# CT Convolution (1B)

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1/29/15

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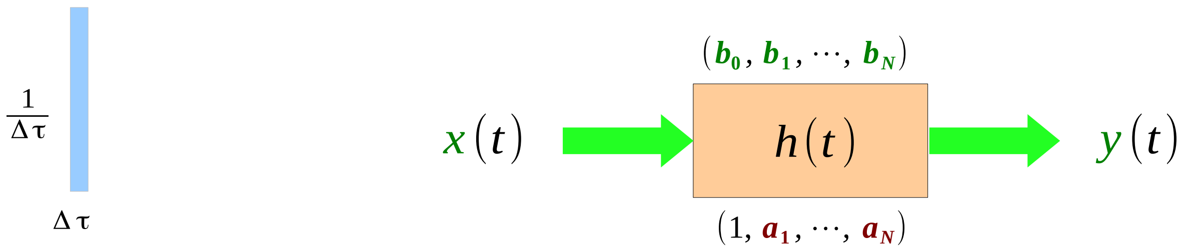
# Convolution

$$x(t) = \lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)\,p(t-n\Delta\tau) = \lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)\frac{p(t-n\Delta\tau)}{\Delta\tau}\Delta\tau = \lim_{\Delta\tau \to 0} \sum_{\tau} x(n\Delta\tau)\,\delta(t-n\Delta\tau)\,\Delta\tau$$




$$\delta(t) \xrightarrow{h(t)} h(t)$$

$$\delta(t-n\Delta\tau) \xrightarrow{h(t)} h(t-n\Delta\tau)$$

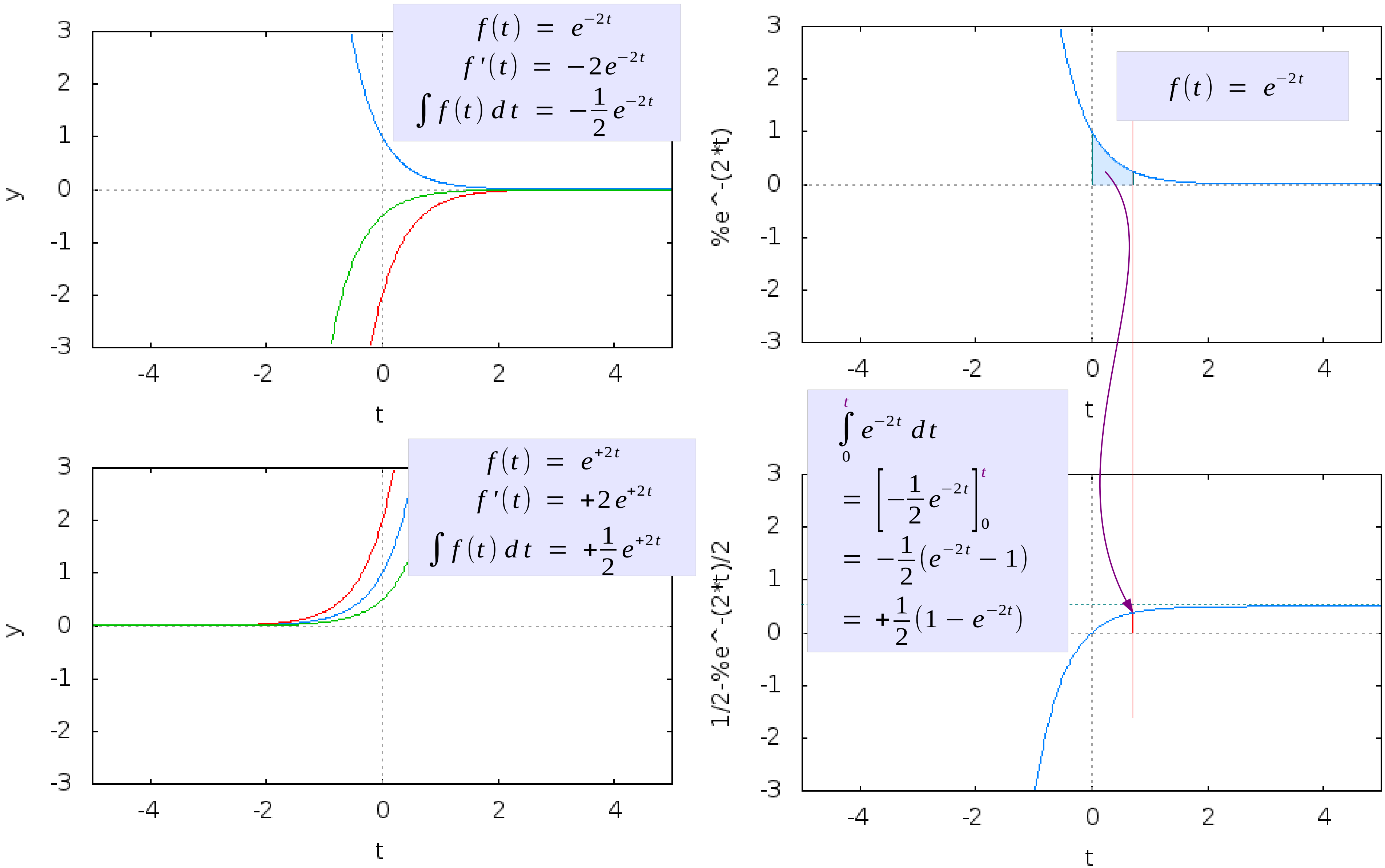
$$x(n\Delta\tau)\,\delta(t-n\Delta\tau)\,\Delta\tau \xrightarrow{h(t)} x(n\Delta\tau)\,h(t-n\Delta\tau)\,\Delta\tau$$

$$\lim_{\Delta\tau \to 0} x(n\Delta\tau)\,\delta(t-n\Delta\tau)\,\Delta\tau \xrightarrow{h(t)} \lim_{\Delta\tau \to 0} x(n\Delta\tau)\,h(t-n\Delta\tau)\,\Delta\tau$$

$$x(t) \xrightarrow{h(t)} y(t)$$

$$y(t) = \lim_{\Delta\tau \to 0} x(n\Delta\tau)\,h(t-n\Delta\tau)\,\Delta\tau = \int_{-\infty}^{+\infty} x(\tau)\,h(t-\tau)\,d\tau$$

# Integration & Differentiation of $e^{at}$



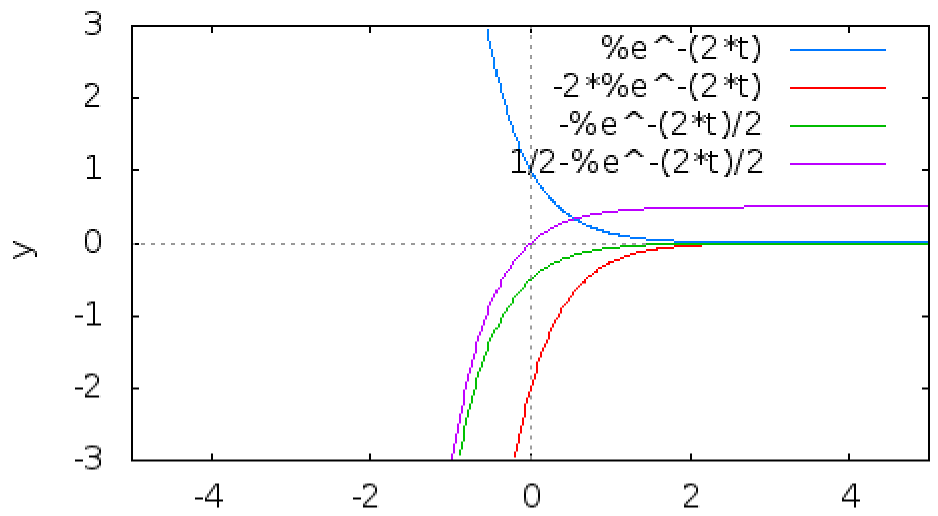

$$f(t) = e^{-2t}$$
$$f'(t) = -2e^{-2t}$$
$$\int f(t)\,dt = -\frac{1}{2}e^{-2t}$$

$$f(t) = e^{+2t}$$
$$f'(t) = +2e^{+2t}$$
$$\int f(t)\,dt = +\frac{1}{2}e^{+2t}$$

$$f(t) = e^{-2t}$$

$$\int_0^t e^{-2t}\,dt$$
$$= \left[-\frac{1}{2}e^{-2t}\right]_0^t$$
$$= -\frac{1}{2}(e^{-2t} - 1)$$
$$= +\frac{1}{2}(1 - e^{-2t})$$

# Definite Integration of $e^{-2t}$

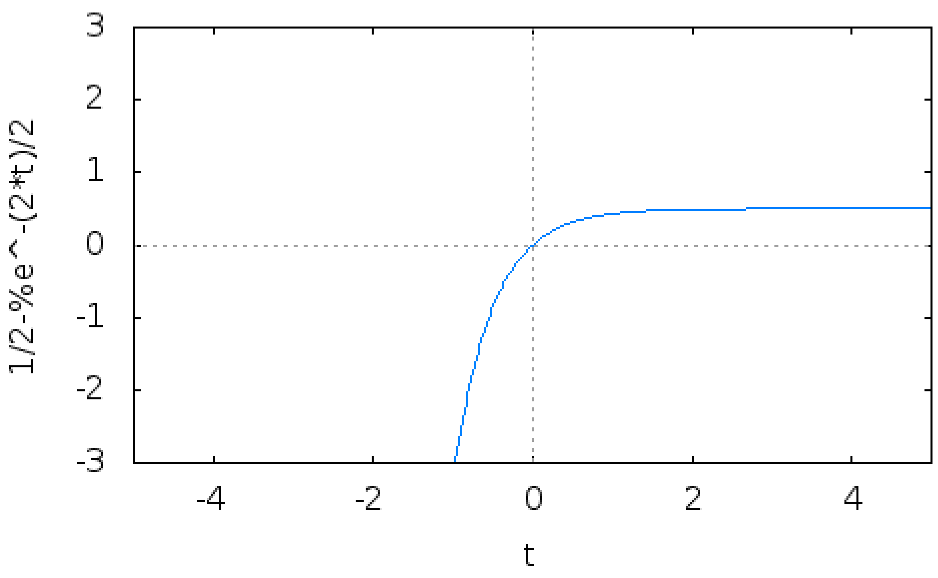



$$e^{-2t}$$

$$\frac{d}{dx} e^{-2t}$$

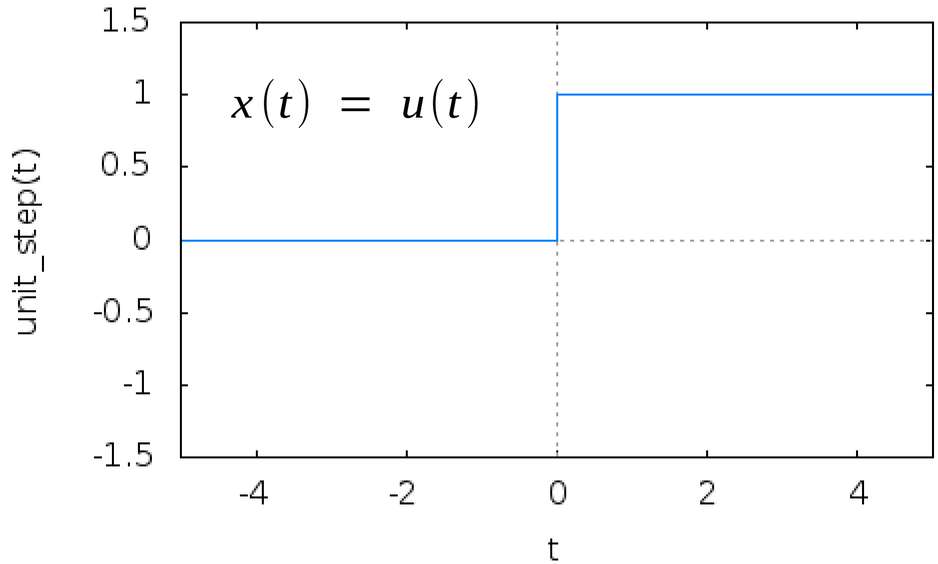
$$\int e^{-2t}\, dt$$

$$\int_0^t e^{-2t}\, dt$$

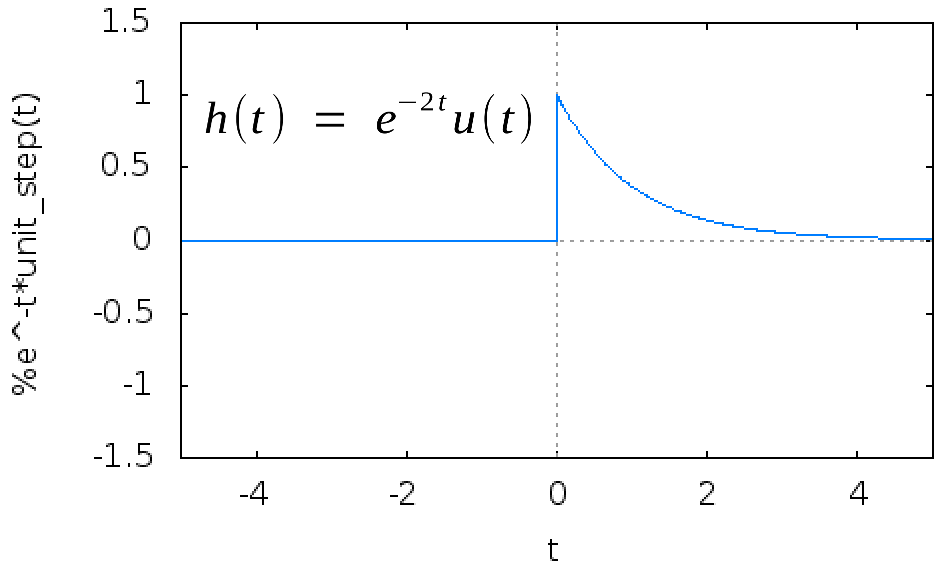



$$\int_0^t e^{-2t}\, dt$$

# u(t) and $e^{-2t}$



1.5
1
0.5
0
-0.5
-1
-1.5
unit_step(t)
$x(t) = u(t)$
-4
-2
0
2
4
t

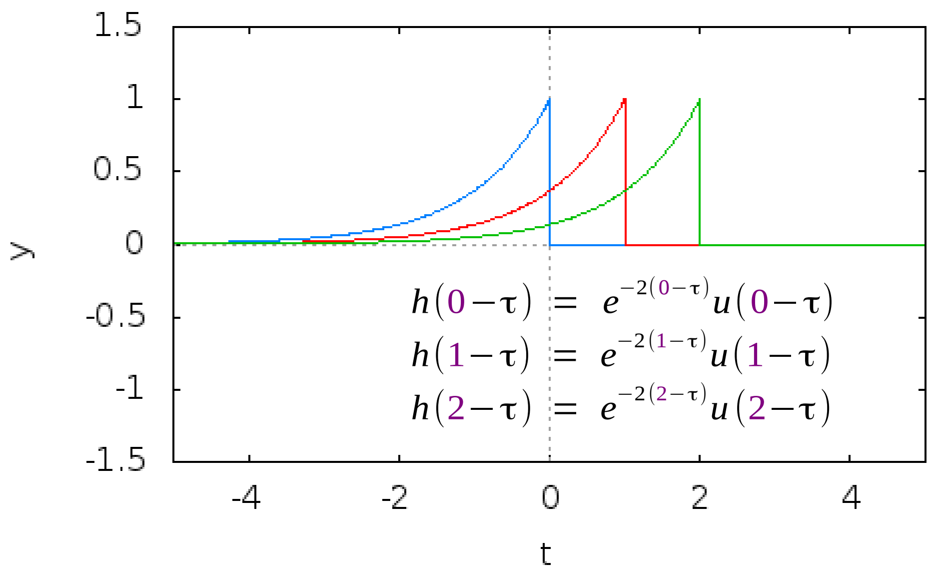



1.5
1
0.5
0
-0.5
-1
-1.5
%e^-t*unit_step(t)
$h(t) = e^{-2t}u(t)$
-4
-2
0
2
4
t

# Graphic Convolution of u(t) and $e^{-2t}$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$$



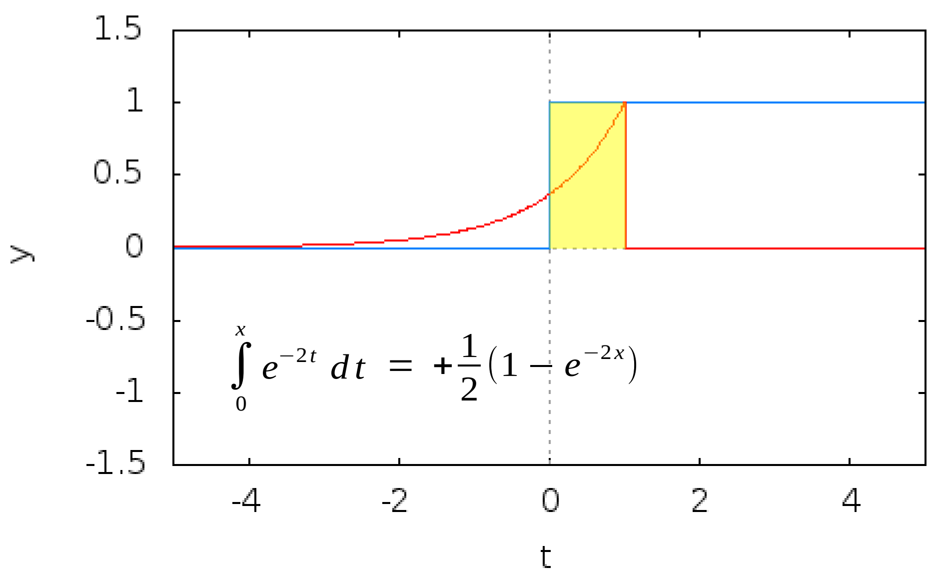

at the given time t

$$= \int_{0}^{t} x(\tau) h(t-\tau)\, d\tau$$

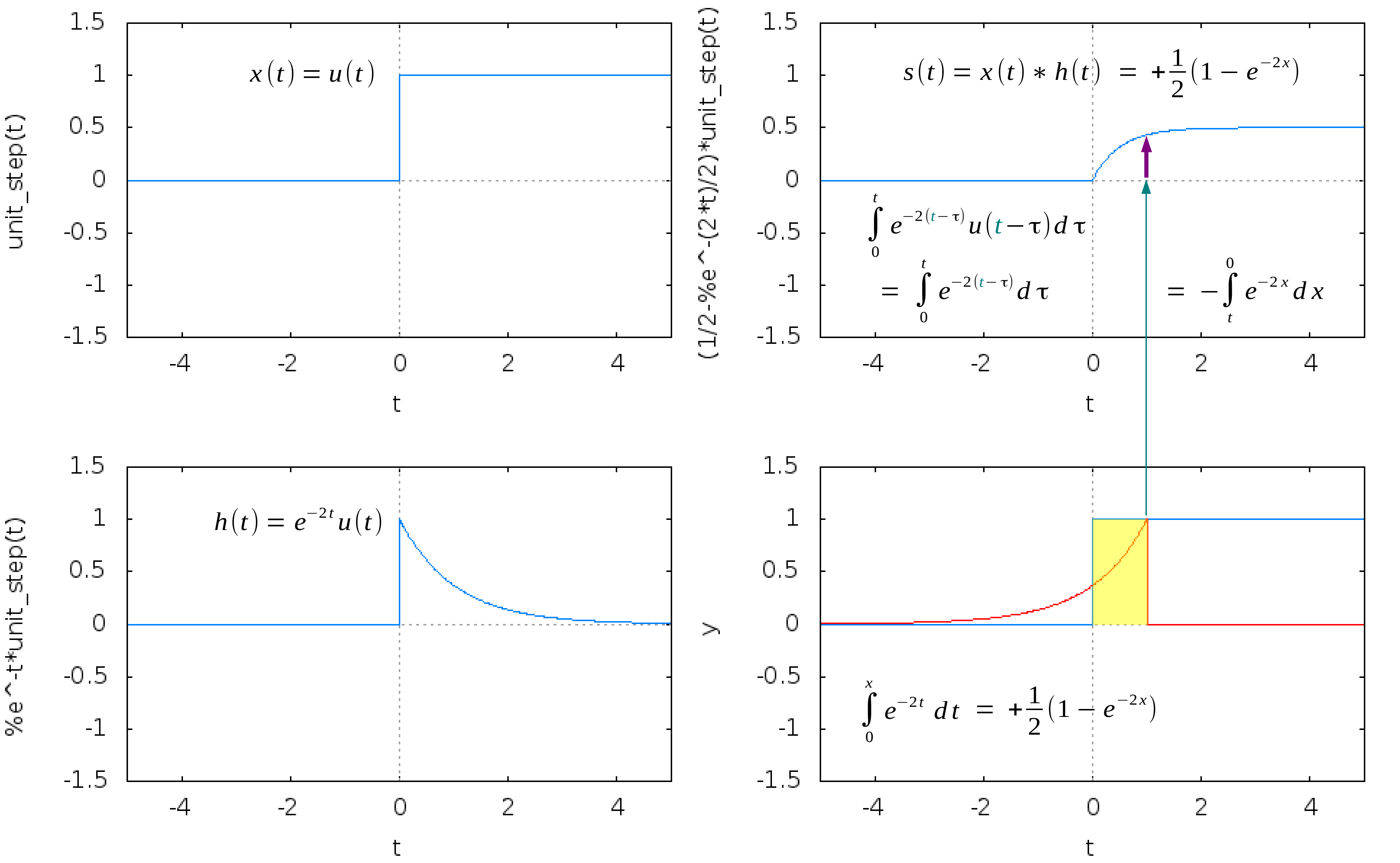
$$= \int_{0}^{t} h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-2\tau}\, d\tau$$

the area of the overlapped region

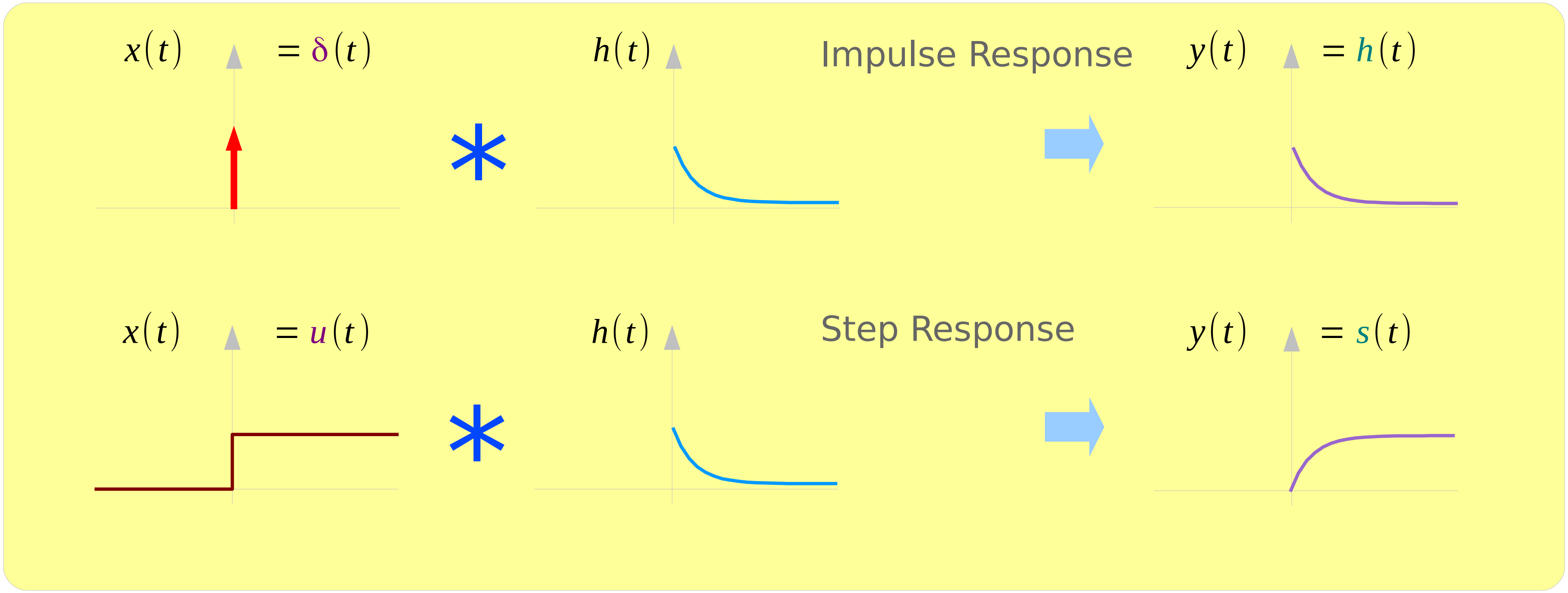


# Convolution of u(t) and $e^{-2t}$

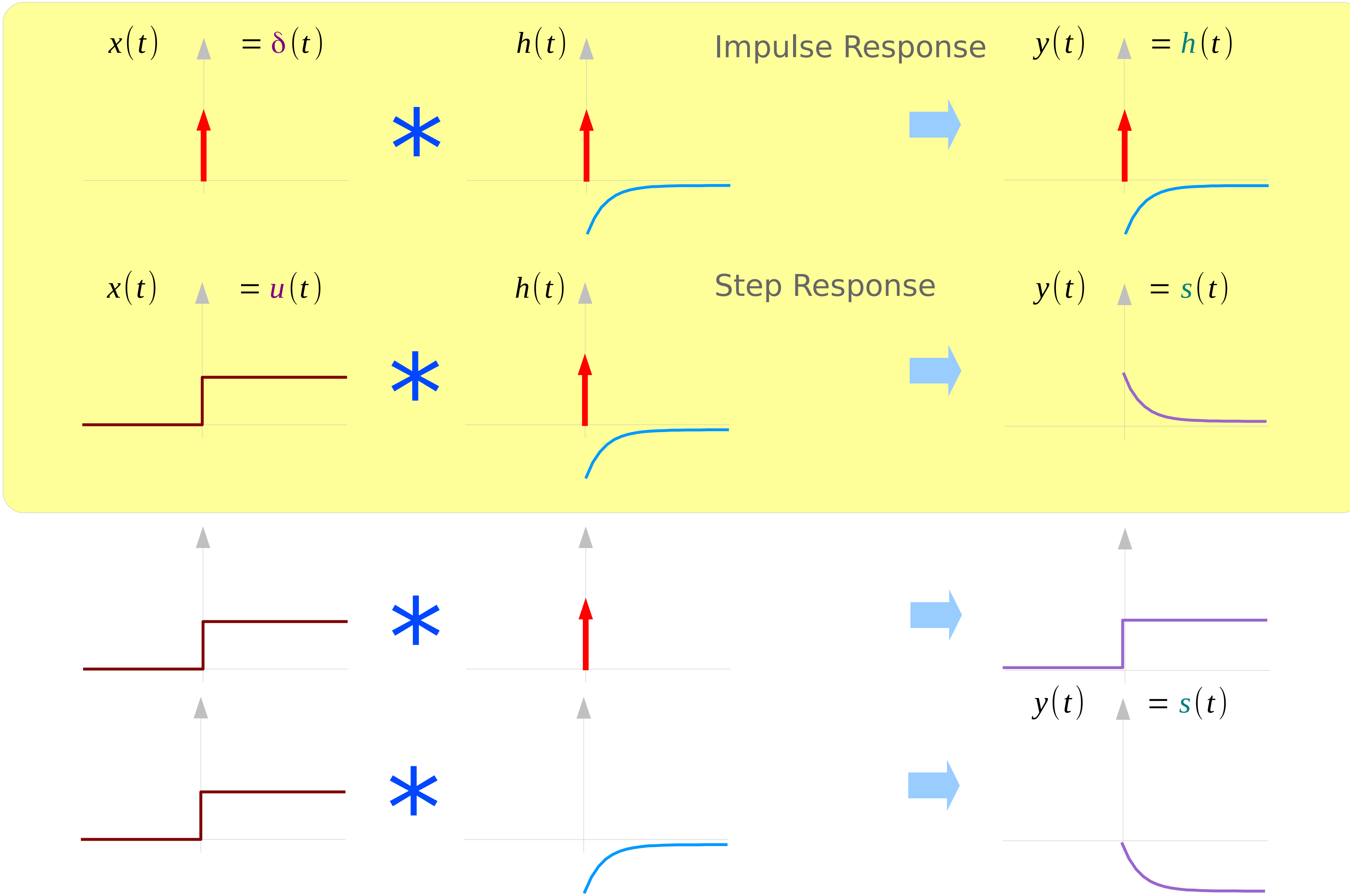


# Convolution Examples (1)

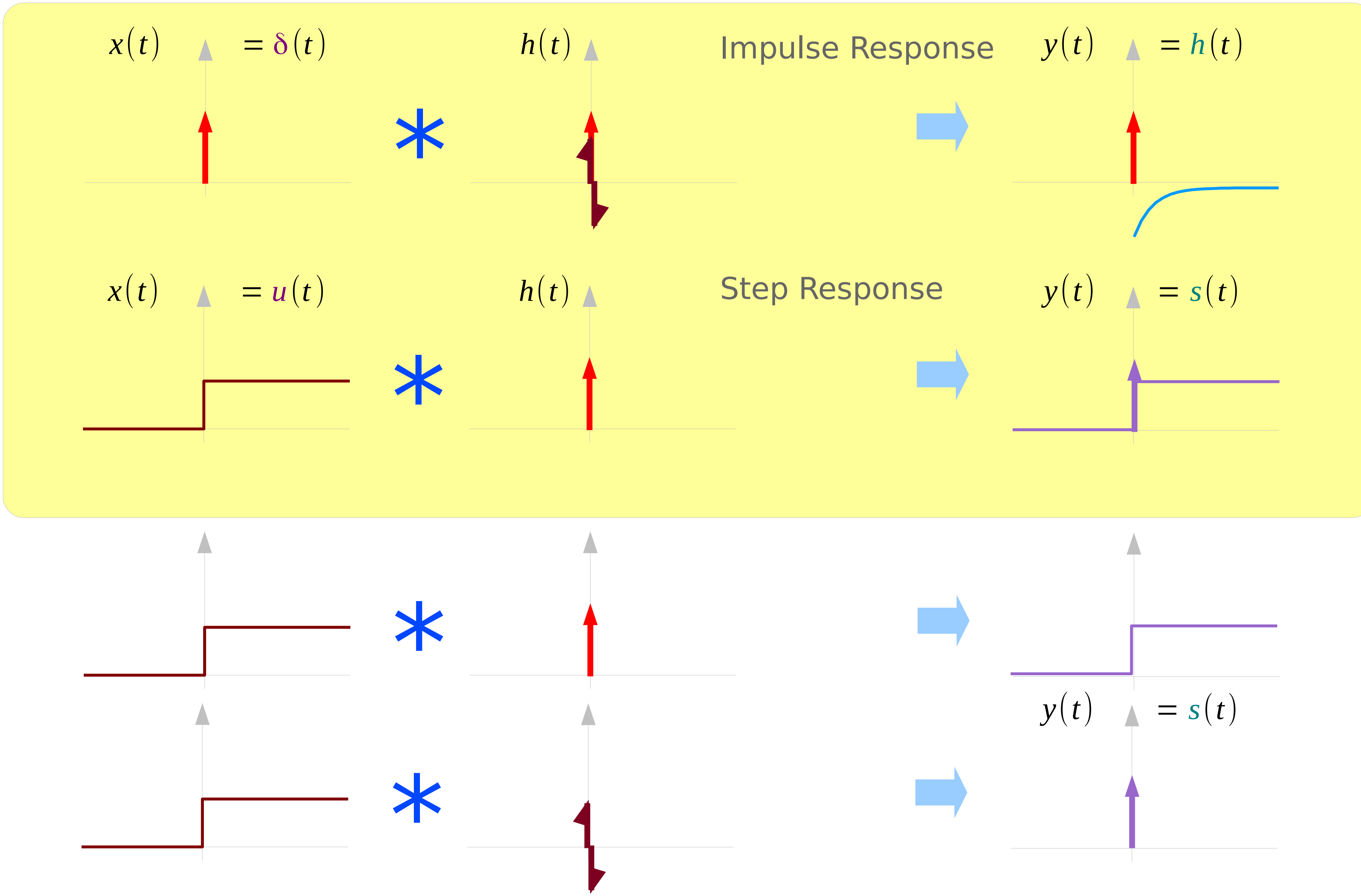




# Convolution Examples (2)

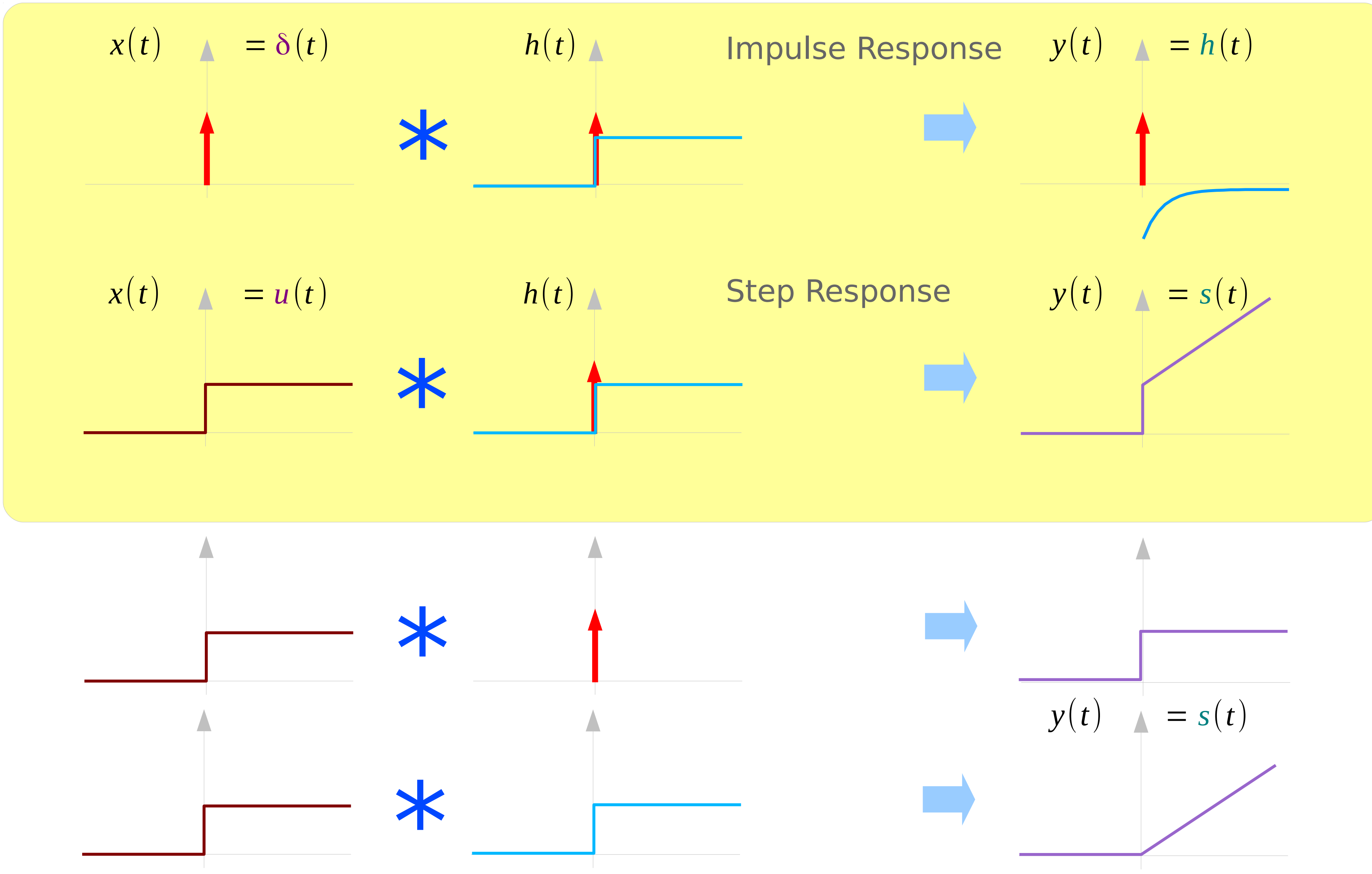


# Convolution Examples (3)

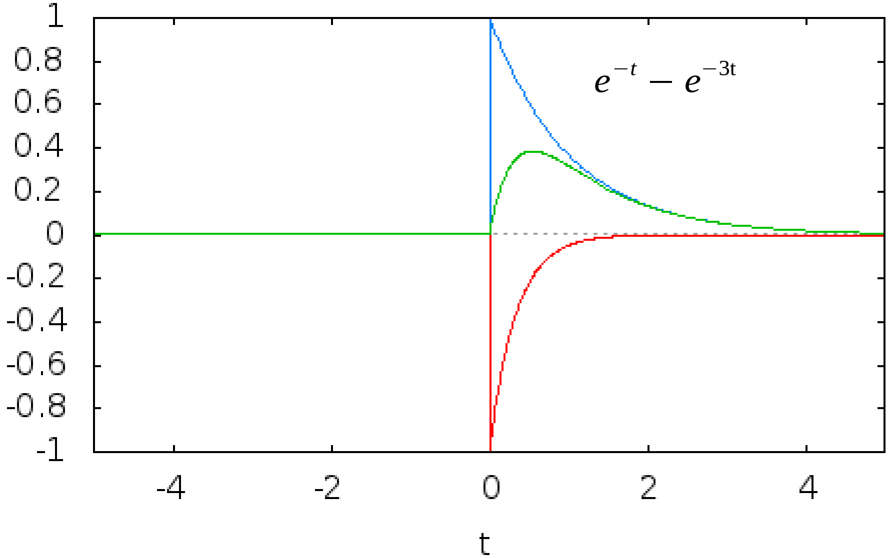


# Convolution Examples (4)

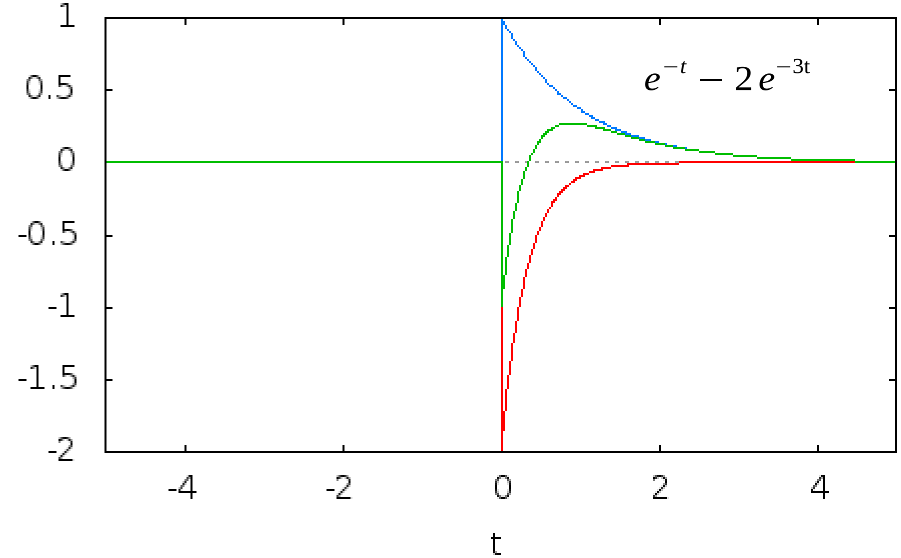




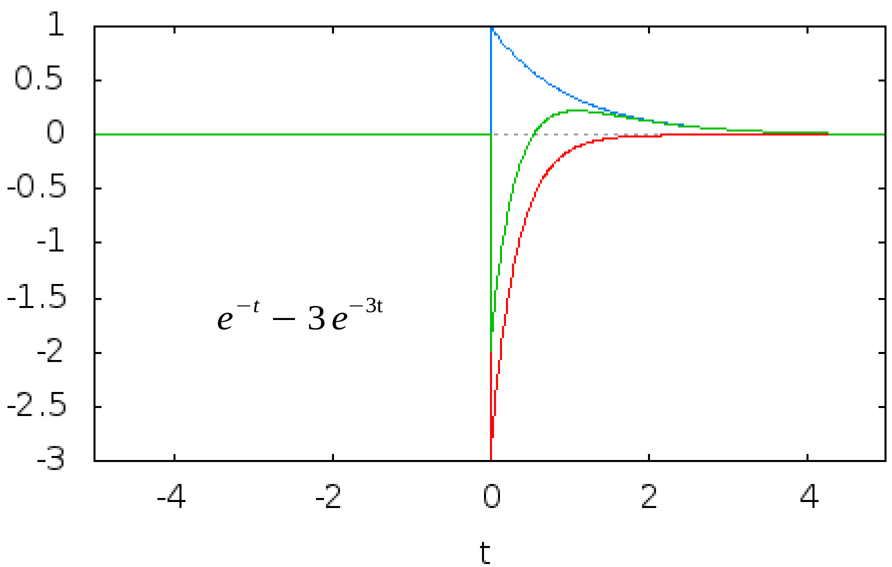
# $c_1 e^{-t} + c_2 e^{-3t}$



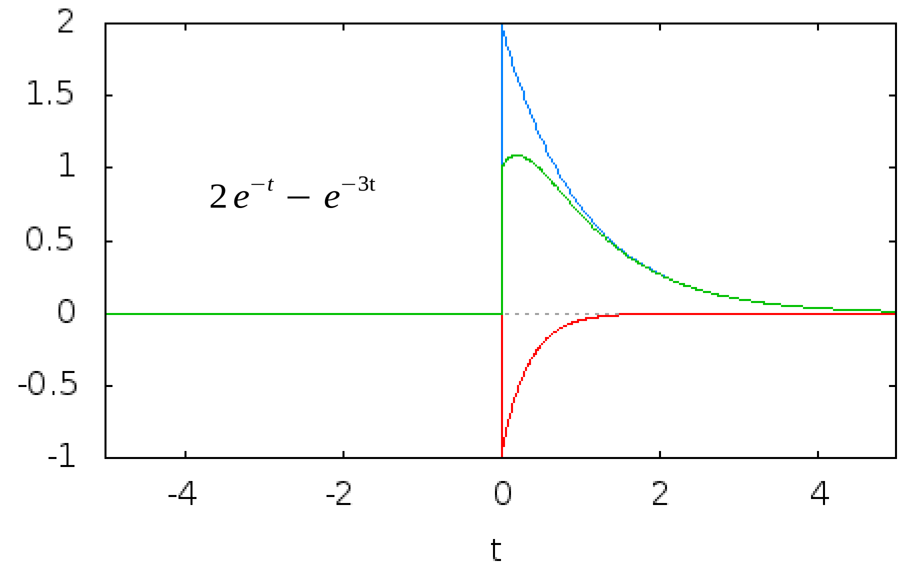
$e^{-t} - e^{-3t}$
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-4
-2
0
2
4
t

$e^{-t} - 2e^{-3t}$
1
0.5
0
-0.5
-1
-1.5
-2
-4
-2
0
2
4
t

$e^{-t} - 3e^{-3t}$
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-4
-2
0
2
4
t

$2e^{-t} - e^{-3t}$
2
1.5
1
0.5
0
-0.5
-1
-4
-2
0
2
4
t

# $h(t) = c_1 e^{-t} + c_2 e^{-3t}$

$$h(t) = (e^{-t} - e^{-3t})u(t)$$

$$\int_0^t h(t)\,dt = \int_0^t e^{-t} - e^{-3t}\,dt$$

$$= \left[-e^{-t} + \frac{1}{3}e^{-3t}\right]_0^t$$

$$s(t) = \left(\frac{2}{3} - e^{-t} + \frac{1}{3}e^{-3t}\right)u(t)$$

$$h(t) = e^{-t} - 2e^{-3t}$$

$$\int_0^t h(t)\,dt = \int_0^t e^{-t} - 2e^{-3t}\,dt$$

$$= \left[-e^{-t} + \frac{2}{3}e^{-3t}\right]_0^t$$

$$s(t) = \left(-\frac{1}{3} - e^{-t} + \frac{2}{3}e^{-3t}\right)u(t)$$

$$h(t) = (e^{-t} - 3e^{-3t})u(t)$$

$$\int_0^t h(t)\,dt = \int_0^t e^{-t} - 3e^{-3t}\,dt$$

$$= \left[-e^{-t} + e^{-3t}\right]_0^t$$

$$s(t) = \left(-e^{-t} + e^{-3t}\right)u(t)$$

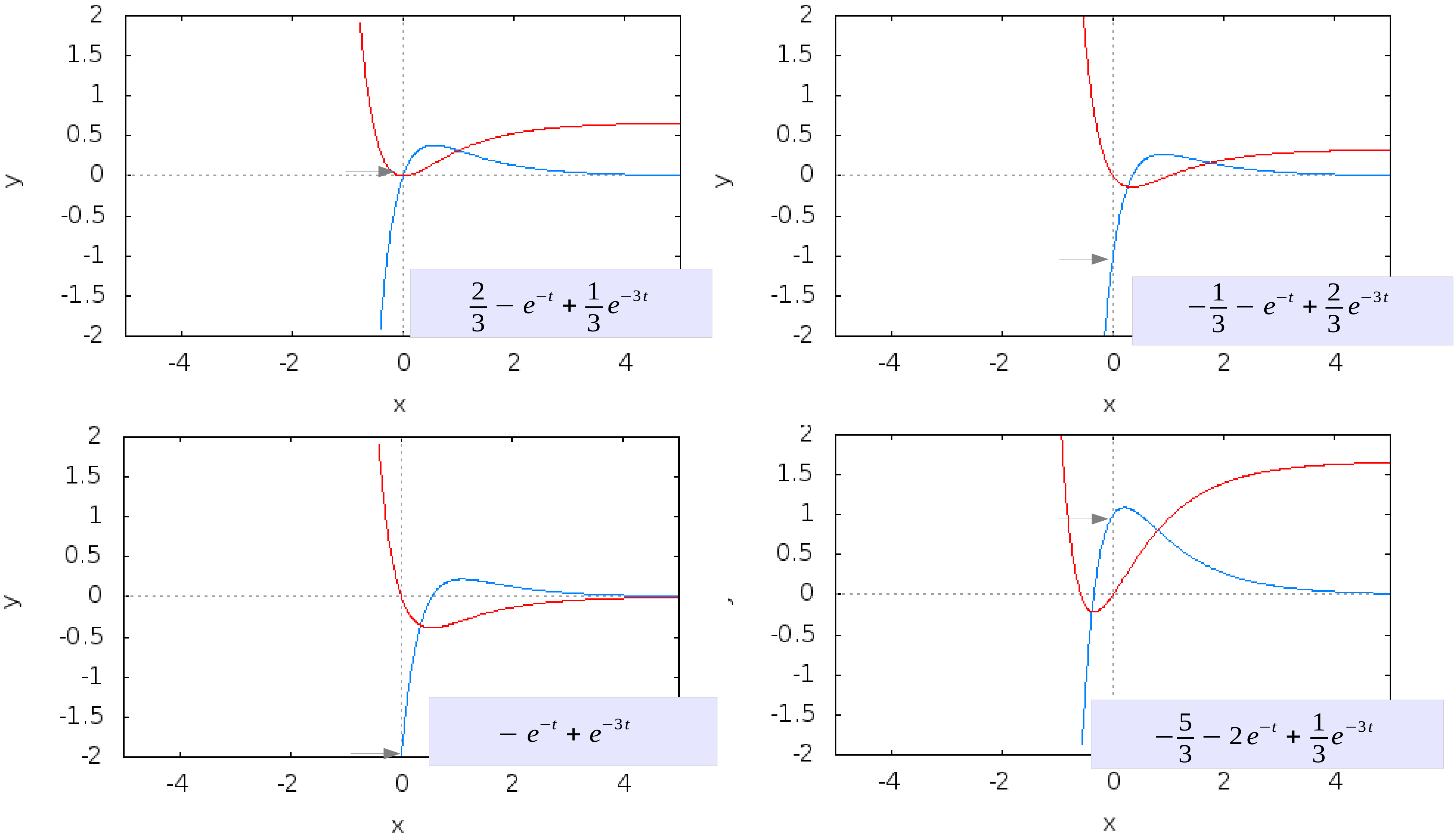
$$h(t) = 2e^{-t} - e^{-3t}$$

$$\int_0^t h(t)\,dt = \int_0^t 2e^{-t} - e^{-3t}\,dt$$

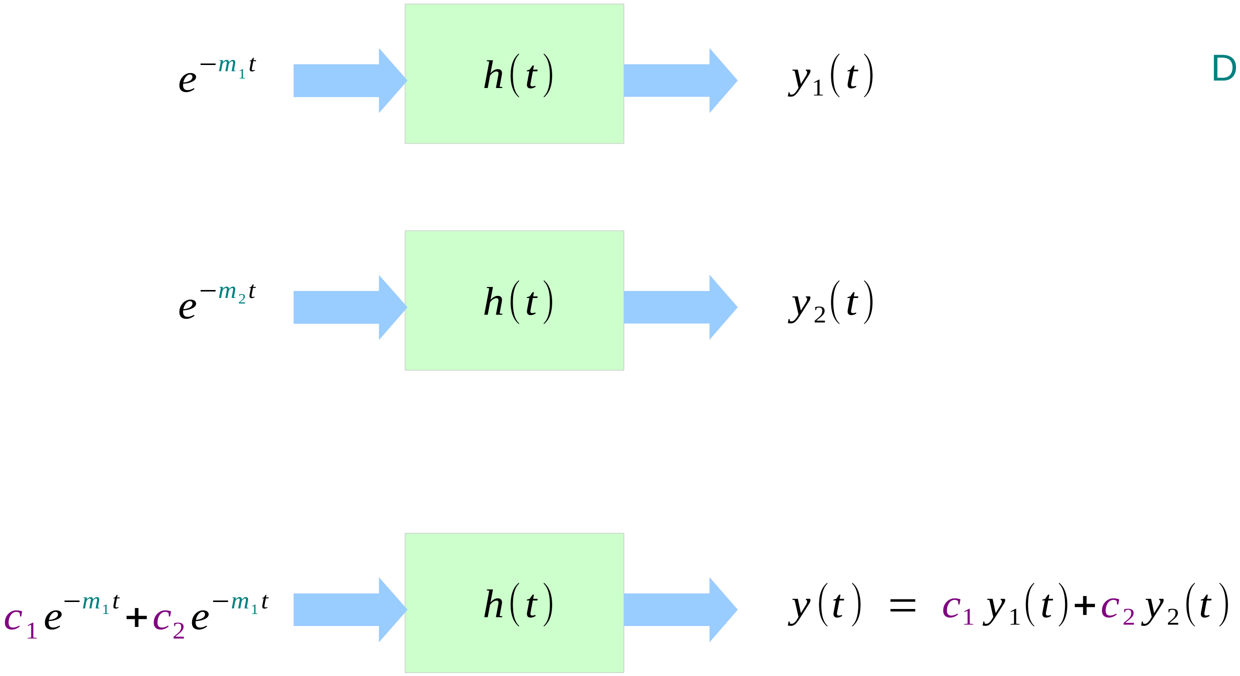
$$= \left[-2e^{-t} + \frac{1}{3}e^{-3t}\right]_0^t$$

$$s(t) = \left(-\frac{5}{3} - 2e^{-t} + \frac{1}{3}e^{-3t}\right)u(t)$$

# s(t) from h(t) = $c_1 e^{-t} + c_2 e^{-3t}$



# Linearity – Overdamped Case




Distinct Real Numbers $m_1, \ m_2$

# Linearity – Underdamped Case

$$
\begin{aligned}
c_1 e^{-m_1 t} + c_2 e^{-m_2 t} &= c_1 e^{-(\sigma+i\omega)t} + c_2 e^{-(\sigma+i\omega)t} \\
&= c_1 e^{-\sigma t} e^{-i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t} \\
&= c_3 e^{-\sigma t}\cos(\omega t) + c_4 e^{-\sigma t}\sin(\omega t)
\end{aligned}
$$

Complex Conjugate Numbers

$$
\begin{aligned}
m_1 &= \sigma + i\omega \\
m_2 &= \sigma - i\omega
\end{aligned}
$$

$$e^{-\sigma t}\cos(\omega t) \rightarrow h(t) \rightarrow y_3(t)$$

$$e^{-\sigma t}\sin(\omega t) \rightarrow h(t) \rightarrow y_4(t)$$

$$c_3 e^{-\sigma t}\cos(\omega t) + c_4 e^{-\sigma t}\sin(\omega t) \rightarrow h(t) \rightarrow y(t) = c_3 y_3(t) + c_4 y_4(t)$$

# Complex Exponentials

$$c_1 e^{-\sigma t} e^{+i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t}$$

$$(c_1 + c_2) = c_3$$

$$i(c_1 - c_2) = c_4$$

$$= e^{-\sigma t}(c_1 e^{-i\omega t} + c_2 e^{-i\omega t})$$

$$= e^{-\sigma t}[c_1(\cos(\omega t) - i\sin(\omega t)) + c_2(\cos(\omega t) + i\sin(\omega t))]$$

$$= e^{-\sigma t}[(c_1 + c_2)\cos(\omega t) + i(c_1 - c_2)\sin(\omega t)]$$

$$= c_3 e^{-\sigma t}\cos(\omega t) + c_4 e^{-\sigma t}\sin(\omega t)$$

$$c_3 e^{-\sigma t}\cos(\omega t) + c_4 e^{-\sigma t}\sin(\omega t)$$

$$\frac{(c_3 - c_4 i)}{2} = c_1$$

$$\frac{(c_3 + c_4 i)}{2} = c_2$$

$$= c_3 e^{-\sigma t}(e^{+i\omega t} + e^{-i\omega t})/2 + c_4 e^{-\sigma t}(e^{+i\omega t} - e^{-i\omega t})/2i$$

$$= c_3 e^{-\sigma t}(e^{+i\omega t} + e^{-i\omega t})/2 + c_4 e^{-\sigma t}(-i e^{+i\omega t} + i e^{-i\omega t})/2$$

$$= \frac{(c_3 - c_4 i)}{2} e^{-\sigma t} e^{+i\omega t} + \frac{(c_3 + c_4 i)}{2} e^{-\sigma t} e^{-i\omega t}$$

$$= c_1 e^{-\sigma t} e^{+i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t}$$

# Complex Exponentials

$$c_1 e^{-\sigma t} e^{+i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t} \quad \rightarrow \quad c_3 e^{-\sigma t} \cos(\omega t) + c_4 e^{-\sigma t} \sin(\omega t)$$

$$(c_1 + c_2) = c_3$$

$$i(c_1 - c_2) = c_4$$

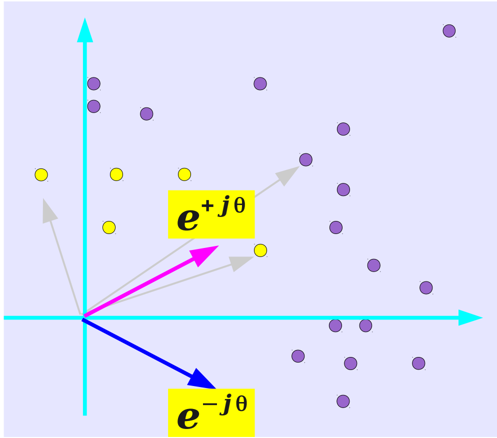
$$c_1 e^{-\sigma t} e^{+i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t} \quad \leftarrow \quad c_3 e^{-\sigma t} \cos(\omega t) + c_4 e^{-\sigma t} \sin(\omega t)$$

$$c_1 = \frac{(c_3 - c_4 i)}{2}$$

$$c_2 = \frac{(c_3 + c_4 i)}{2}$$

# Basis of the Complex Plane

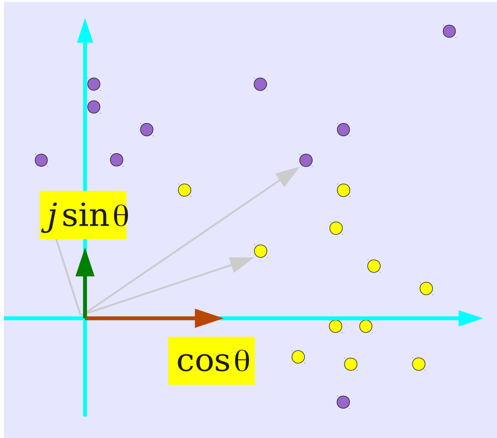
**Basis : a set of linear independent spanning vectors**




every complex number can be represented by

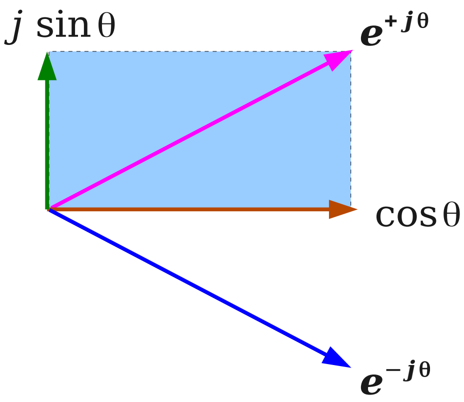
$$k_1 e^{+j\theta} + k_2 e^{+j\theta}$$

linear combination of $e^{+j\theta}$ and $e^{+j\theta}$
which are one set of linear independent
two vectors




every complex number can also be represented by

$$l_1 \cos\theta + l_2 j \sin\theta$$



# Real Coefficients $C_1$ & $C_2$

$$c_1 e^{-\sigma t} e^{+i\omega t} + c_2 e^{-\sigma t} e^{-i\omega t} \quad \longleftrightarrow \quad c_3 e^{-\sigma t} \cos(\omega t) + c_4 e^{-\sigma t} \sin(\omega t)$$

| | | | |
|---|---|---|---|
| *real number* | $c_1 = (c_3 - c_4 i)/2$ | $c_3 = (c_1 + c_2)$ | *real number* |
| *real number* | $c_2 = (c_3 + c_4 i)/2$ | $c_4 = i(c_1 - c_2)$ | *complex number* |

$$1 \cdot e^{-\sigma t} e^{+i\omega t} + 0 \cdot e^{-\sigma t} e^{-i\omega t} \qquad 1 \cdot e^{-\sigma t} \cos(\omega t) + 1 i \cdot e^{-\sigma t} \sin(\omega t)$$

$$\tfrac{1}{\sqrt{2}} \cdot e^{-\sigma t} e^{+i\omega t} + \tfrac{1}{\sqrt{2}} \cdot e^{-\sigma t} e^{-i\omega t} \qquad \sqrt{2} \cdot e^{-\sigma t} \cos(\omega t) + 0 i \cdot e^{-\sigma t} \sin(\omega t)$$

$$0 \cdot e^{-\sigma t} e^{+i\omega t} + 1 \cdot e^{-\sigma t} e^{-i\omega t} \qquad 1 \cdot e^{-\sigma t} \cos(\omega t) - 1 i \cdot e^{-\sigma t} \sin(\omega t)$$

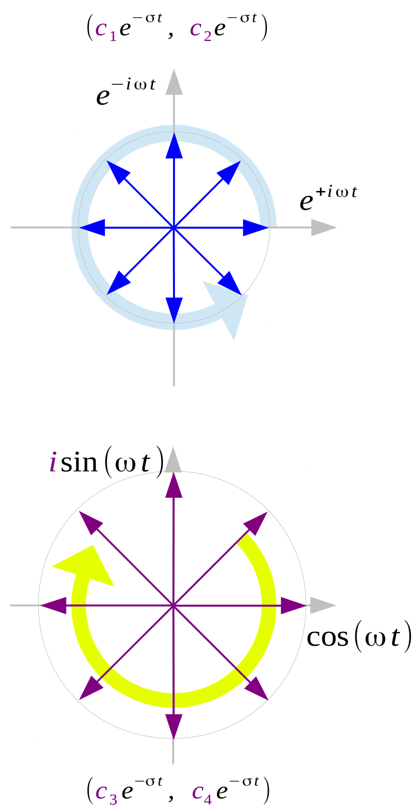
$$\tfrac{-1}{\sqrt{2}} \cdot e^{-\sigma t} e^{+i\omega t} + \tfrac{1}{\sqrt{2}} \cdot e^{-\sigma t} e^{-i\omega t} \qquad 0 \cdot e^{-\sigma t} \cos(\omega t) - \sqrt{2} i \cdot e^{-\sigma t} \sin(\omega t)$$

$$-1 \cdot e^{-\sigma t} e^{+i\omega t} + 0 \cdot e^{-\sigma t} e^{-i\omega t} \qquad -1 \cdot e^{-\sigma t} \cos(\omega t) - 1 i \cdot e^{-\sigma t} \sin(\omega t)$$

$$\tfrac{-1}{\sqrt{2}} \cdot e^{-\sigma t} e^{+i\omega t} + \tfrac{-1}{\sqrt{2}} \cdot e^{-\sigma t} e^{-i\omega t} \qquad -\sqrt{2} \cdot e^{-\sigma t} \cos(\omega t) - 0 i \cdot e^{-\sigma t} \sin(\omega t)$$

$$0 \cdot e^{-\sigma t} e^{+i\omega t} - 1 \cdot e^{-\sigma t} e^{-i\omega t} \qquad -1 \cdot e^{-\sigma t} \cos(\omega t) + 1 i \cdot e^{-\sigma t} \sin(\omega t)$$

$$\tfrac{1}{\sqrt{2}} \cdot e^{-\sigma t} e^{+i\omega t} + \tfrac{-1}{\sqrt{2}} \cdot e^{-\sigma t} e^{-i\omega t} \qquad 0 \cdot e^{-\sigma t} \cos(\omega t) + \sqrt{2} i \cdot e^{-\sigma t} \sin(\omega t)$$



# Complex Plane Basis

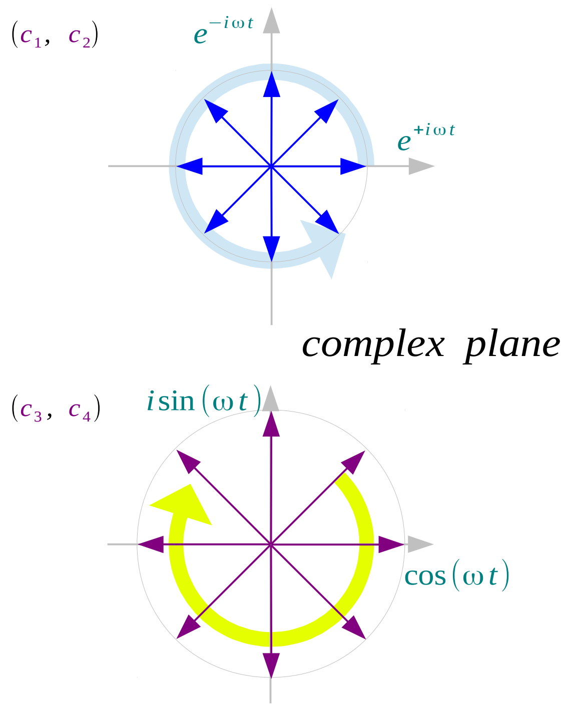
$$c_1 e^{+i\omega t} + c_2 e^{-i\omega t} \quad \Longleftrightarrow \quad c_3 \cos(\omega t) + c_4\, i \sin(\omega t)$$

$$c_1 = (c_3 + c_4)/2 \qquad \textit{real number}$$
$$c_2 = (c_3 - c_4)/2 \qquad \textit{real number}$$

$$\mathbb{C}^1$$

$$c_3 = (c_1 + c_2) \qquad \textit{real number}$$
$$c_4 = (c_1 - c_2) \qquad \textit{real number}$$

$$1 \cdot e^{+i\omega t} + 0 \cdot e^{-i\omega t}$$
$$\tfrac{1}{\sqrt{2}} \cdot e^{+i\omega t} + \tfrac{1}{\sqrt{2}} \cdot e^{-i\omega t}$$
$$0 \cdot e^{+i\omega t} + 1 \cdot e^{-i\omega t}$$
$$\tfrac{-1}{\sqrt{2}} \cdot e^{+i\omega t} + \tfrac{1}{\sqrt{2}} \cdot e^{-i\omega t}$$
$$-1 \cdot e^{+i\omega t} + 0 \cdot e^{-i\omega t}$$
$$\tfrac{-1}{\sqrt{2}} \cdot e^{+i\omega t} + \tfrac{-1}{\sqrt{2}} \cdot e^{-i\omega t}$$
$$0 \cdot e^{+i\omega t} - 1 \cdot e^{-i\omega t}$$
$$\tfrac{1}{\sqrt{2}} \cdot e^{+i\omega t} + \tfrac{-1}{\sqrt{2}} \cdot e^{-i\omega t}$$




$$1 \cdot \cos(\omega t) + 1 i \cdot \sin(\omega t)$$
$$\sqrt{2} \cdot \cos(\omega t) + 0 i \cdot \sin(\omega t)$$
$$1 \cdot \cos(\omega t) - 1 i \cdot \sin(\omega t)$$
$$0 \cdot \cos(\omega t) - \sqrt{2} i \cdot \sin(\omega t)$$
$$-1 \cdot \cos(\omega t) - 1 i \cdot \sin(\omega t)$$
$$-\sqrt{2} \cdot \cos(\omega t) - 0 i \cdot \sin(\omega t)$$
$$-1 \cdot \cos(\omega t) + 1 i \cdot \sin(\omega t)$$
$$0 \cdot \cos(\omega t) + \sqrt{2} i \cdot \sin(\omega t)$$

# Real Coefficients $C_3$ & $C_4$

$$c_1 e^{+i\omega t} + c_2 e^{-i\omega t} \quad \Longleftrightarrow \quad c_3 \cos(\omega t) + c_4 \sin(\omega t)$$

$$c_1 = (c_3 - c_4 i)/2$$
$$c_2 = (c_3 + c_4 i)/2$$

*conjugate complex number*

$R^2$

$+2 * real\ part$

$-2 * imag\ part$

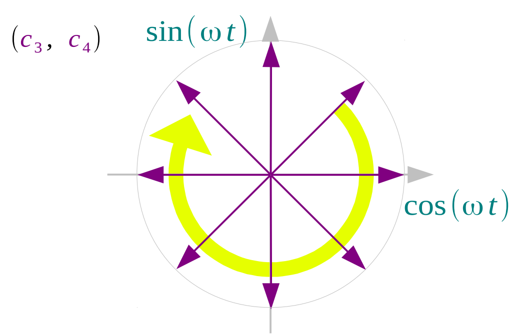
$$c_3 = (c_1 + c_2)$$
$$c_4 = i(c_1 - c_2)$$

*real number*

*real number*

| | |
|---|---|
| $\frac{(+1-0i)}{2} \cdot e^{+i\omega t} + \frac{(+1+0i)}{2} \cdot e^{-i\omega t}$ | $1 \cdot \cos(\omega t) + 0 \cdot \sin(\omega t)$ |
| $\frac{(+1-i)}{2\sqrt{2}} \cdot e^{+i\omega t} + \frac{(+1+i)}{2\sqrt{2}} \cdot e^{-i\omega t}$ | $\frac{1}{\sqrt{2}} \cdot \cos(\omega t) + \frac{1}{\sqrt{2}} \cdot \sin(\omega t)$ |
| $\frac{(0-i)}{2} \cdot e^{+i\omega t} + \frac{(0+i)}{2} \cdot e^{-i\omega t}$ | $0 \cdot \cos(\omega t) + 1 \cdot \sin(\omega t)$ |
| $\frac{(-1-i)}{2\sqrt{2}} \cdot e^{+i\omega t} + \frac{(-1+i)}{2\sqrt{2}} \cdot e^{-i\omega t}$ | $\frac{-1}{\sqrt{2}} \cdot \cos(\omega t) + \frac{1}{\sqrt{2}} \cdot \sin(\omega t)$ |
| $\frac{(-1-0i)}{2} \cdot e^{+i\omega t} + \frac{(-1+0i)}{2} \cdot e^{-i\omega t}$ | $-1 \cdot \cos(\omega t) + 0 \cdot \sin(\omega t)$ |
| $\frac{(-1+i)}{2\sqrt{2}} \cdot e^{+i\omega t} + \frac{(-1-i)}{2\sqrt{2}} \cdot e^{-i\omega t}$ | $\frac{-1}{\sqrt{2}} \cdot \cos(\omega t) + \frac{-1}{\sqrt{2}} \cdot \sin(\omega t)$ |
| $\frac{(0+i)}{2} \cdot e^{+i\omega t} + \frac{(0-i)}{2} \cdot e^{-i\omega t}$ | $0 \cdot \cos(\omega t) - 1 \cdot \sin(\omega t)$ |
| $\frac{(+1+i)}{2\sqrt{2}} \cdot e^{+i\omega t} + \frac{(+1-i)}{2\sqrt{2}} \cdot e^{-i\omega t}$ | $\frac{1}{\sqrt{2}} \cdot \cos(\omega t) - \frac{1}{\sqrt{2}} \cdot \sin(\omega t)$ |

*real plane*



# Real Coefficients $C_3$ & $C_4$

$$c_1 e^{+i\omega t} + c_2 e^{-i\omega t} \quad \Longleftrightarrow \quad c_3 \cos(\omega t) + c_4 \sin(\omega t)$$

$$c_1 = (c_3 - c_4 i)/2$$
$$c_2 = (c_3 + c_4 i)/2$$

conjugate complex number

$R^2$

$+2*real\ part$ $\quad c_3 = (c_1 + c_2)$ — real number

$-2*imag\ part$ $\quad c_4 = i(c_1 - c_2)$ — real number

$$A\cos(\omega t - \varphi) \quad \Longleftrightarrow \quad c_3 \cos(\omega t) + c_4 \sin(\omega t)$$

$$\sqrt{c_3^2 + c_4^2} = A$$

$$\frac{c_3}{\sqrt{c_3^2 + c_4^2}} = \cos(\varphi)$$

$$\frac{c_4}{\sqrt{c_3^2 + c_4^2}} = \sin(\varphi)$$

# $C^1$ and $R^2$ Spaces

$c_1 e^{+i\omega t} + c_2 e^{-i\omega t}$ ⟷ $c_3 \cos(\omega t) + c_4 i \sin(\omega t)$

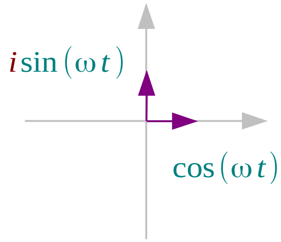
$c_1 = (c_3 + c_4)/2$ *real number*

$c_2 = (c_3 - c_4)/2$ *real number*

$c_3 = (c_1 + c_2)$ *real number*

$c_4 = (c_1 - c_2)$ *real number*

$C^1$




$c_1 e^{+i\omega t} + c_2 e^{-i\omega t}$ ⟷ $c_3 \cos(\omega t) + c_4 \sin(\omega t)$

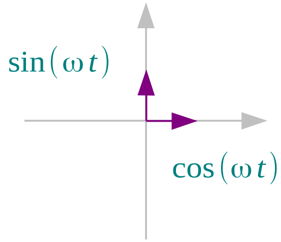
$c_1 = (c_3 - c_4 i)/2$ *conjugate*

$c_2 = (c_3 + c_4 i)/2$ *complex number*

$+2*real\ part$ $c_3 = (c_1 + c_2)$ *real number*

$-2*imag\ part$ $c_4 = i(c_1 - c_2)$ *real number*

$R^2$



# Signal Spaces and Phasors

$c_1 e^{+i\omega t} + c_2 e^{-i\omega t}$ ⟷ $c_3 \cos(\omega t) + c_4 \sin(\omega t)$

$c_1 = (c_3 - c_4 i)/2$

$c_2 = (c_3 + c_4 i)/2$

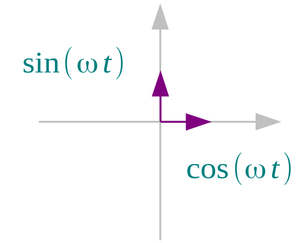
*conjugate*

*complex number*

$+2 * real\ part$ $c_3 = (c_1 + c_2)$ *real number*

$-2 * imag\ part$ $c_4 = i(c_1 - c_2)$ *real number*

$R^2$



# Causality

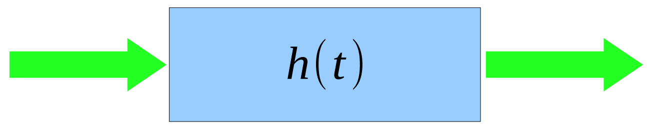
# Convolution Properties

# System Response to standard signals

# Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$x(t) = A\, e^{j\Phi}\, e^{j\omega t}$$




$$y(t) = H(jw) \cdot A\, e^{j\Phi}\, e^{j\omega t}$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jw(t-\tau)}\, d\tau$$

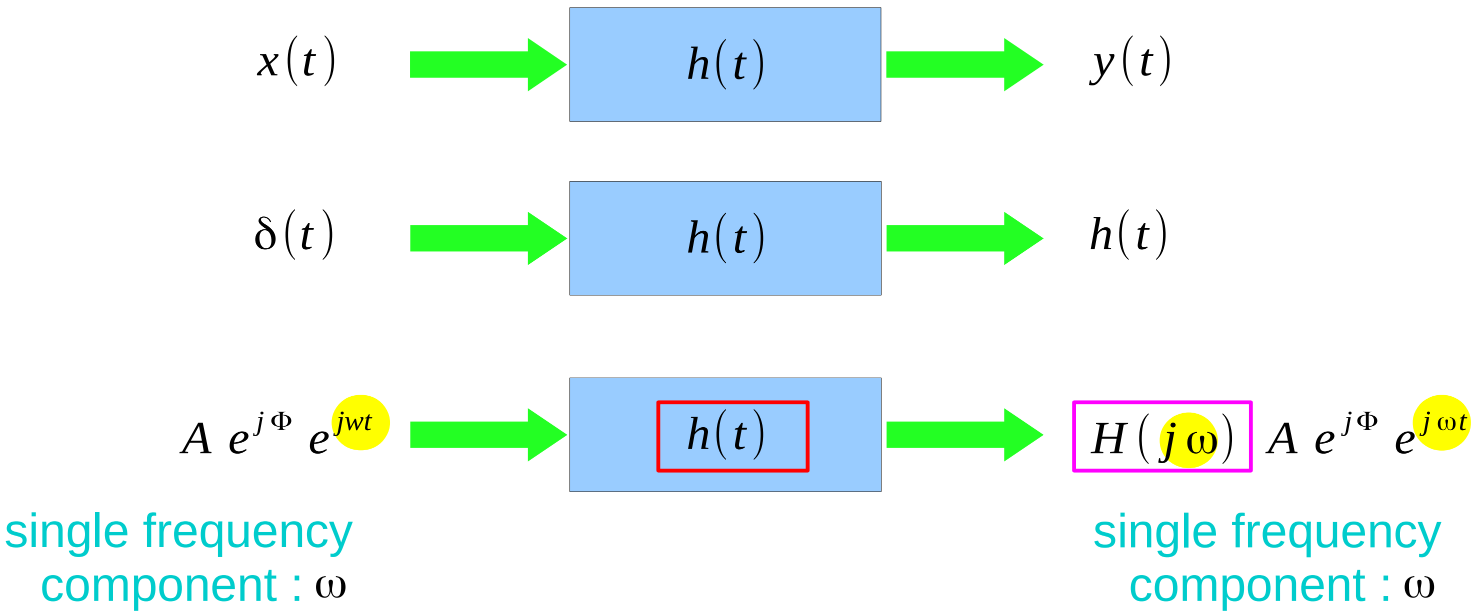
$$= \int_{-\infty}^{+\infty} h(\tau)\, A\, e^{j\Phi}\, e^{jwt}\, e^{-j\omega\tau}\, d\tau$$

$$= \underline{A\, e^{j\Phi}\, e^{jwt}} \cdot \underline{\int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau}$$

$$= \underline{x(t)} \cdot \underline{H(jw)}$$

# Frequency Response

$$y(t) = h(t) * x(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$




$$H(j\omega) = \int_{-\infty}^{+\infty} h(\tau)\, e^{-j\omega\tau}\, d\tau$$

# Transfer Function

# Block Diagram of Differential Equation

# Zero State Response & Convolution

# Green's Function & Convolution

# Green's Function

$$y'' + P(x)y' + Q(x)y = 0$$

$$y'' + P(x)y' + Q(x)y = f(x)$$

$$y'' + P(x)y' + Q(x)y = g(x)$$

$$y'' + P(x)y' + Q(x)y = h(x)$$

$y_1, \; y_2$ $\qquad W(t) = \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix}$

***the same Green's function***

$$G(x, \; t) = \left[ \frac{y_1(t) y_2(x) - y_1(x) y_2(t)}{W(t)} \right]$$

$$y_p = u_1(x) y_1 + u_2(x) y_2 = \int_{x_0}^{x} G(x, \; t) f(t) dt = \int_{x_0}^{x} \left[ \frac{y_1(t) y_2(x) - y_1(x) y_2(t)}{W(t)} \right] f(t) dt$$

# ZSR and Green's Function

$$y'' + 5y' + 6y = f(x)$$
$$y(x_0) = 0$$
$$y'(x_0) = 0$$

$$m^2 + 5m + 6 = (m+2)(m+3) = 0$$

$$m = -2, \ -3$$

$$y_1 = e^{-2t} \qquad y_2 = e^{-3t}$$

$$W(t) = \begin{vmatrix} e^{-2t} & e^{-3t} \\ -2e^{-2t} & -3e^{-3t} \end{vmatrix} = -e^{-5t}$$

$$G(x, t) = \left[\frac{y_1(t)y_2(x) - y_1(x)y_2(t)}{W(t)}\right]$$

$$= \left[\frac{e^{-2t}e^{-3x} - e^{-2x}e^{-3t}}{-e^{-5t}}\right]$$

$$= \left[-e^{3t}e^{-3x} + e^{-2x}e^{+2t}\right]$$

$$= \left[e^{-2(x-t)} - e^{-3(x-t)}\right]$$

$$= h(x-t)$$

$$y_p = \int_{x_0}^{x} h(x-t)f(t)\,dt$$

# Impulse Response → Zero Input Response

## References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003