Phase Locked Loop Systems

Young W Lim

March 5, 2020

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Definition

$$e_L(t) = K_P\left[\theta_1(t_1) - \theta_2(t)\right]$$

Definition

$$VCO_{output} = A_V \cos[\omega_0 t + \theta_0 + \theta_V(t)]$$

$$= A_V \cos\left[\omega_0 t + \theta_0 + k_V \int S_R(t)dt\right]$$

$$= A_V \cos[\theta_2(t)]$$

$$\theta_2(t) = \omega_0 t + \theta_0 + k_V \int S_R(t)dt$$

$$\theta_V(t) = k_V \int S_R(t)dt$$

Definition

$$\theta_1(t) = \omega_0 t + \theta_0 + \theta_1(t)$$

$$\theta_2(t) = \omega_0 t + \theta_0 + k_V \int S_R(t)dt$$

$$e_L(t) = K_P\left[\theta_1(t_1) - \theta_2(t)\right]$$

$$= K_P\left[\omega_0 t + \theta_0 + \theta_i(t) - \omega_0 t - \theta_0 - k_V \int S_R(t)dt\right]$$

$$= K_P\left[\theta_i(t) - k_V \int S_R(t)dt\right]$$

Definition

$$e_L(t) \Longleftrightarrow E_L(\omega)$$

$$\theta_i(t) \Longleftrightarrow \Theta_i(\omega)$$

$$Z_R(t) \Longleftrightarrow Z_R(\omega)$$

$$E_L(\omega) = K_P \left[\Theta_i(\omega) - \frac{k_V Z_R(\omega)}{j\omega} \right]$$

Loop Transfer Function (4)

N Gaussian random variables

Definition

$$E_L(\omega) = \frac{Z_R(\omega)}{H_L(\omega)}$$

$$H_T(\omega) = \frac{Z_R(\omega)}{\Theta_I(\omega)} = \frac{K_P j\omega H_L(\omega)}{j\omega + K_P k_V H_L(\omega)}$$

$$H_T(\omega) = \frac{j\omega}{k_V} H(\omega)$$

$$H(\omega) = \frac{K_P k_V H_L(\omega)}{j\omega + K_P k_V H_L(\omega)}$$

Definition

$$Z_R(\omega) = H_T(\omega)\Theta_i(\omega)$$

$$Z_R(t) = \int_{-\infty}^{\infty} h_T(t-\xi)\theta_i(\xi)d\xi$$

$$h_T(t) \Longleftrightarrow H_T(\omega)$$

Definition

$$N_i(t) = N_c(t)\cos(\omega_0 t) - N_s(t)\sin(\omega_0 t)$$

$$A_i \cos\left[\omega_0 t + \theta_0 + k_{FM}\int X(t)dt\right]$$

$$+N_c(t)\cos(\omega_0 t) - N_s(t)\sin(\omega_0 t)$$

Loop Noise Performance (2)

N Gaussian random variables

Definition

$$R(t)\cos[\omega_0 t + \theta_0 + \theta_{FM}(t) + \theta_N(t)]$$

$$\theta_{FM}(t) = k_{FM}\int X(t)dt$$

$$\theta_i(t) = \theta_{FM}(t) + \theta_N(t)$$

$$= \theta_{FM}(t) + \frac{N_c(t)}{A_i}$$

Loop Noise Performance (3)

N Gaussian random variables

Definition

$$S_{\theta_i\theta_i}(\omega) = \frac{k_{FM}^2 S_{XX}(\omega)}{\omega^2} + \frac{S_{N_cN_c}(\omega)}{A_i^2}$$

$$S_{Z_RZ_R}(\omega) = S_{\theta_i\theta_i}(\omega)\left|H_T(\omega)\right|^2$$

$$= S_{XX}(\omega)\left(\frac{k_{FM}}{k_V}\right)^2 \left|H_T(\omega)\right|^2 + S_{N_cN_c}(\omega)\frac{\omega^2}{A_i^2 k_V^2}\left|H_T(\omega)\right|^2$$

Definition

$$S_0 = \frac{1}{2\pi}\int_{-W_X}^{W_X} S_{N_c N_c}(\omega)\frac{\omega^2}{A_i^2 k_V^2}\left|H_T(\omega)\right|^2 d\omega$$

$$\approx \left(\frac{k_{FM}}{k_V}\right)^2 \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega) d\omega$$

$$= \left(\frac{k_{FM}}{k_V}\right)^2 \overline{X^2(t)}$$

Definition

$$N_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(\omega)\left(\frac{k_{FM}}{k_V}\right)^2 |H_T(\omega)|^2\, d\omega$$

$$\approx \frac{\mathcal{N}_0}{2\pi A_i^2 k_V^2}\int_{-W_X}^{W_X} \omega^2\, d\omega$$

$$= \frac{\mathcal{N}_0 W_X^3}{3\pi A_i^2 k_V^2}$$

Definition

$$S_0 == \left(\frac{k_{FM}}{k_V}\right)^2 \overline{X^2(t)}$$

$$N_0 = \frac{\mathcal{N}_0 W_X^3}{3\pi A_i^2 k_V^2}$$

$$A_i = AG_{ch}$$

$$\left(\frac{S_0}{N_0}\right)_{FM} = \frac{3\pi G_{ch}^2 A_{FM}^2 \overline{X^2(t)}}{\mathcal{N}_0 W_X^3}$$