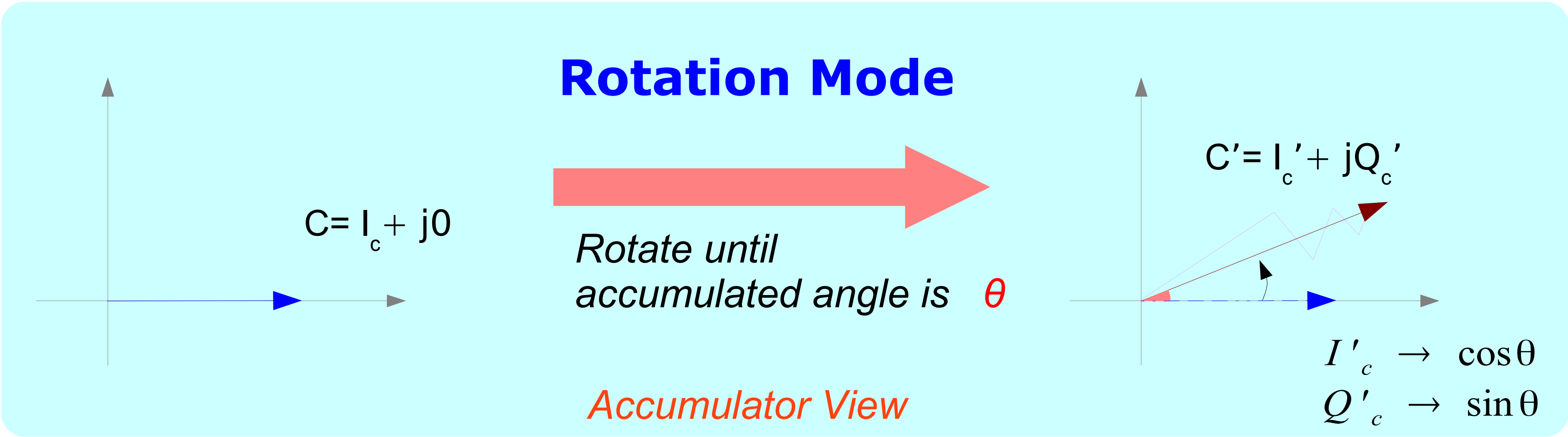


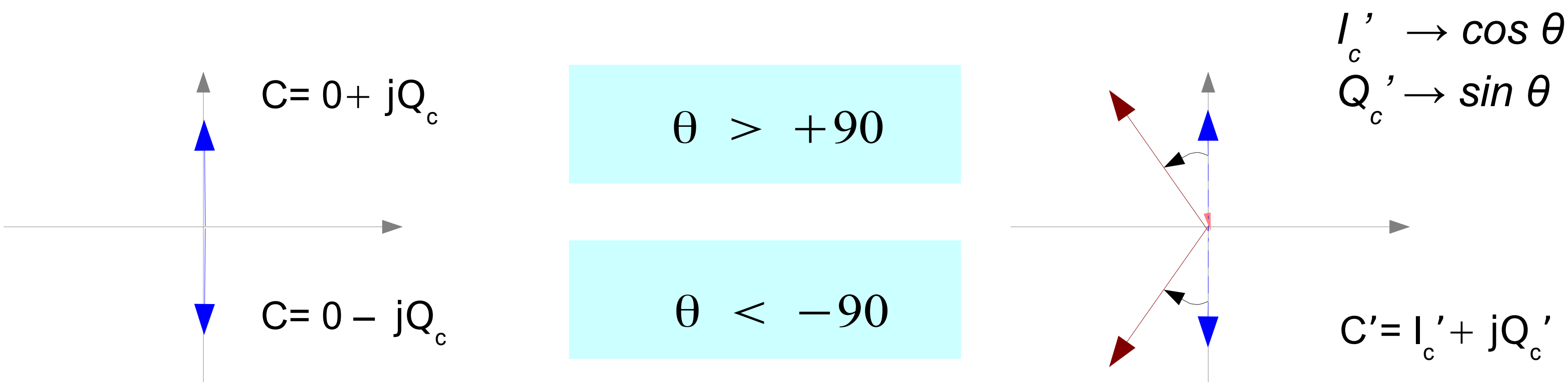


$$(I_c - 2^{-l} Q_c) + j(Q_c + 2^{-l} I_c)$$

After iterations, the result ➡ **$I_c + j\,0$**

Rotation Mode – Calculating Sine and Cosine

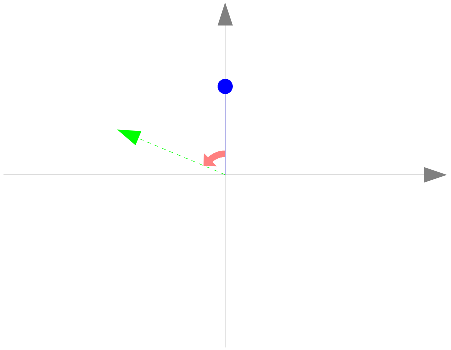




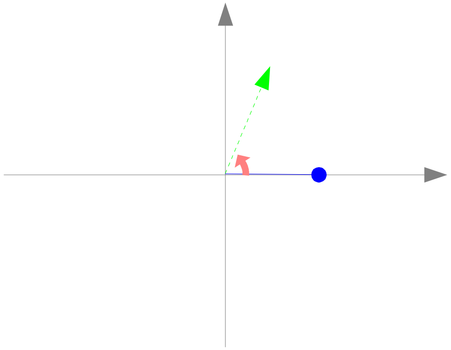
Young Won Lim
04/09/2012

Initial Vector

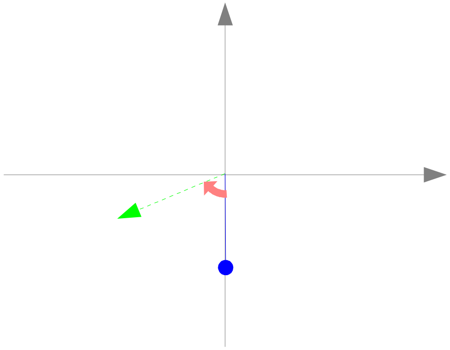
$\Theta > +90$ ➡ *starting from* *0 + j1*



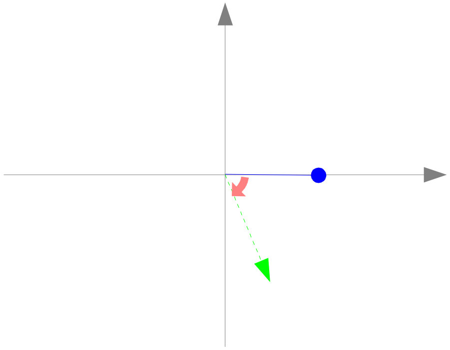
$\Theta < +90$ ➡ *starting from* *1 + j0*



$\Theta < -90$ ➡ *starting from* *0 – j1*



$\Theta > -90$ ➡ *starting from* *1 – j0*



Initialize the accumulate rotation ➡ -90, +90, 0

Angle Update in Rotation Mode

In each iteration

(Θ – *the accumulated rotation*) *< 0*

➡ *then add the next angle*

(Θ – *the accumulated rotation*) *> 0*

➡ *then subtract the next angle*

The final

I_c ➡ *cosine*

Q_c ➡ *sine*

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann
[4] CORDIC FAQ, www.dspguru.com