Radar

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Definition

$$P_{fa} = \int_{W_T}^{\infty} f_0(w)dw$$

$$P_d = \int_{W_T}^{\infty} f_1(w)dw$$

Definition

$$f_R(r) = \frac{u(r)}{\sigma^2} r e^{-r^2/2\sigma^2}$$

$$W_t = g(R_r)$$

$$R_T = g^{-1}(W_T)$$

Definition

$$P_{fa} = \int_{W_f}^{\infty} f_0(w)dw = \int_{R_T}^{\infty} f_R(r)dr$$

$$= \int_{R_T}^{\infty} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} dr = e^{-R_f^2/2\sigma^2}$$

$$R_T = \left[2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)\right]^{1/2}$$

$$W_T = g\left(\left[2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)\right]^{1/2}\right)$$

Definition

$$P_d = \int_{W_f}^{\infty} f_1(w)dw = \int_{R_T}^{\infty} f_R(r)dr$$

$$= \int_{\left[2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)\right]^{1/2}}^{\infty} \frac{r}{\sigma^2} I_o\left(\frac{r^2 A_0}{\sigma^2}\right) e^{-(r^2+A_0^2)/2\sigma^2} dr$$

$$= Q\left[\sqrt{\frac{A_0^2}{\sigma^2}}, \sqrt{2\ln\left(\frac{1}{P_{fa}}\right)}\right]$$

$$Q(\alpha,\beta) = \int_{\beta}^{\infty} \xi I_0(\alpha\xi) e^{-(\xi^2+\alpha^2)/2} d\xi$$

Definition

$$P_d = Q\left[\sqrt{\frac{A_0^2}{\sigma^2}}, \sqrt{2\ln\left(\frac{1}{P_{fa}}\right)}\right]$$

$$Q(\alpha, \beta) = \int_\beta^\infty \xi I_0(\alpha\xi) e^{-(\xi^2+\alpha^2)/2} d\xi$$

$$P_d \approx F\left[\frac{A_0^2}{\sigma^2} - \sqrt{2\ln\left(\frac{1}{P_{fa}}\right)}\right]$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^\infty e^{-\xi^2/2} d\xi$$