Signal Processing

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

Based on

Signal Processing with Free Software : Practical Experiments
F. Auger

filter (1)

```
: y = filter (b, a, x)
: [y, sf] = filter (b, a, x, si)
: [y, sf] = filter (b, a, x, [], dim)
: [y, sf] = filter (b, a, x, si, dim)
```

https://octave.sourceforge.io/octave/function/filter.html

filter (2)

Apply a 1-D digital filter to the data x.

filter returns the solution to the following
linear, time-invariant difference equation:

$$\sum_{k=0}^{N} a(k+1)\, y(n-k) \;=\; \sum_{k=0}^{M} b(k+1)\, x(n-k) \qquad \mathit{for}\;\; 1 \le n \le \mathit{length}(x)$$

where N=length(a)-1 and M=length(b)-1.

$$\boldsymbol{a} = [a(1), a(2), \cdots, a(N+1)]$$
$$\boldsymbol{b} = [b(1), b(2), \cdots, b(M+1)]$$
$$\mathit{length}(\boldsymbol{a}) = N+1$$
$$\mathit{length}(\boldsymbol{b}) = M+1$$

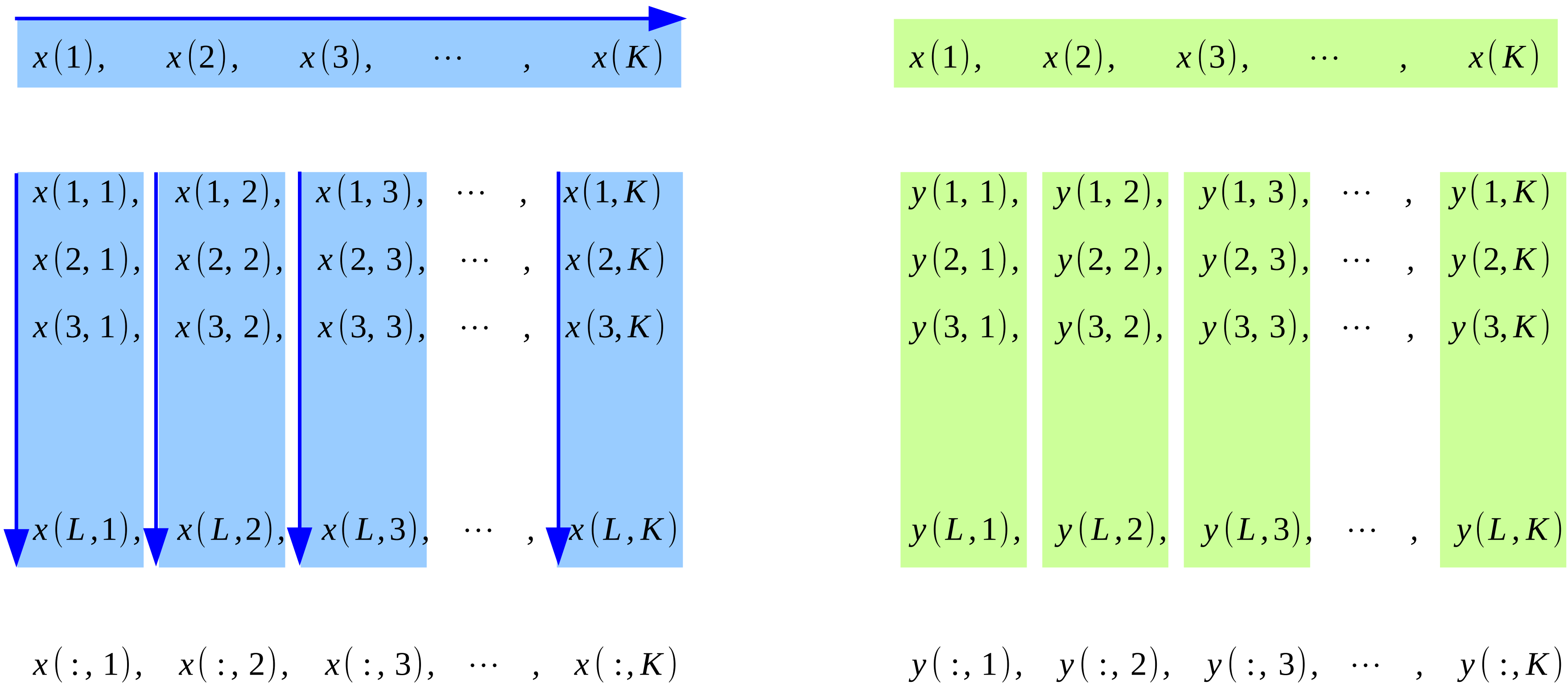
$$\boldsymbol{x} = [x(1), x(2), \cdots, x(L+1)]$$
$$\mathit{length}(\boldsymbol{x}) = L+1$$

$$1 \le n \le L+1$$

https://octave.sourceforge.io/octave/function/filter.html

filter (3)

The result is calculated over the **first** non-singleton dimension of x
or over **dim** if supplied.



https://octave.sourceforge.io/octave/function/filter.html

filter (4)

$$\sum_{k=0}^{N} a(k+1)\,y(n-k) = \sum_{k=0}^{M} b(k+1)\,x(n-k) \qquad \mathit{for}\ \ 1 \le n \le \mathit{length}(x)$$

$$a(1)\,y(n) + \sum_{k=1}^{N} a(k+1)\,y(n-k) = \sum_{k=0}^{M} b(k+1)\,x(n-k)$$

$$a(1)\,y(n) = -\sum_{k=1}^{N} a(k+1)\,y(n-k) + \sum_{k=0}^{M} b(k+1)\,x(n-k)$$

$$y(n) = -\sum_{k=1}^{N} \frac{a(k+1)}{a(1)}\,y(n-k) + \sum_{k=0}^{M} \frac{b(k+1)}{a(1)}\,x(n-k)$$

$$y(n) = -\sum_{k=1}^{N} c(k+1)\,y(n-k) + \sum_{k=0}^{M} d(k+1)\,x(n-k) \qquad \mathit{for}\,1 \le n \le \mathit{length}(x)$$

where c = a/a(1) and d = b/a(1).

https://octave.sourceforge.io/octave/function/filter.html

filter (5)

si : the initial state of the system
sf : the final state

the state vector is a column vector
whose length is equal to the length of
the longest coefficient vector – 1

No **si** is presented, the zero initial state.

in terms of the z transform,
y is the result of passing the discrete-time signal **x**
through a system characterized
by the following rational system function:

$$H(z) = \frac{\sum_{k=0}^{M} d(k+1) z^{-k}}{1 + \sum_{k=1}^{N} c(k+1) z^{-k}}$$

https://octave.sourceforge.io/octave/function/filter.html

freqz (1)

```
: [h, w] = freqz (b, a, n, "whole")
: [h, w] = freqz (b)
: [h, w] = freqz (b, a)
: [h, w] = freqz (b, a, n)
: h = freqz (b, a, w)
: [h, w] = freqz (..., Fs)
: freqz (...)
```

https://octave.sourceforge.io/octave/function/freqz.html

freqz (2)

Return the complex frequency response **h**
of the rational **IIR** filter
with the numerator coefficients **b** and
the denominator coefficients **a**

The response is evaluated
at **n** angular frequencies between **0** and **2*pi**.

The output value **w** is a vector of the frequencies.

h : the frequency response vector
w : the frequency vector

https://octave.sourceforge.io/octave/function/freqz.html

freqz (3)

If **a** is omitted, the denominator is assumed to be **1**
(this corresponds to a simple **FIR** filter).

If n is omitted, a value of 512 is assumed.
For fastest computation, n should factor
into a small number of small primes.

If the fourth argument, "**whole**", is omitted
the response is evaluated
at frequencies between **0** and **pi**.

https://octave.sourceforge.io/octave/function/freqz.html

freqz (4)

freqz (**b**, **a**, **w**)

Evaluate the response
at the specific frequencies in the vector **w**.
The values for **w** are measured in radians.

freqz (…)

Plot the magnitude and phase response
of **h** rather than returning them.

https://octave.sourceforge.io/octave/function/freqz.html

freqz (5)

[…] = **freqz** (…, Fs)

Return frequencies in Hz instead of radians
assuming a sampling rate Fs.
If you are evaluating the response
at specific frequencies **w**,
those frequencies should be requested
in Hz rather than radians.

[**h**, **w**] = **freqz** (**b**, **a**, n, "whole", Fs)
[**h**, **w**] = **freqz** (**b**, Fs)
[**h**, **w**] = **freqz** (**b**, **a**, Fs)
[**h**, **w**] = **freqz** (**b**, **a**, n, Fs)
h = **freqz** (**b**, **a**, **w**, Fs)

https://octave.sourceforge.io/octave/function/freqz.html

freqz_plot

: **freqz_plot** (**w**, **h**)
: **freqz_plot** (**w**, **h**, freq_norm)

Plot the magnitude and phase response of **h**.

If the optional freq_norm argument is true,
the frequency vector **w** is in units of normalized radians.
If freq_norm is false, or not given,
then **w** is measured in Hertz.

https://octave.sourceforge.io/octave/function/freqz_plot.html

conv

: **conv** (**a**, **b**)
: **conv** (**a**, **b**, shape)

Convolve two vectors **a** and **b**.

The output convolution is a vector
with length equal to length (**a**) + length (**b**) - 1.
When **a** and **b** are the coefficient vectors of two polynomials,
the convolution represents
the coefficient vector of the product polynomial.

The optional shape argument may be
shape = "full"
Return the full convolution. (default)
shape = "same"
Return the central part of the convolution with the length(**a**).

https://octave.sourceforge.io/octave/function/conv.html

fftconv

```
: fftconv (x, y)
: fftconv (x, y, n)
```

Convolve two vectors using the FFT for computation.

c = **fftconv** (**x**, **y**) returns
a vector of length equal to length(**x**) + length(**y**) - 1

If **x** and **y** are the coefficient vectors of two polynomials,
the returned value is the coefficient vector of the product polynomial.

The computation uses the FFT
by calling the function **fftfilt**.

If the optional argument n is specified,
an n-point FFT is used.

https://octave.sourceforge.io/octave/function/fftconv.html

deconv

: **deconv** (**y**, **a**)

Deconvolve two vectors.

[**b**, **r**] = **deconv** (**y**, **a**) solves for **b** and **r** such that **y** = **conv** (**a**, **b**) + **r**.

If **y** and **a** are polynomial coefficient vectors,
b will contain the coefficients of the polynomial quotient and
r will be a remainder polynomial of lowest order.

https://octave.sourceforge.io/octave/function/deconv.html

Moving Average Filter

```
t = 0: 1/100 : 1;
s = sin(2 * pi * t);
x  = (s > 0);                    % 1 or 0
x = (x – 0.5) * 2;               % -1 or +1
x = [x 0 0 0];                   % zero padding for i+1, i+2, i+3

for i=1 : length(x)-3
    y(i) = (x(i) + x(i+1) + x(i+2) + x(i+3)) / 4;
endfor

hold
plot(t, s, ‘g’)
plot(t, x)
plot(t, y, ‘m--’);
```

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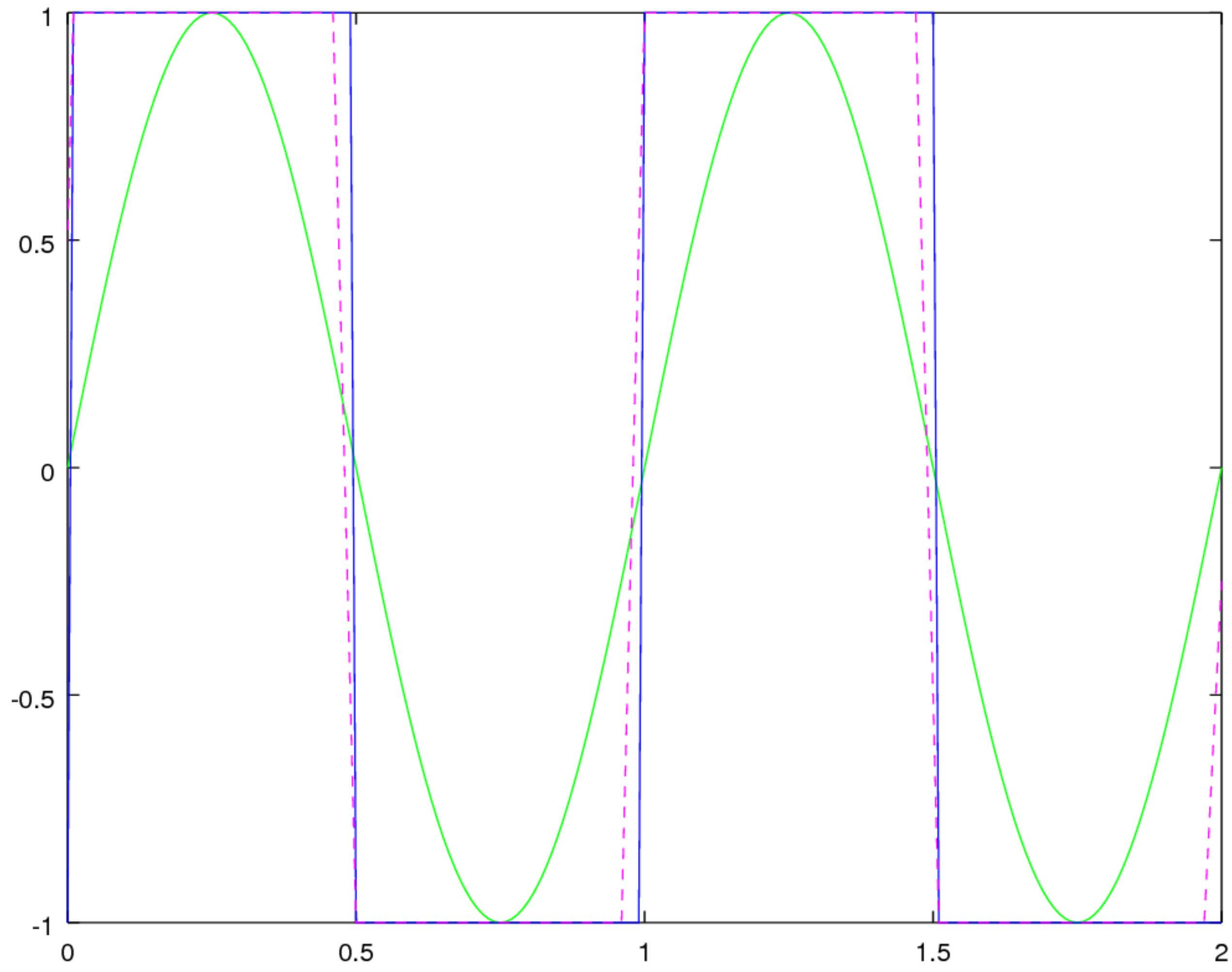
Moving Average Filter



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Impulse Response

```
fs=44100;
t=0: 1/fs :1-1/fs;
k = fs * t ;

x=sin(2*pi*10*t);

i=zeros(1, length(t));
i(1)=0.5;

I=fft(i);
X=fft(x);
Y = I .* X;
y = real(ifft(Y));
```

```
clf;
plot(t, x, t, y, 'g')

source "../0.util.octave/util.m"
pause
clf;

TimeFreqPlot(t, x, k, X, 'half')
pause;
TimeFreqPlot(t, i, k, I, 'half')
pause;
```

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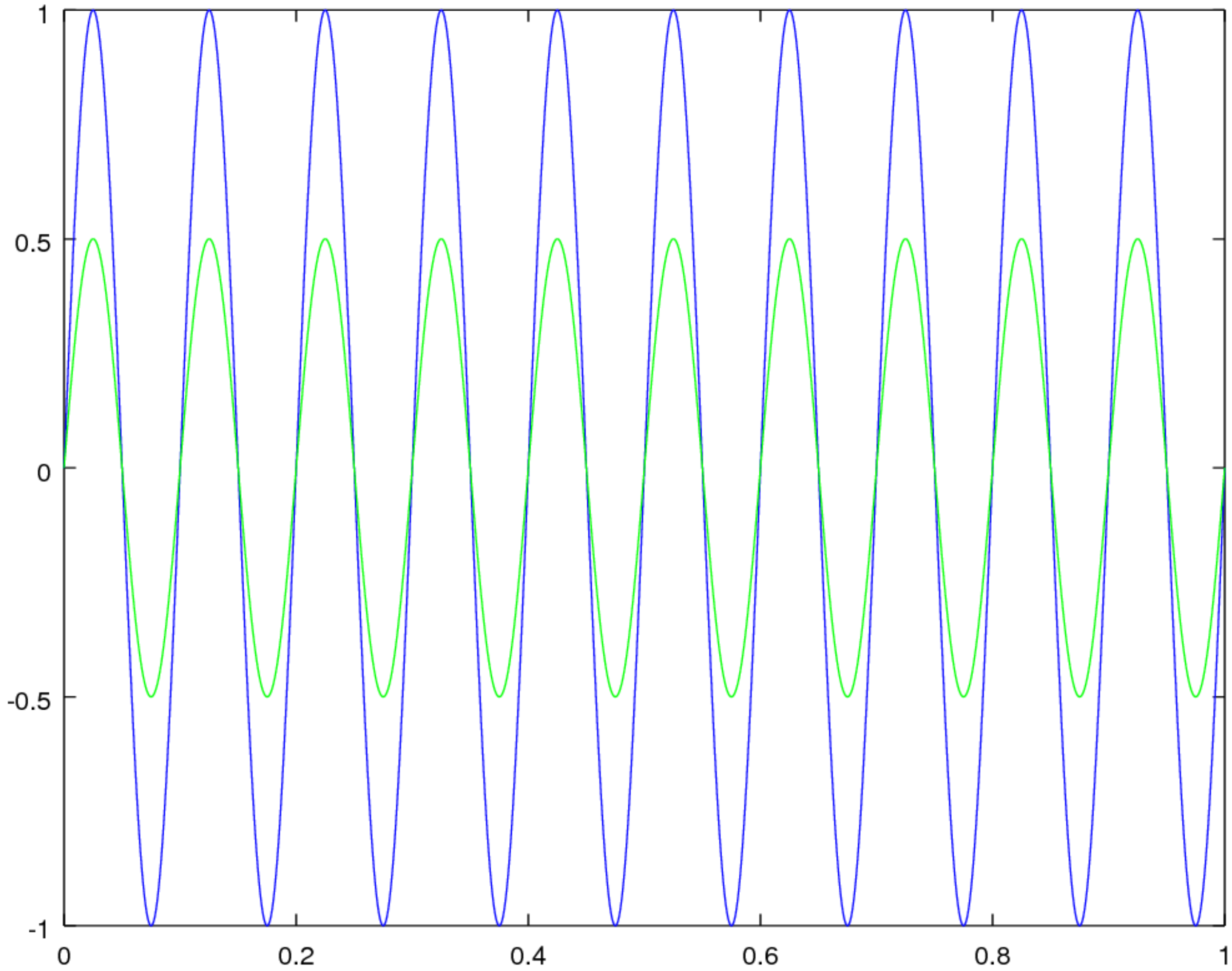
Impulse Response



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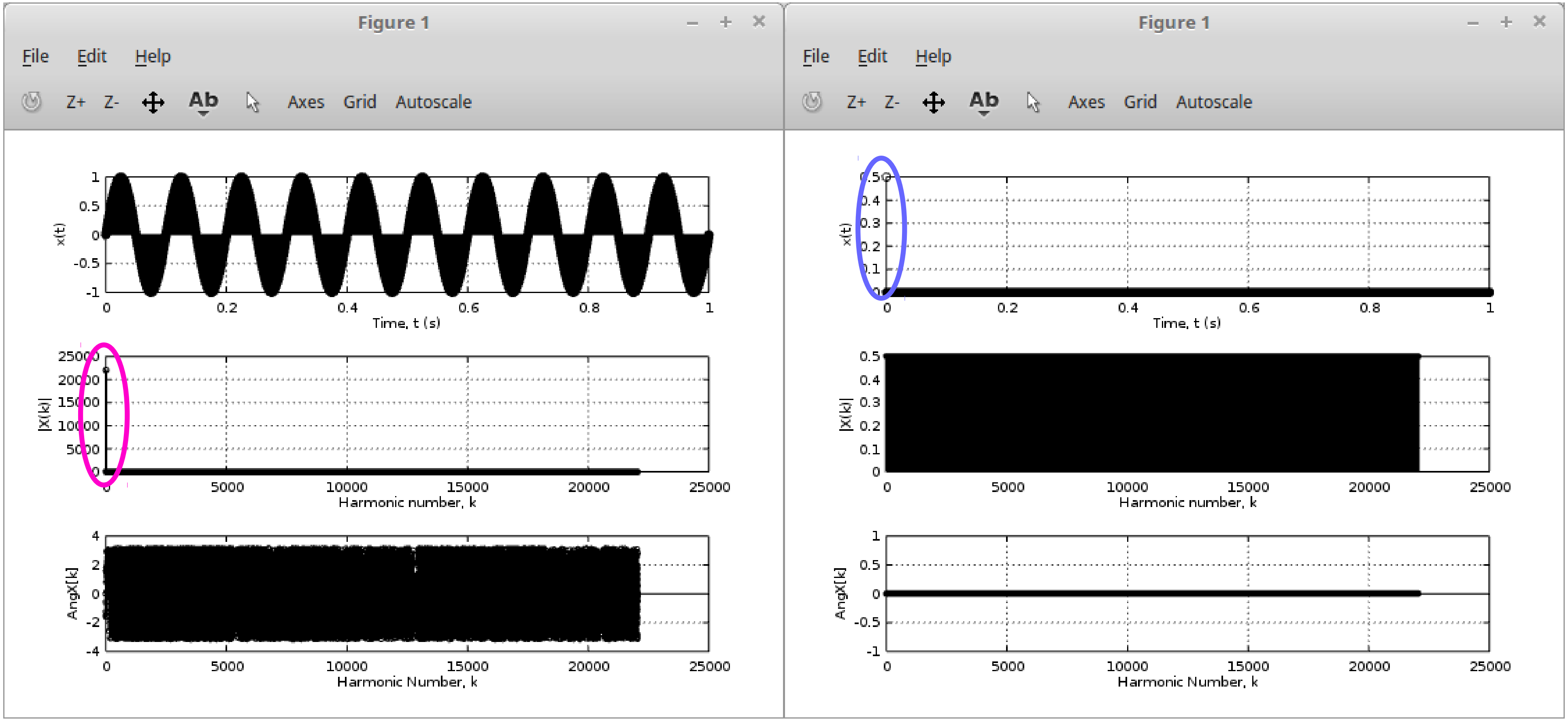
Impulse Response



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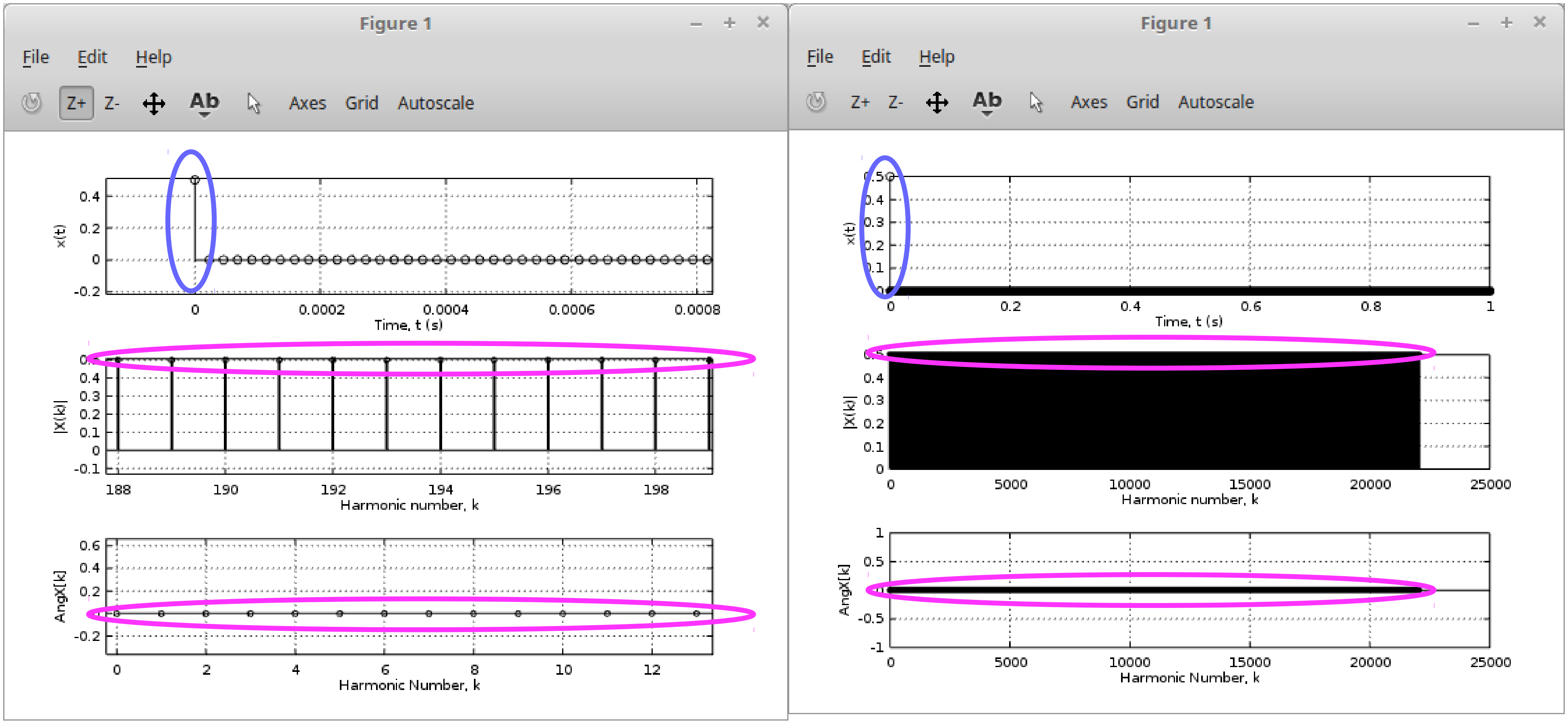
Impulse Response



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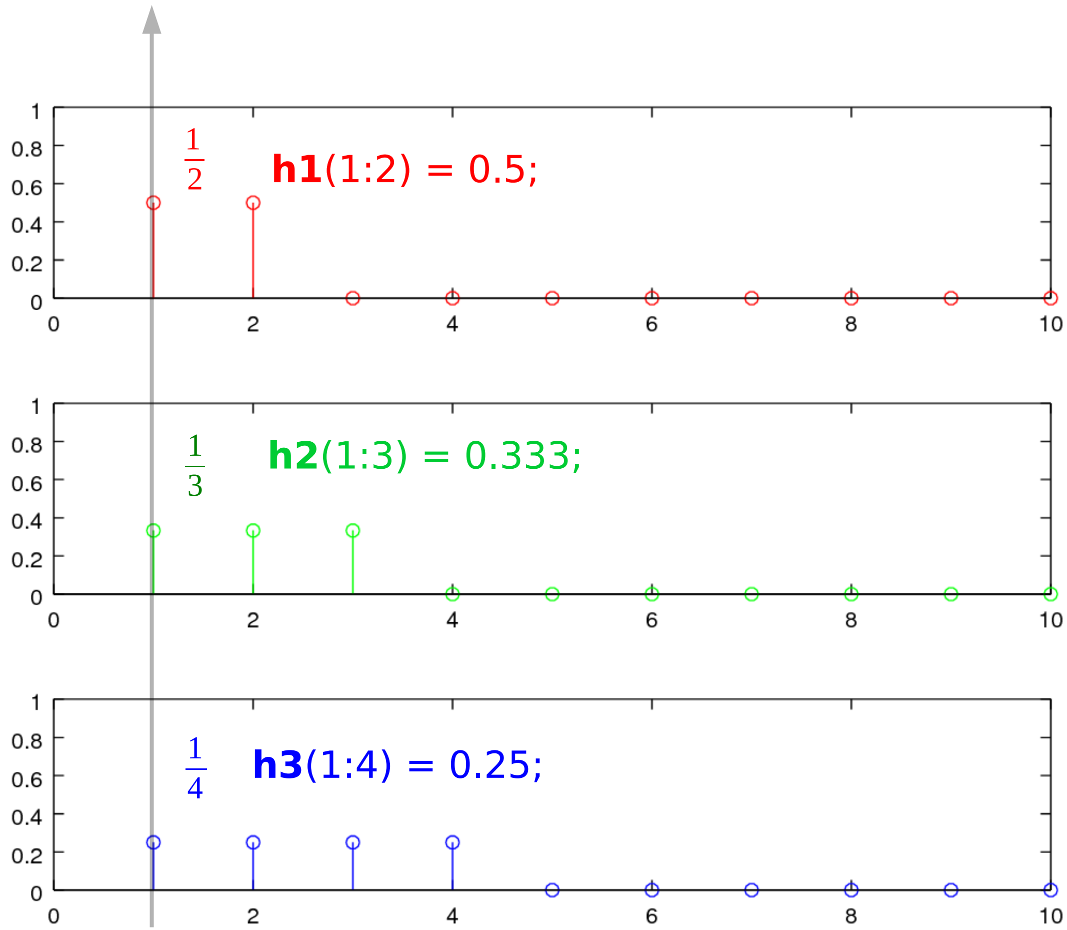
FIR - Low Pass filter

```
h1 = zeros(1, 44100);
h1(1:2) = 0.5;
H1 = abs(fft(h1));
H1 = H1(1: 22050);
plot(H1, 'r'); hold on;
```

```
h2 = zeros(1, 44100);
h2(1:3) = 0.333;
H2 = abs(fft(h2));
H2 = H2(1: 22050);
plot(H2, 'g'); hold on;
```

```
h3 = zeros(1, 44100);
h3(1:4) = 0.25;
H3 = abs(fft(h3));
H3 = H3(1: 22050);
plot(H3, 'b'); hold on;
```



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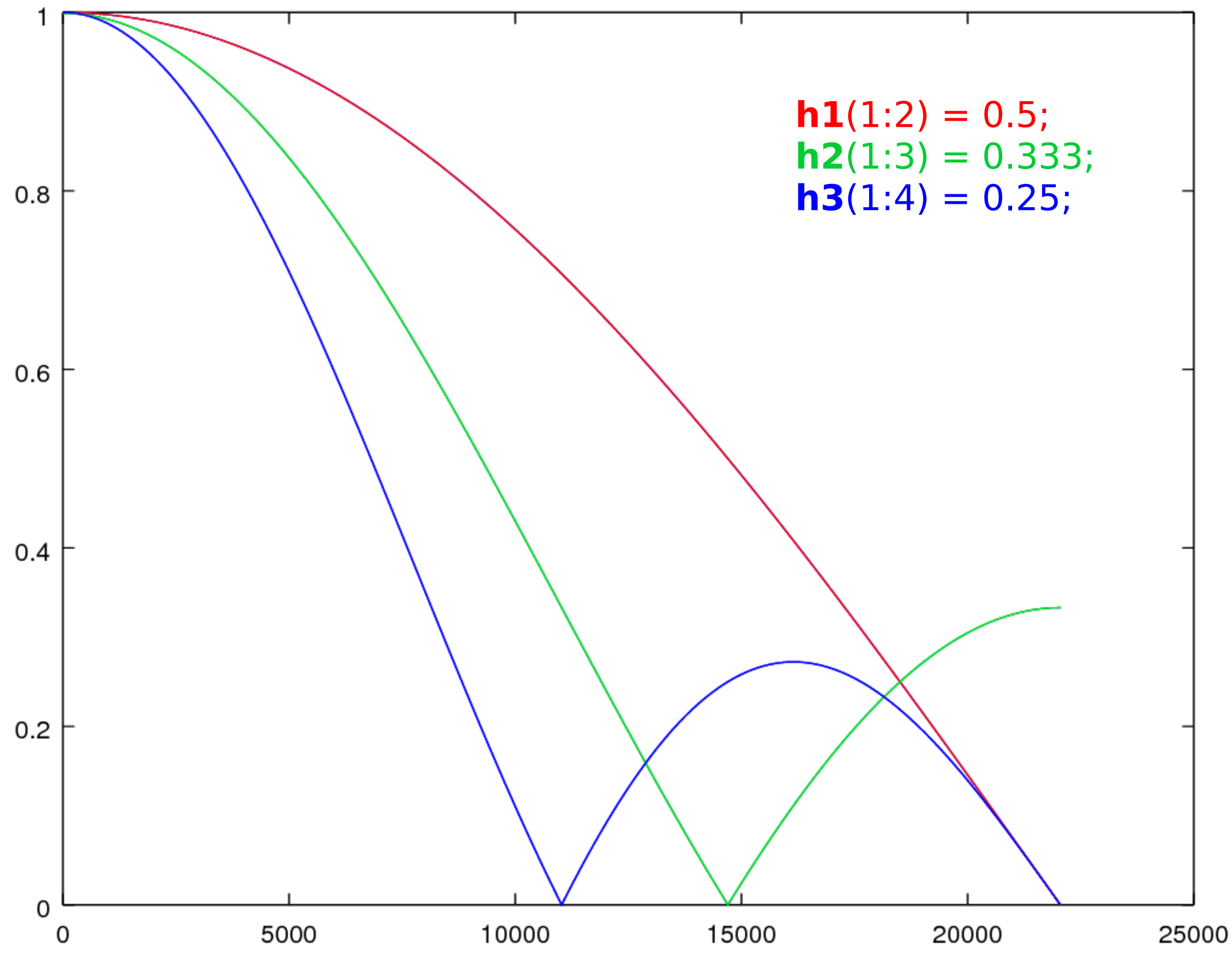
FIR - Low Pass filter



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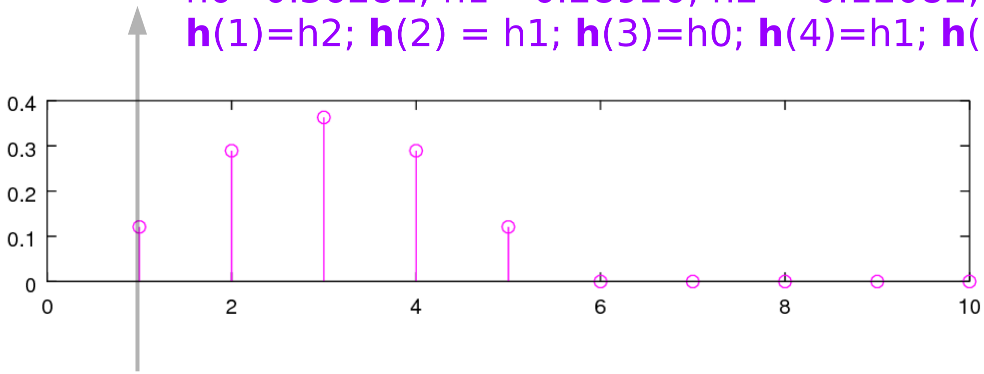
FIR - Low Pass filter

```
h0=0.36281; h1= 0.28920; h2 = 0.12082;

h = zeros(1, 44100);

h(1)=h2; h(2) = h1; h(3)=h0; h(4)=h1; h(5)=h2;
H = abs( fft(h) );
H = H(1: 44100/2);
plot(H, 'v')
```

h0=0.36281; h1= 0.28920; h2 = 0.12082;
h(1)=h2; **h**(2) = h1; **h**(3)=h0; **h**(4)=h1; **h**(5)=h2;



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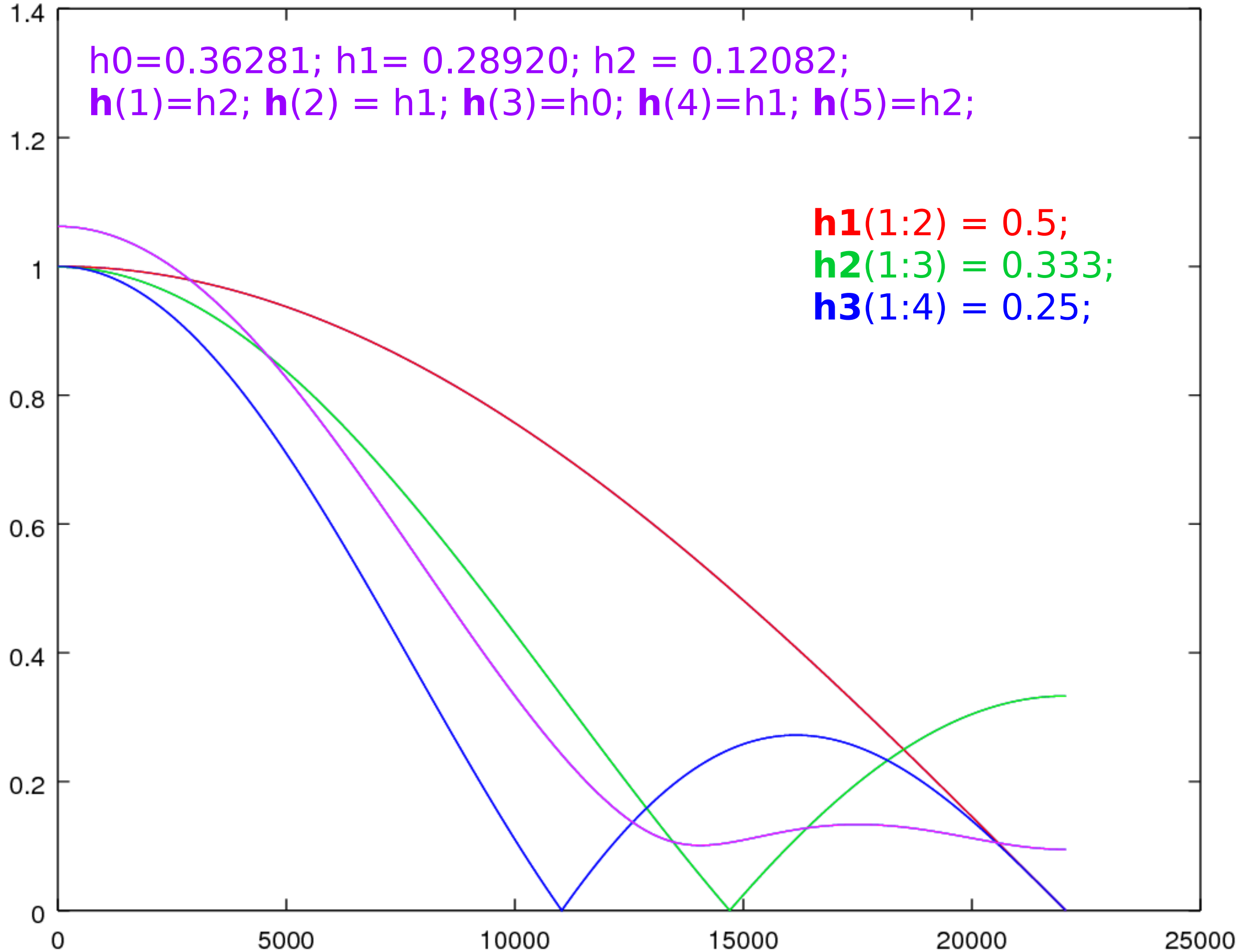
FIR - Low Pass filter



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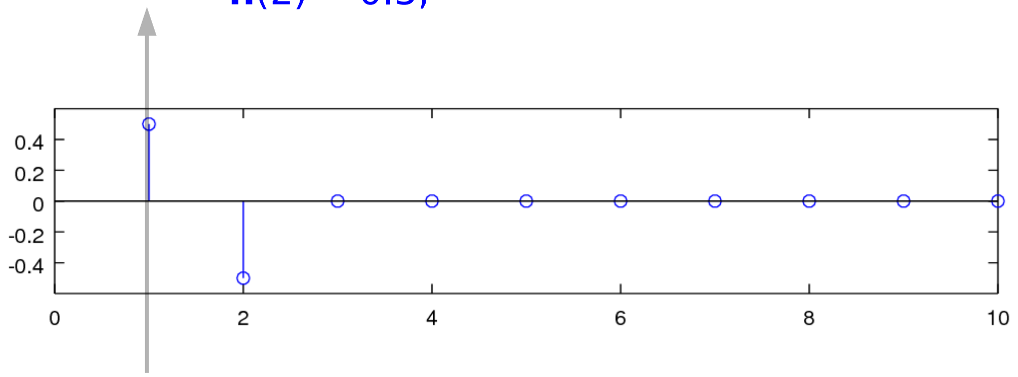
FIR - High Pass filter

```
h=zeros(1, 44100);
h(1)=0.5;
h(2)=-0.5;
H=abs(fft(h));
H = H(1: 22050);
plot(H);
```

h(1)=0.5;
h(2)=**-**0.5;



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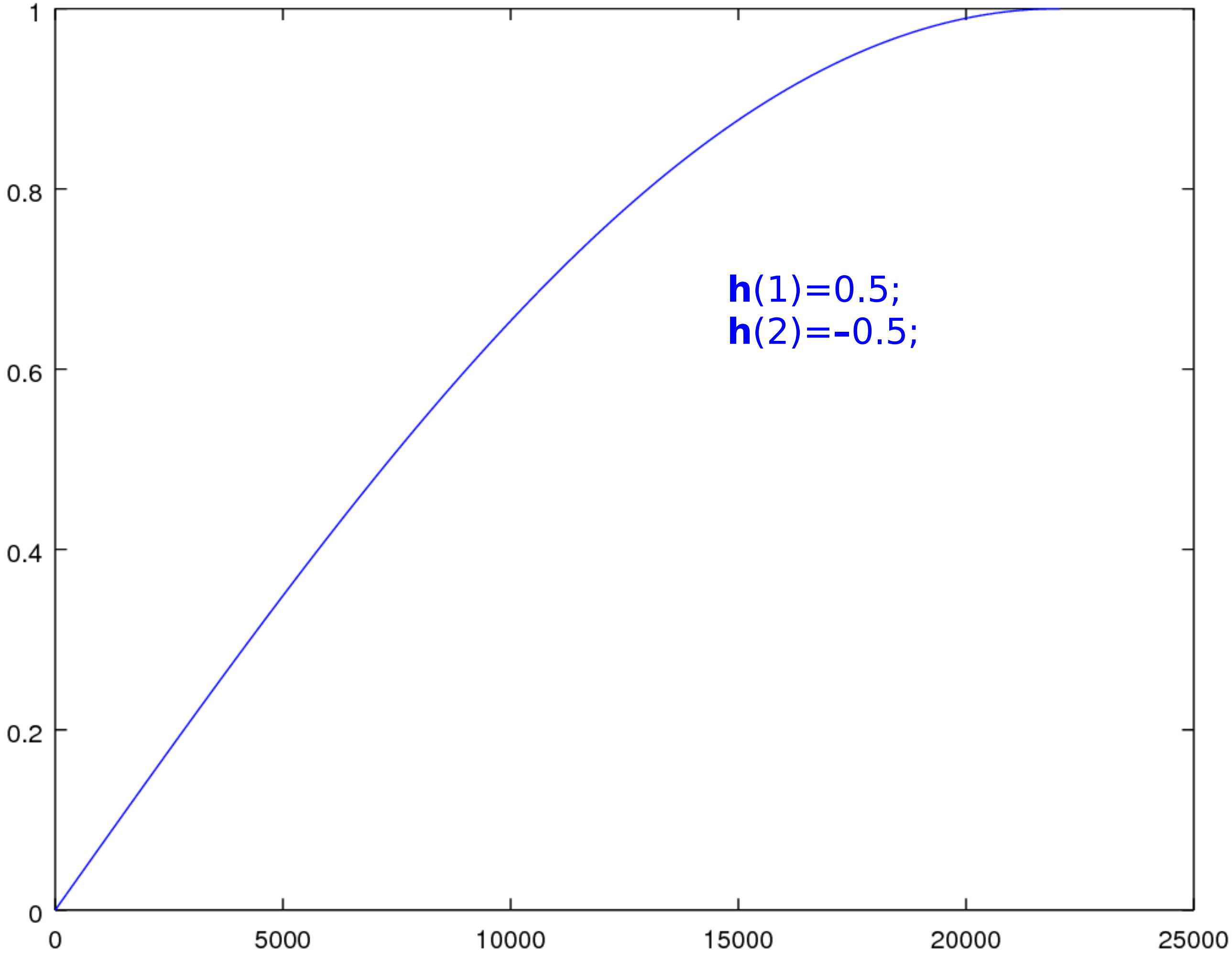
FIR - High Pass filter



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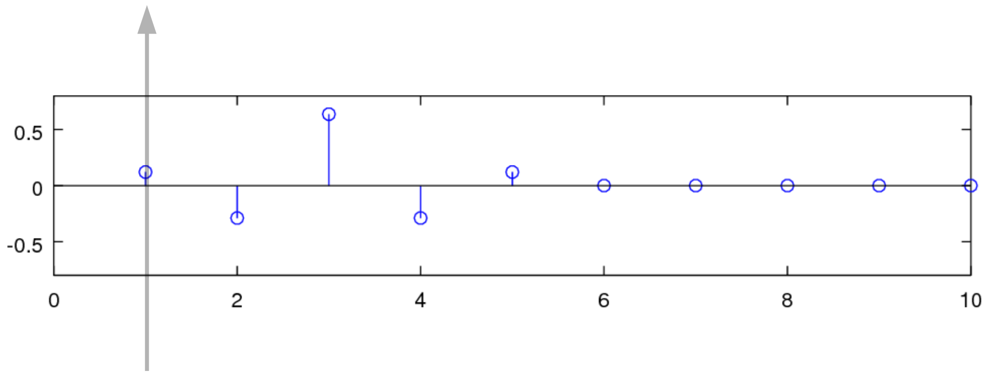
FIR - High Pass filter

```
h0=0.63719; h1 = 0.28920; h2 = 0.12082;
h = zeros(1, 44100);
h(1) = h2; h(2) = -h1; h(3) = h0; h(4)=-h1; h(5)=h2;
H = abs(fft(h));
H = H(1: 44100/2);
plot(H);
```



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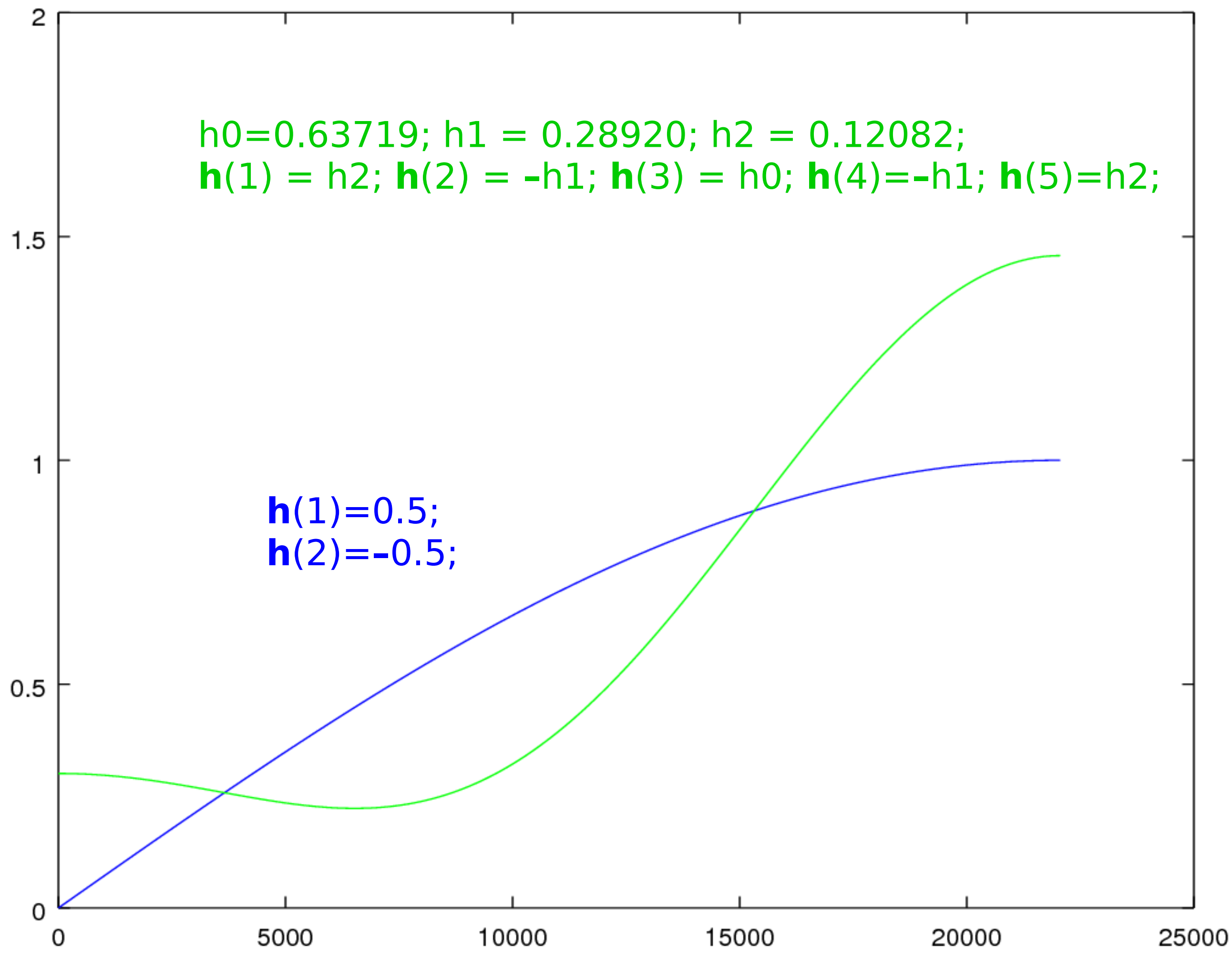
FIR - High Pass filter



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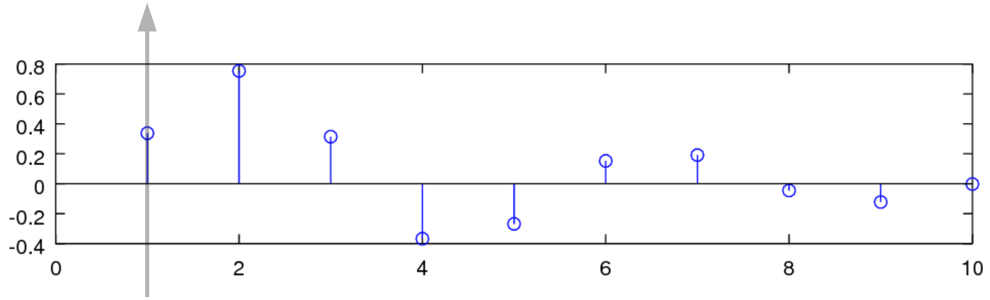
IIR - Low Pass filter

```
a0 = 0.338240392840684;
a1 = 0.676480785681368;
a2 = 0.338240392840684;
b1 =-0.230411611089973;
b2 = 0.58337275845273;

x= zeros(1, 44100);
x(1)=1;

y(1) = a0*x(1);
y(2) = a0*x(2) + a1*x(1) – b1*y(1);
for i=3:length(x)
  y(i) = a0*x(i) + a1*x(i-1) + a2*x(i-2) – b1*y(i-1) – b2*y(i-2);
endfor

Y= abs(fft(y));
Y= Y(1:length(y)/2);
plot(Y);
```



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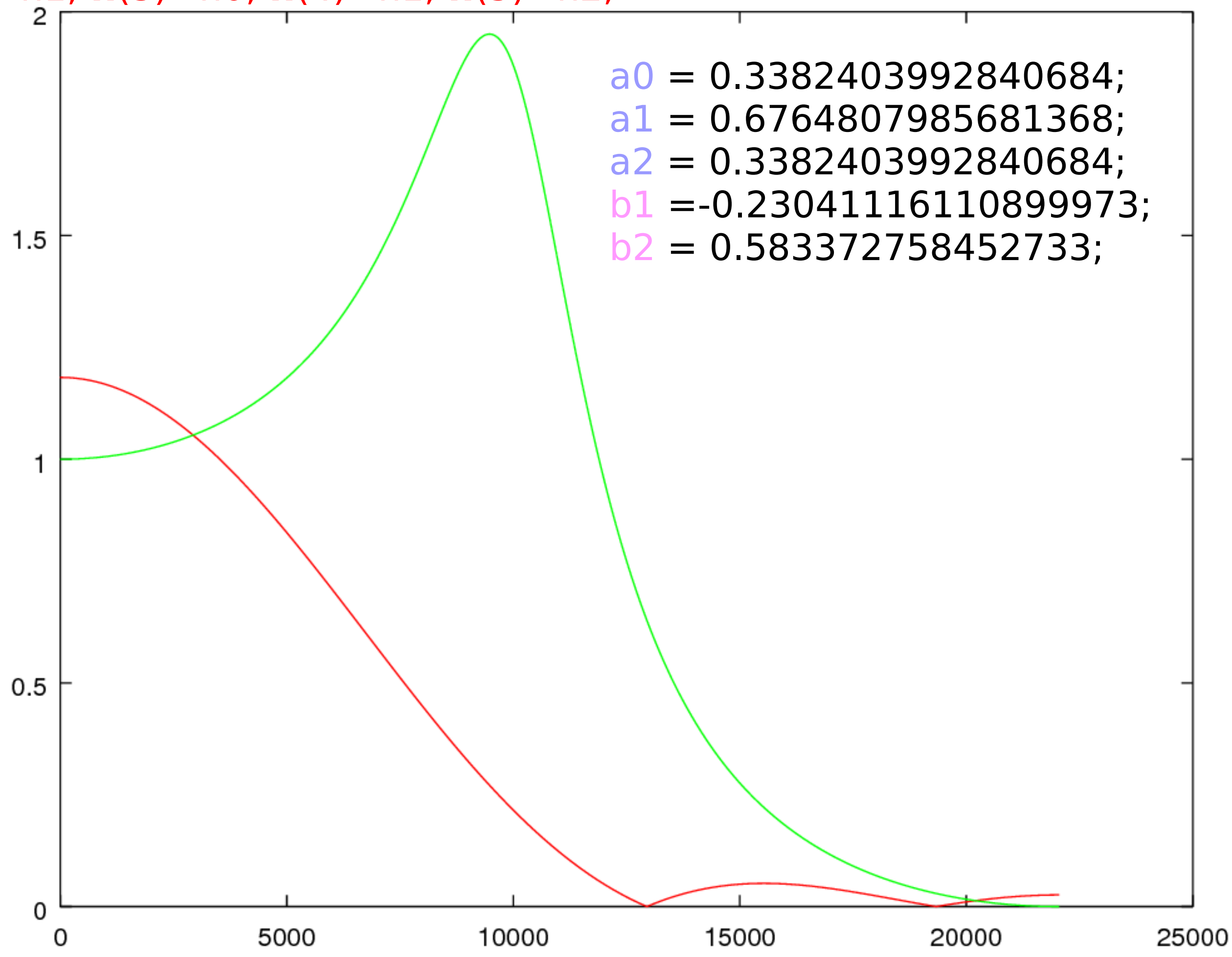
IIR - Low Pass filter

h0=0.36281; h1= 0.28920; h2 = 0.12082;
h(1)=h2; **h**(2) = h1; **h**(3)=h0; **h**(4)=h1; **h**(5)=h2;



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Moving Average Filter

```
a0 = 0.5;
a1 = 0.5;

x= zeros(1, 44100);
x(1)=1;

y(1) = a0*x(1);
for i=2:length(x)
  y(i) = a0*x(i) + a1*x(i-1);
endfor

Y= abs(fft(y));
Y= Y(1:length(y)/2);
plot(Y);
```

```
[y, fs]= waveread('t.wav');
y=y';
y2=[y 0];

for i=1:length(y)
  yy(i) = (y2(i) + y2(i+1)) / 2;
endfor

hold
plot(y, 'linewidth', 5);
plot(yy, 'm', 'linewidth', 2);
axis([100, 140, -1, 1])

sound(y, fs)
sound(yy, fs);
```

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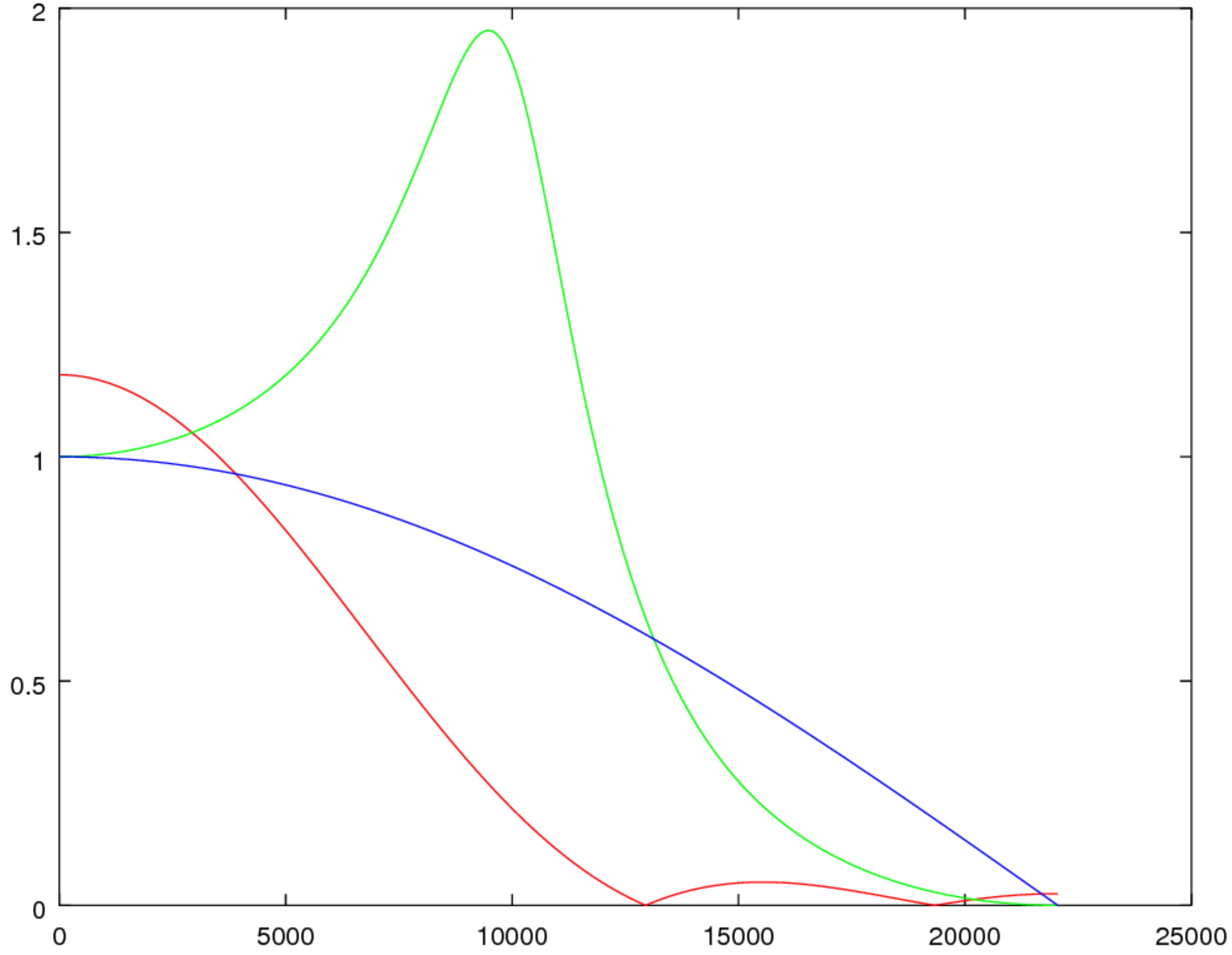
Moving Average Filter



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Moving Average Filter - FFT

```
Y = abs(fft(y));
Y= Y(1:length(y)/2);
YY= abs(fft(yy));
YY= YY(1:length(yy)/2);

f=fs*(0:length(y)/2 -1)/length(y)

hold;
plot(f, Y, 'linewidth', 2);
plot(f, YY, 'y', 'linewidth', 0.5)
axis([3200, 15000, 0, 410]);
```

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Ideal Low Pass Filter

$$\frac{\sin(x)}{x} \qquad \frac{\sin(\pi x)}{\pi x}$$

$$H(f) = \operatorname{rect}\left(\frac{f}{2B}\right)$$

$$h(t) = \mathcal{F}^{-1}\{H(f)\} = 2B\frac{\sin(2\pi Bt)}{2\pi Bt} = 2B\,\operatorname{sinc}(2Bt)$$

$$h_{LPF}(t) = 2B_L\,\operatorname{sinc}(2B_L t)$$

$$H_{LPF}(f) = \operatorname{rect}\left(\frac{f}{2B_L}\right).$$

$$W_L = 2\pi\frac{8000}{44100} = 1.1.398$$

$$H_0 = \frac{W_L}{\pi} = 0.36281$$

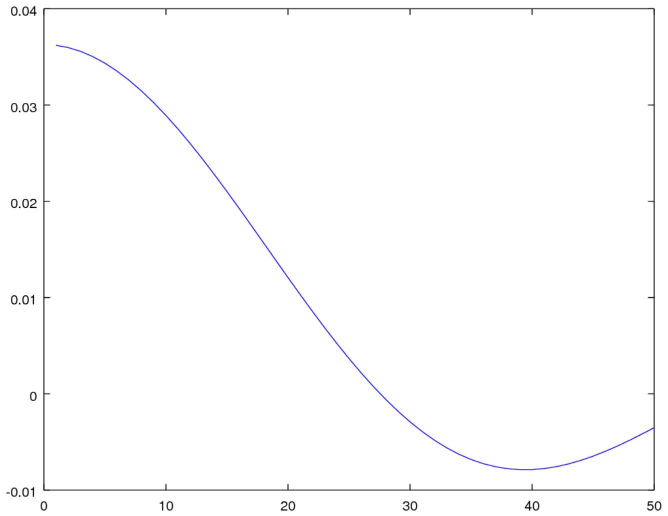
$$H_m = H_{-m} = \frac{\sin(m\,W_L)}{m\pi}$$

https://en.wikipedia.org/wiki/Sinc_filter

Low Pass Filter - Manual

```
w = 2 * pi * (800/44100);
h0 = w / pi;
for i=1:50
  h(i) = sin(i*w) / (i*pi);
endfor
g=zeros(1, 44100);
g(51)=h0;

for i=1:101
  if (i>51)
     g(i) = h(i-51);
  elseif (i <51)
     g(i) = h(51-i);
  endif
endfor
```




```
G=abs(fft(g));
G=G(1:22050);
plot(G);
```

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Low Pass Filter

```
fc = input("input fc: ");
delay = input("input delay number : ");
dv=ceil(delay/2 +1);
w = 2*pi*(fc/44100);

for i=1:ceil(delay/2)
  h(i) = sin(i*w) / (i*pi);
endfor

h_lo=zeros(1, 44100);
h_lo(dv)= h0;
for d=1:(delay+1)
  if (d>dv)
    h_lo(d) = h(dv-d);
  endif
endfor
```

```
H_lo=abs(fft(lo));
H_lo=H_lo(1:22050);
plot(H_lo)
pritnf("save lpf");
disp("");
```

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High Pass Filter

```
fc = input("input fc: ");
delay = input("input delay number : ");
g = input("input gain : ");
w1 = 2*pi*(fc/44100);
w2 = w2 / pi;

for i=1:ceil(delay/2)
  h(i) = sin(i*w2) / (i*pi);
endfor

h_hi=zeros(1, 44100);
h_hi(dv)= h0;
for d=1:(delay+1)
  if (d>dv)
    h_hi(d) = h(dv-d);
  endif
endfor
```

```
for n=1:length(hi);
  if (mod(n,2) == 0)
    h_hi(n) = -1 * h_hi(n);
  endif
endfor

H_hi = abs(fft(h_hi));
H_hi = g*H_hi(1:22050);
plot(H_hi)
printf("save hpf")
disp("");
```

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HPF and LPF

```
Y = 4 * (H_hi * H_lo);
plot(Y);
axis([fc -1000, fc+1000, 0, g])
```

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Peak Filter

```
a0 = 1.0130557885385833;
a1 = -1.465715424005588;
a2=0.923649194879148;
b1=-1.465715424005588;
b2=0.9367049680264983;

x = zeros(1, 44100);
x(1)=1;
y(1)=a0*x(1);
y(2)=a0*x(2) + a*x(1) – b1*y(1);
for i=3:length(x)
  y(i)=a0*x(i) + a1*x(i-1) + a2*x(i-2)
        -b1*y(i-1) – b2*y(i-2);
endfor

Y = abs(fft(y));
Y = Y(1:length(y)/2);
plot(Y);
```

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One Pole Filter

```
a0=0.4344299454321238;
a1=0;
a2=0;b1=-0.56558005455678762;
B2=0;

x=zeros(1, 44100);
x(1)=1;
y(1)=a0*x(1);
y(2)=a0*x(2) + a1*x(1) – b1*y(1);
for i=3:length(x)
  y(i)=a0*x(i) + a1*x(i-1) + a2*x(i-2)
        - b1*y(i-1) – b2*y(i-2);
endfor

Y=abs(fft(y));
Y= Y(1:length(y)/2);
```

```
[xn, fs] = waveread('t.wav');
xn = xn';
XN=abs(fft(xn));
XN=XN(1:22050);
XN=XN/max(XN);

XF= XN .* Y;
hold
plot(XN, 'm');
plot(XF, 'g');
plot(Y, 'linewidth', 5);

y2=real(ifft(XF));

wavewrite(y2, 44100, 'noise.wav');
```

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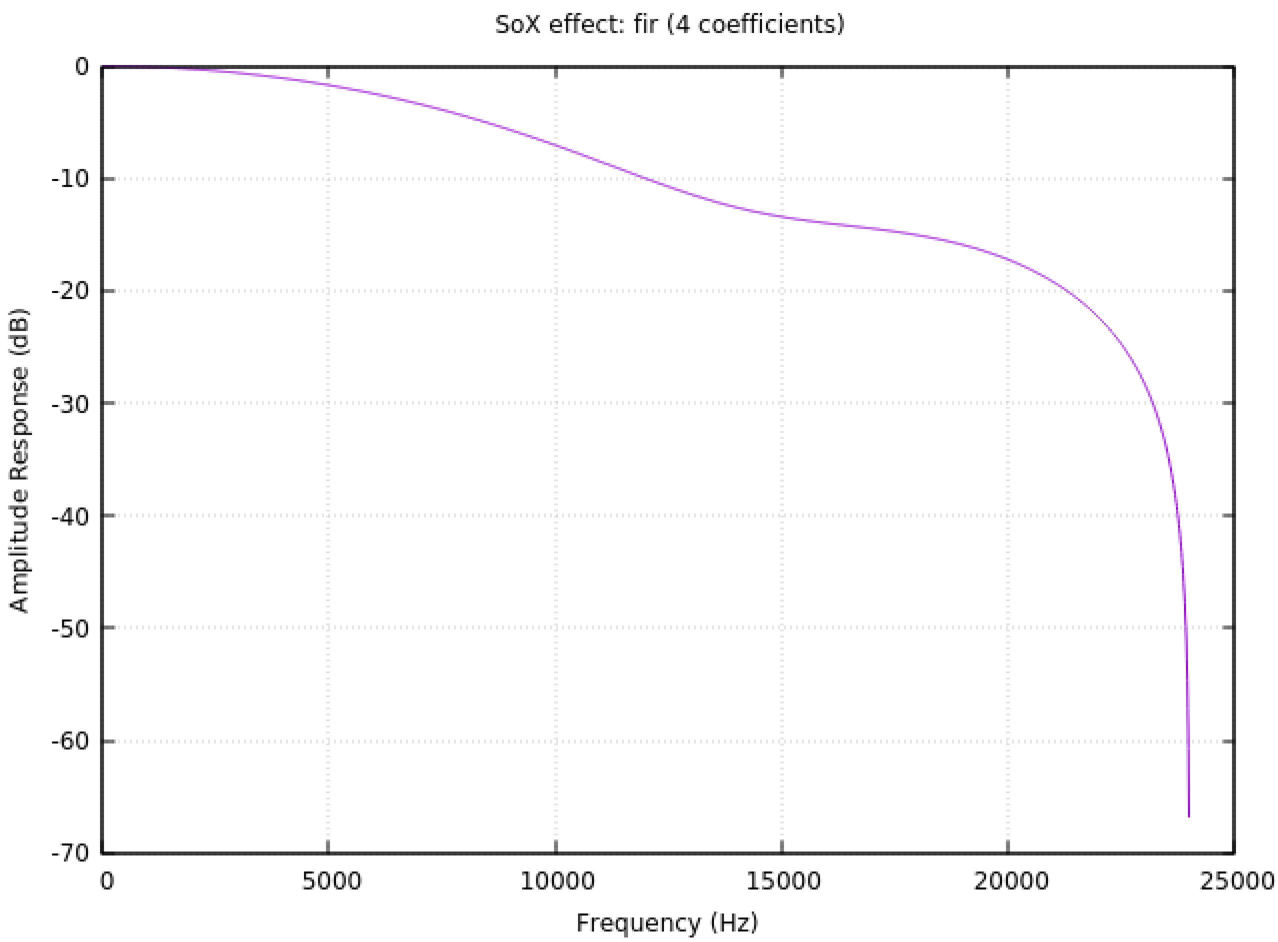
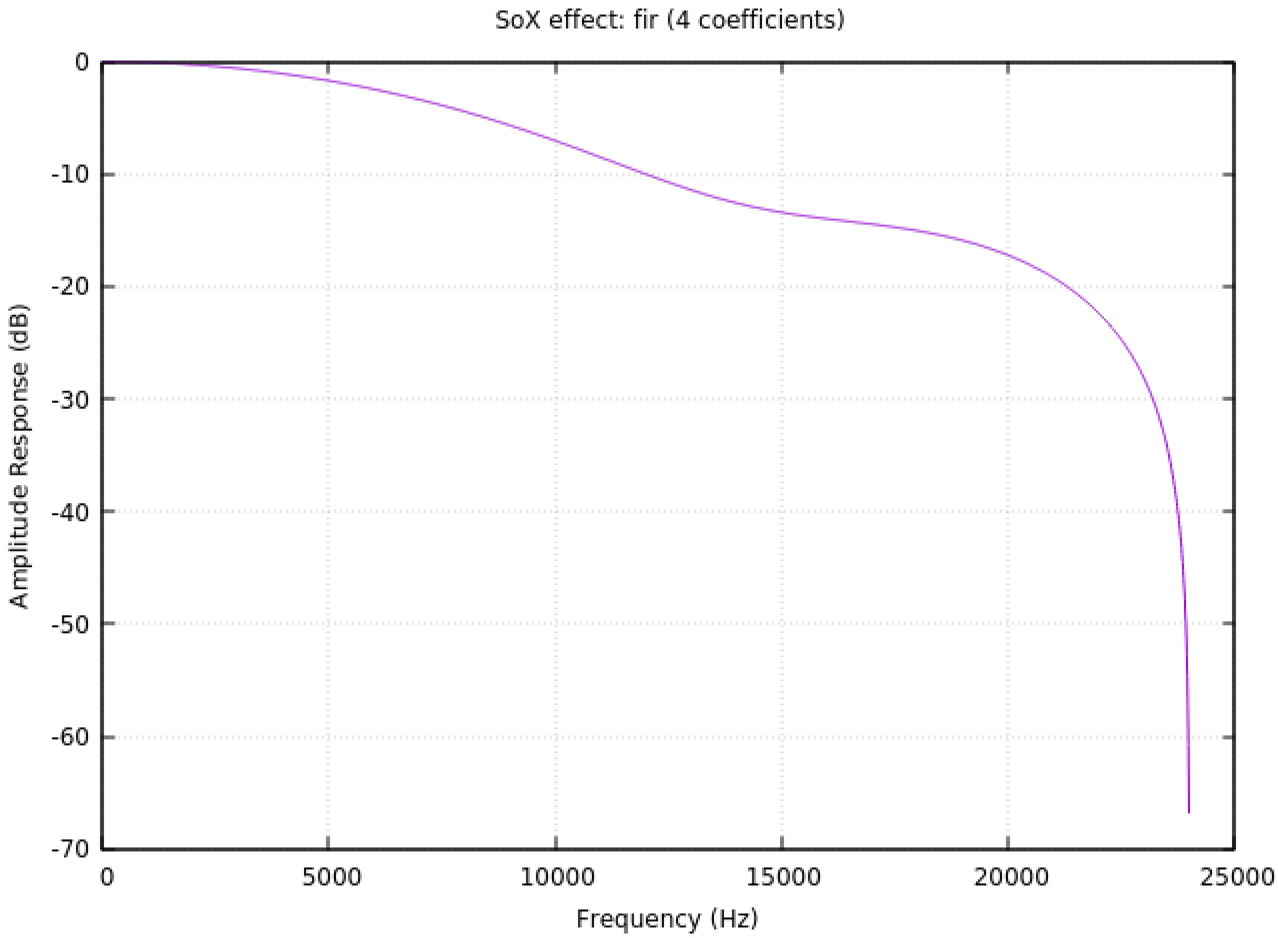
--plot gnuplot | octave

```
sox --plot gnuplot s6s.wav -n fir 0.1 0.2 0.4 0.3            >fir1.plt
sox --plot gnuplot s6s.wav -n fir coeff.txt                  >fir2.plt
sox --plot gnuplot s6s.wav -n biquad .6 .2 .4 1 -1.5 .6 >fir3.plt
sox --plot gnuplot s6s.wav -n fir 0.2 0.2 0.2 0.2 0.2    >fir4.plt
```

--plot gnuplot | octave



--plot gnuplot | octave



References

[1] F. Auger, Signal Processing with Free Software : Practical Experiments