DFT Frequency (9A)

- Each Row of the DFT Matrix
-

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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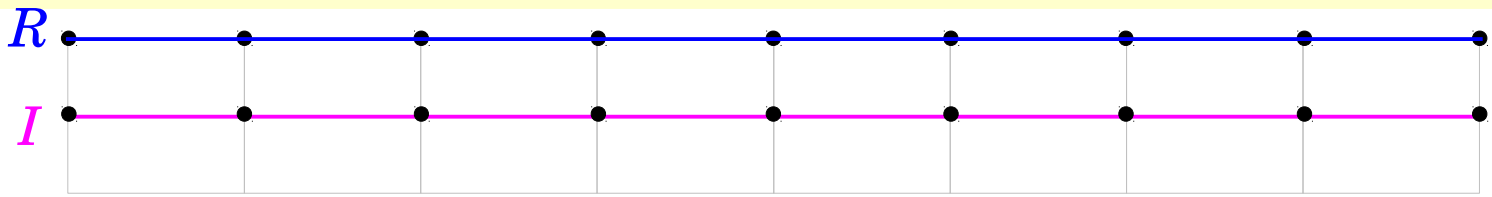
N=8 DFT : The 1st Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 0} \end{pmatrix}$$



0 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 0, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{0}{8})\cdot f_s\cdot t$$

X*[0]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

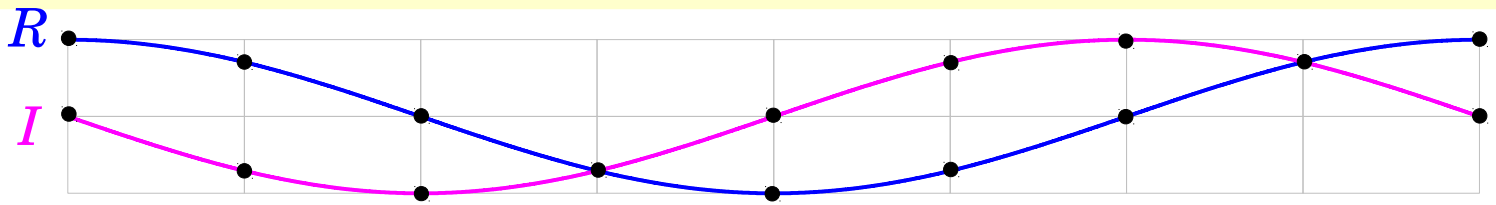
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

Zero Frequency

N=8 DFT : The 2nd Row of the DFT Matrix

$$\left(e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 1} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 3} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 5} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 7} \right)$$



1 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 1, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{1}{8})\cdot f_s\cdot t$$

X*[1]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

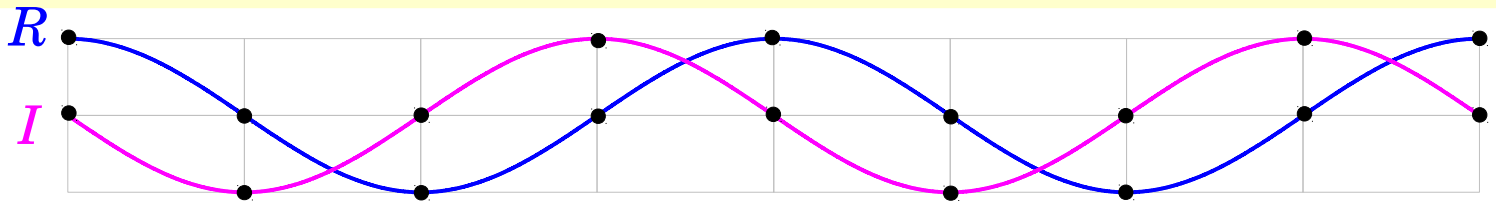
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

N=8 DFT : The 3rd Row of the DFT Matrix

$$\left(e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \right)$$



2 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{2}{8})\cdot f_s\cdot t$$

X*[2]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

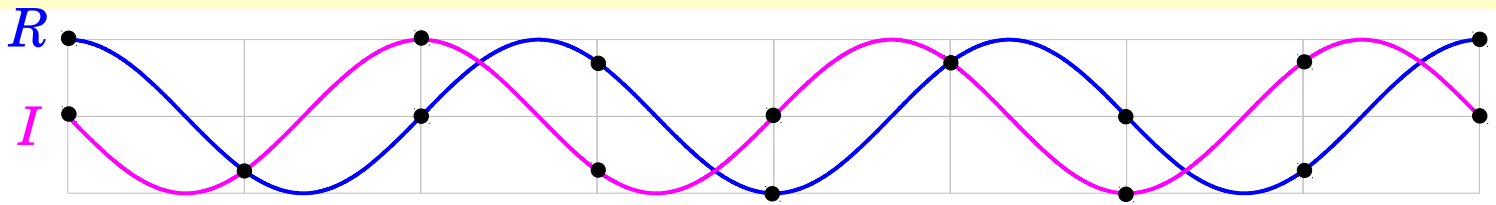
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : The 4th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 3} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 5} \end{pmatrix}$$



3 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 3, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{3}{8})\cdot f_s\cdot t$$

X*[3]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

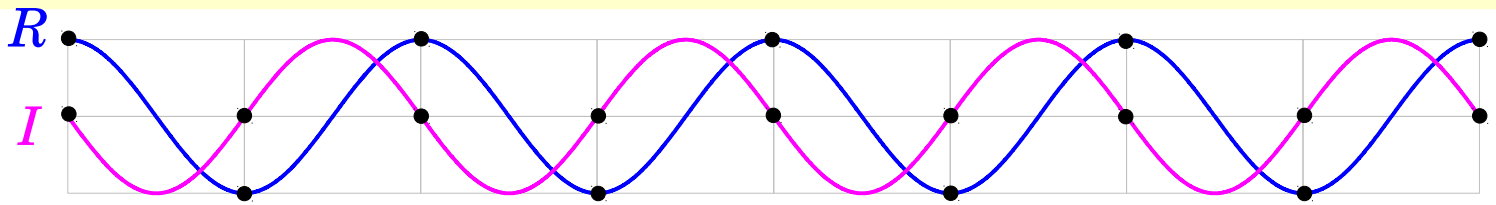
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

N=8 DFT : The 5th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 4} \end{pmatrix}$$



4 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 4, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi\cdot(\tfrac{4}{8})\cdot f_s\cdot t$$

X[4] *measures how much of the above signal component is present in* ***x***.

Sampling Time τ

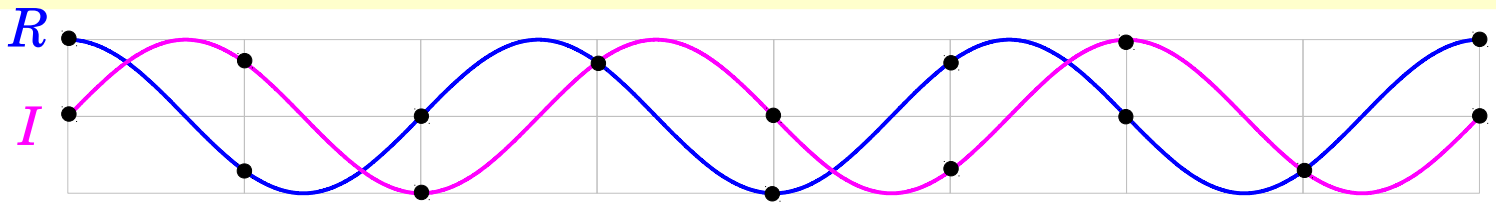
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

4th Harmonic Freq $f_4 = \frac{4}{T} = \frac{4}{N\tau} = \frac{4 f_s}{N}$

N=8 DFT : The 6th Row of the DFT Matrix

$$\begin{pmatrix} e^{-j\cdot\frac{\pi}{4}\cdot 0} & e^{-j\cdot\frac{\pi}{4}\cdot 5} & e^{-j\cdot\frac{\pi}{4}\cdot 2} & e^{-j\cdot\frac{\pi}{4}\cdot 7} & e^{-j\cdot\frac{\pi}{4}\cdot 4} & e^{-j\cdot\frac{\pi}{4}\cdot 1} & e^{-j\cdot\frac{\pi}{4}\cdot 6} & e^{-j\cdot\frac{\pi}{4}\cdot 3} \end{pmatrix}$$



3 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 5, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$
$$2\pi\cdot(\tfrac{-3}{8})\cdot f_s\cdot t$$

X*[5]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

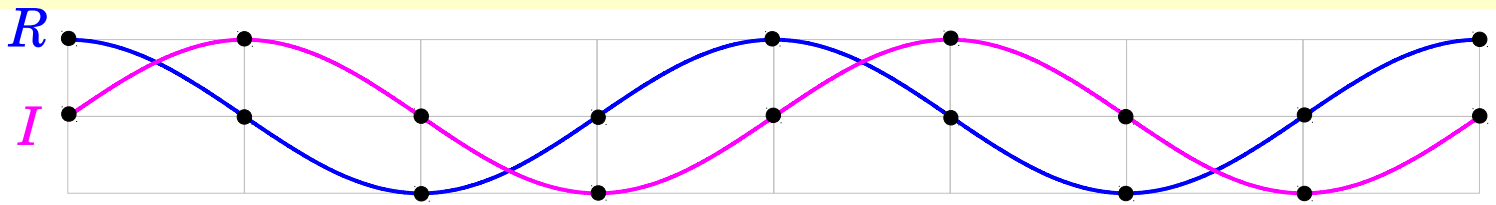
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

3rd Harmonic Freq $f_3 = \frac{3}{T} = \frac{3}{N\tau} = \frac{3 f_s}{N}$

N=8 DFT : The 7th Row of the DFT Matrix

$$\left(e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \right)$$



2 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$
$$2\pi\cdot(\tfrac{-2}{8})\cdot f_s\cdot t$$

X*[6]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

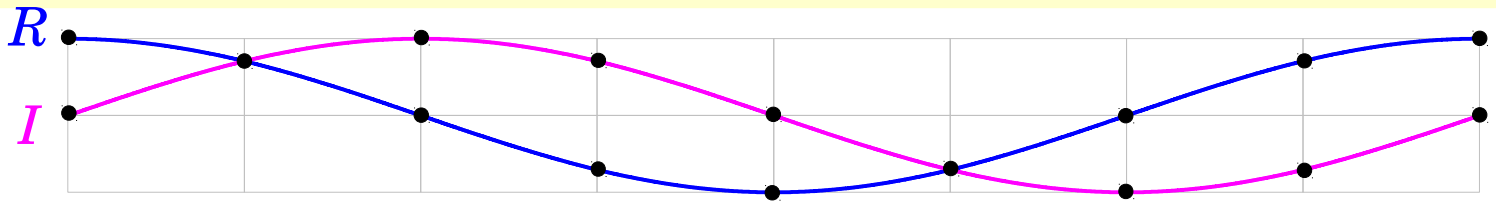
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : The 8th Row of the DFT Matrix

$$\left(e^{-j\cdot\frac{\pi}{4}\cdot 0} \quad e^{-j\cdot\frac{\pi}{4}\cdot 7} \quad e^{-j\cdot\frac{\pi}{4}\cdot 6} \quad e^{-j\cdot\frac{\pi}{4}\cdot 5} \quad e^{-j\cdot\frac{\pi}{4}\cdot 4} \quad e^{-j\cdot\frac{\pi}{4}\cdot 3} \quad e^{-j\cdot\frac{\pi}{4}\cdot 2} \quad e^{-j\cdot\frac{\pi}{4}\cdot 1} \right)$$



1 cycle

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 7, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(\omega' t) = \cos(-(-\omega)t)$

I ➡ *sampled values of* $\sin(\omega' t) = \sin(-(-\omega)t)$

$$-\omega t = -2\pi f t$$
$$2\pi\cdot(\tfrac{-1}{8})\cdot f_s\cdot t$$

X*[7]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

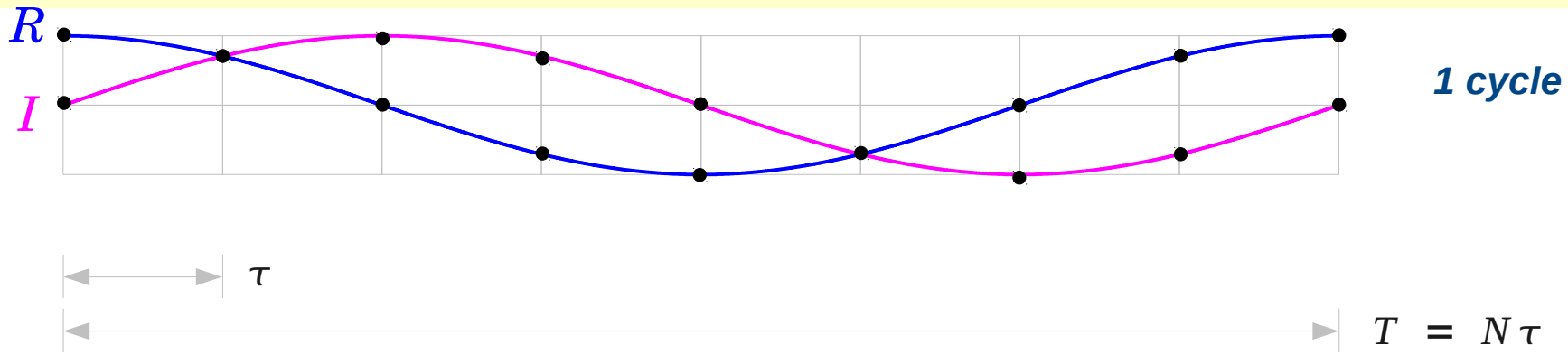
Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

Fundamental Frequency

$$\left(e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \right)$$



1 cycle

Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \frac{1}{\tau}$

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

$$f_2 = 2\cdot f_1$$
$$f_3 = 3\cdot f_1$$
$$\cdots$$
$$f_{N-1} = (N-1)\cdot f_1$$

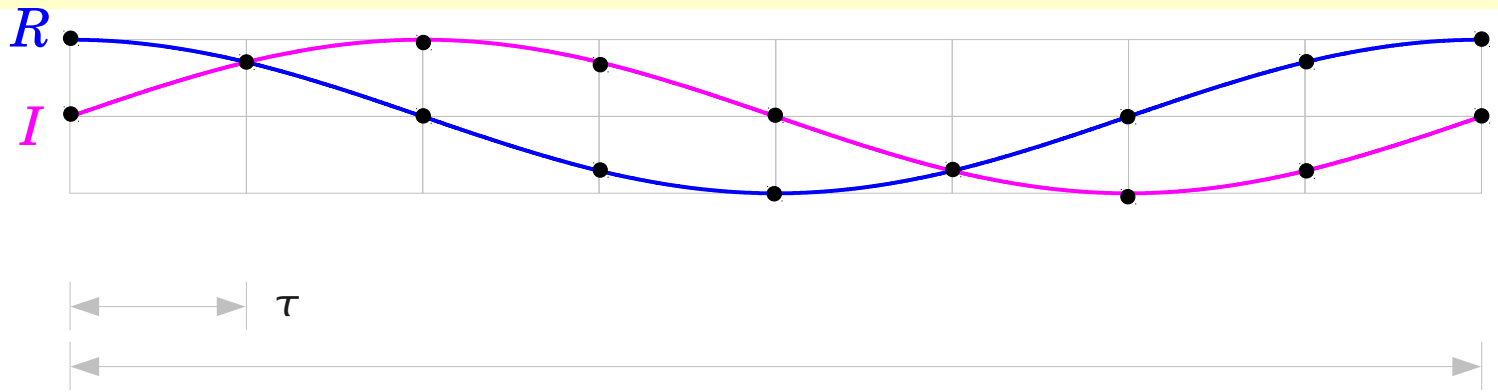
Fundamental Frequency f_0

The Lowest Frequency in a harmonic series.

$$f_0 = f_1 = \frac{f_s}{N}$$

Normalized Frequency

$$\left[\, e^{+j\cdot\frac{\pi}{4}\cdot 0} \quad e^{+j\cdot\frac{\pi}{4}\cdot 1} \quad e^{+j\cdot\frac{\pi}{4}\cdot 2} \quad e^{+j\cdot\frac{\pi}{4}\cdot 3} \quad e^{+j\cdot\frac{\pi}{4}\cdot 4} \quad e^{+j\cdot\frac{\pi}{4}\cdot 5} \quad e^{+j\cdot\frac{\pi}{4}\cdot 6} \quad e^{+j\cdot\frac{\pi}{4}\cdot 7} \,\right]$$



1 cycle

Sampling Time τ

Sequence Time Length $T = N\tau$

Sampling Frequency $f_s = \frac{1}{\tau}$ *(samples per second)*

1st Harmonic Freq $f_1 = \frac{1}{T} = \frac{1}{N\tau} = \frac{f_s}{N}$

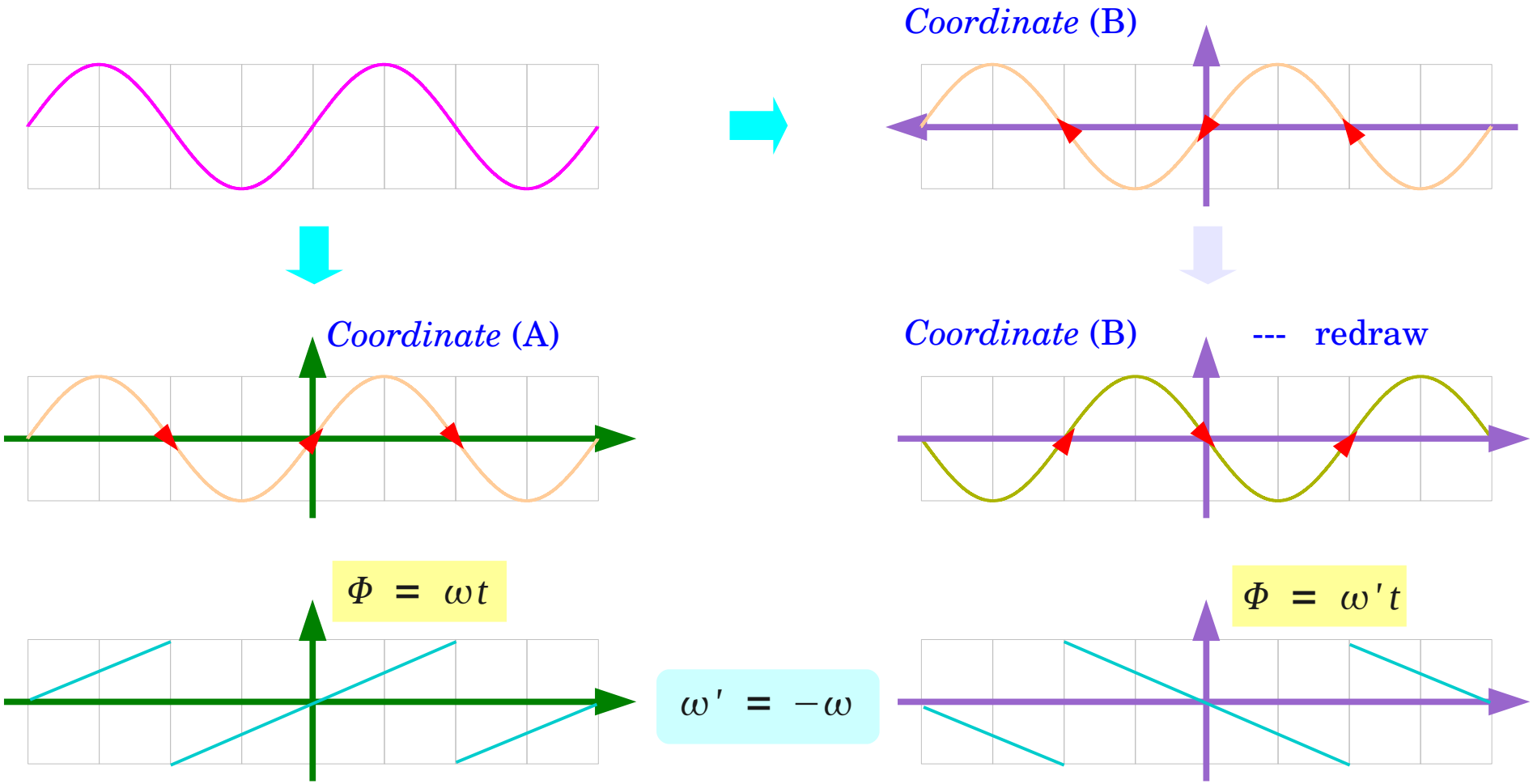
$$f_1 = 1\cdot f_1 \qquad 1/N$$
$$f_2 = 2\cdot f_1 \qquad 2/N$$
$$f_3 = 3\cdot f_1 \qquad 3/N$$
$$\ldots$$
$$f_{N-1} = (N-1)\cdot f_1 \qquad (N-1)/N$$

$$n = 0,\ 1,\ 2,\ \ldots,\ N-1$$

Normalized Frequency *(cycles per sample)*

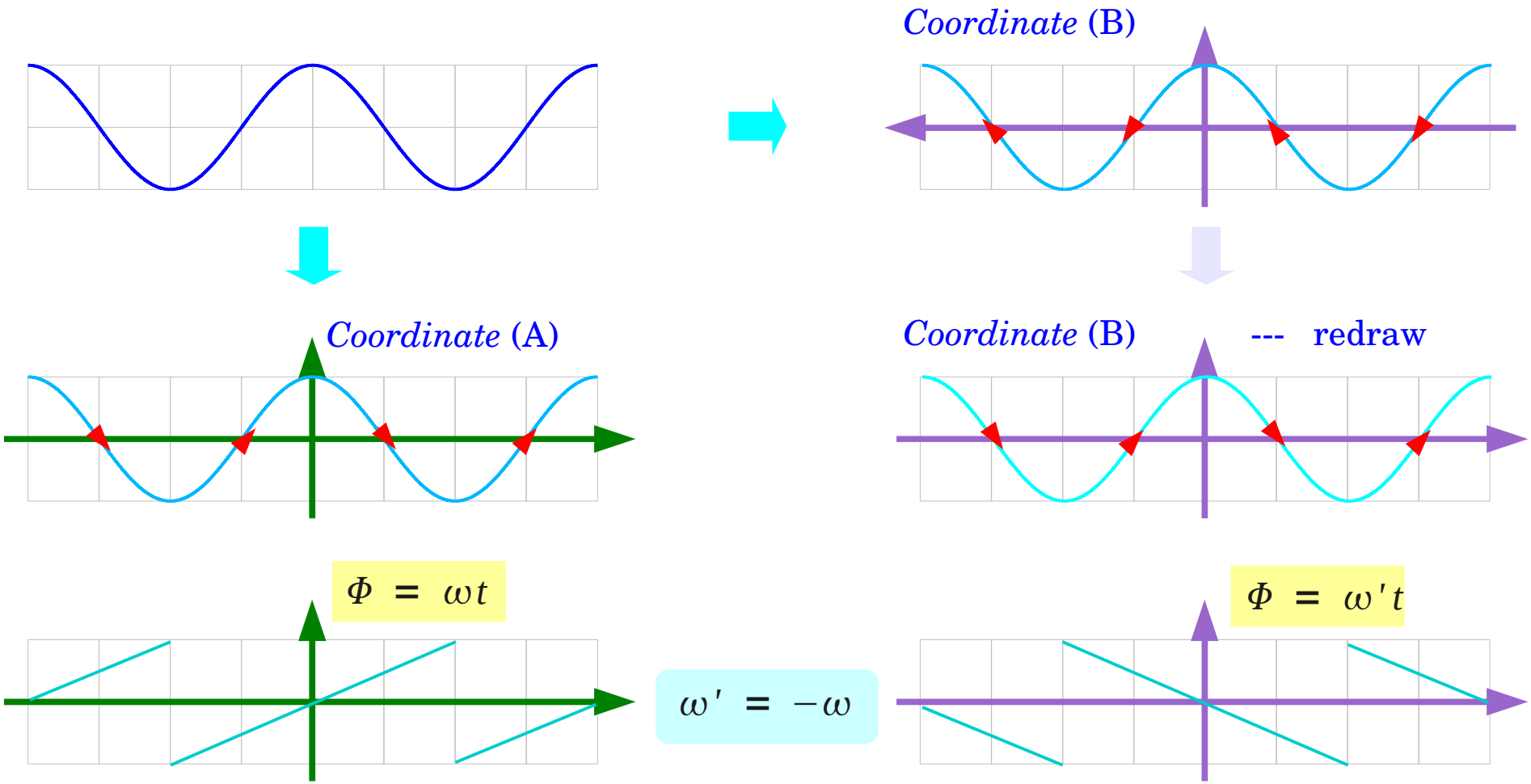
$$f_n = \frac{n\cdot f_s}{N} \quad \Rightarrow \quad \frac{f_n}{f_s} = \frac{n}{N}$$

Negative Frequency (1)



$$\sin(\omega' t) = -\sin(-\omega t)$$

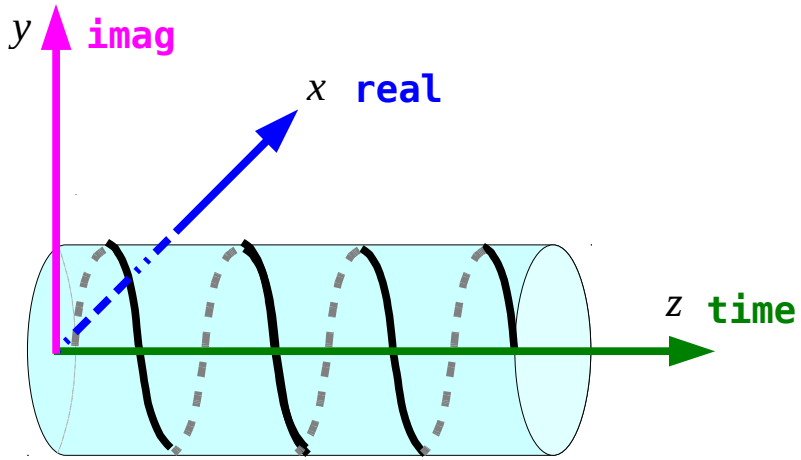
Negative Frequency (2)

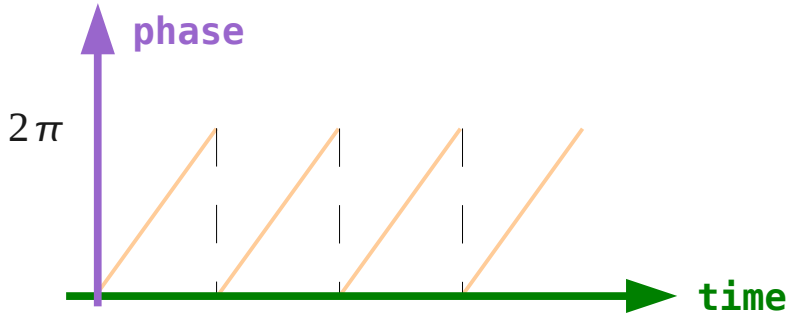


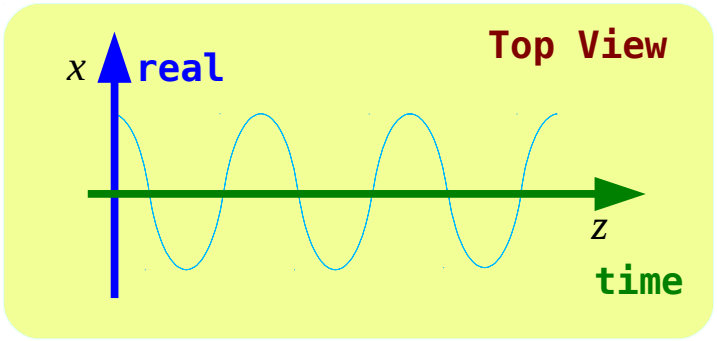
$$\cos(\omega' t) = \cos(-\omega t)$$

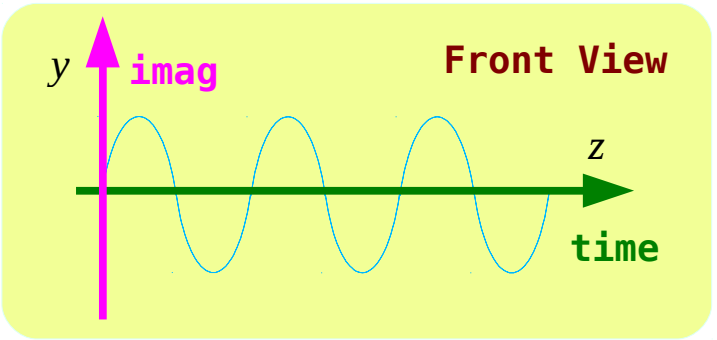
Euler Equation (1)

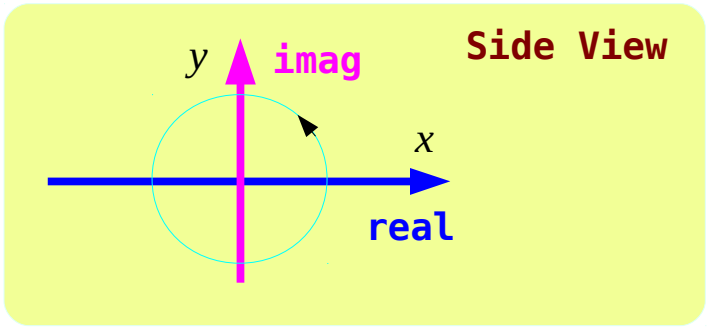
$$e^{+j\omega t} = \cos\omega t + j\sin\omega t$$



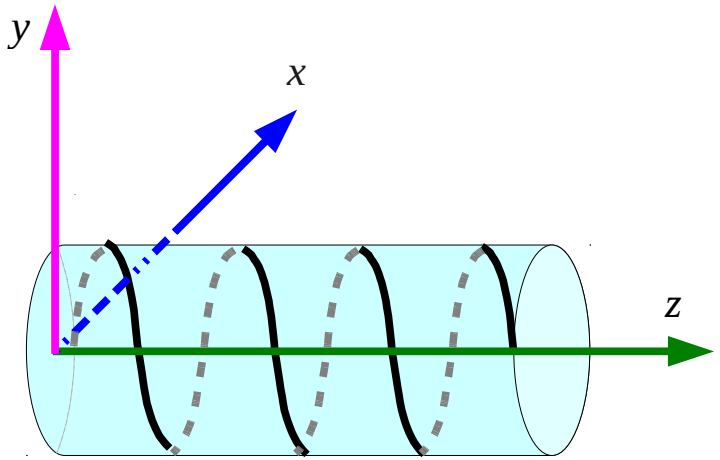


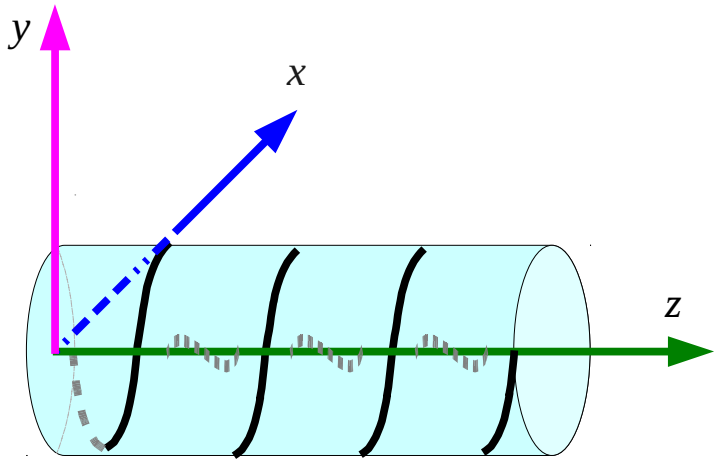




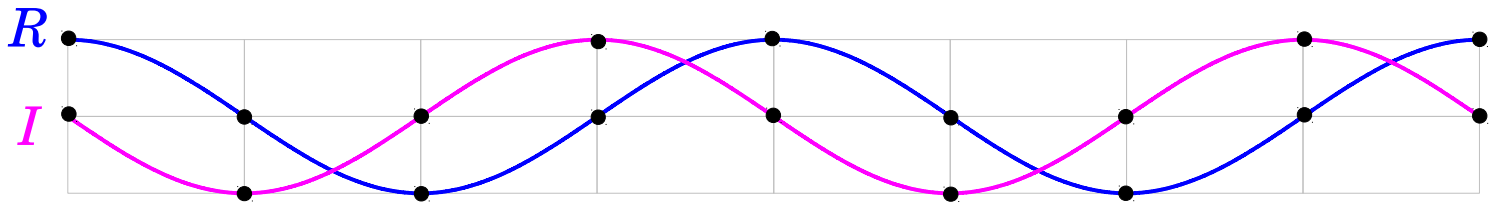


Negative Frequency (3)





Negative Frequency



2 cycles

$$W_8^{kn} = e^{-j(\frac{2\pi}{8})kn} \qquad k = 2, \quad n = 0, 1, \ldots, 7$$

R ➡ *sampled values of* $\cos(-\omega t) = \cos(\omega t)$

I ➡ *sampled values of* $\sin(-\omega t) = -\sin(\omega t)$

$$\omega t = 2\pi f t$$
$$2\pi \cdot (\tfrac{2}{8}) \cdot f_s \cdot t$$

X*[2]** *measures how much of the above signal component is present in* ***x.

Sampling Time τ

Sampling Frequency $f_s = \frac{1}{\tau}$

Sequence Time Length $T = N\tau$

2nd Harmonic Freq $f_2 = \frac{2}{T} = \frac{2}{N\tau} = \frac{2 f_s}{N}$

N=8 DFT : DFT Matrix in + or – Frequencies

$$\omega_0 = 2\pi \cdot \frac{f_s}{N}$$

0th row:	*samples of*	$\cos(\ \ 0\omega_0)t + j \cdot \sin(\ \ 0\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(-5\omega_0)t + j \cdot \sin(-5\omega_0)t$	(5 cycles)
6th row:	*samples of*	$\cos(-6\omega_0)t + j \cdot \sin(-6\omega_0)t$	(6 cycles)
7th row:	*samples of*	$\cos(-7\omega_0)t + j \cdot \sin(-7\omega_0)t$	(7 cycles)

=

0th row:	*samples of*	$\cos(\ \ 0\omega_0)t + j \cdot \sin(\ \ 0\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(+7\omega_0)t + j \cdot \sin(+7\omega_0)t$	(7 cycles)
2th row:	*samples of*	$\cos(+6\omega_0)t + j \cdot \sin(+6\omega_0)t$	(6 cycles)
3th row:	*samples of*	$\cos(+5\omega_0)t + j \cdot \sin(+5\omega_0)t$	(5 cycles)
4th row:	*samples of*	$\cos(+4\omega_0)t + j \cdot \sin(+4\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(+3\omega_0)t + j \cdot \sin(+3\omega_0)t$	(3 cycles)
6th row:	*samples of*	$\cos(+2\omega_0)t + j \cdot \sin(+2\omega_0)t$	(2 cycles)
7th row:	*samples of*	$\cos(+1\omega_0)t + j \cdot \sin(+1\omega_0)t$	(1 cycles)

N=8 DFT : DFT Matrix in Both Frequencies

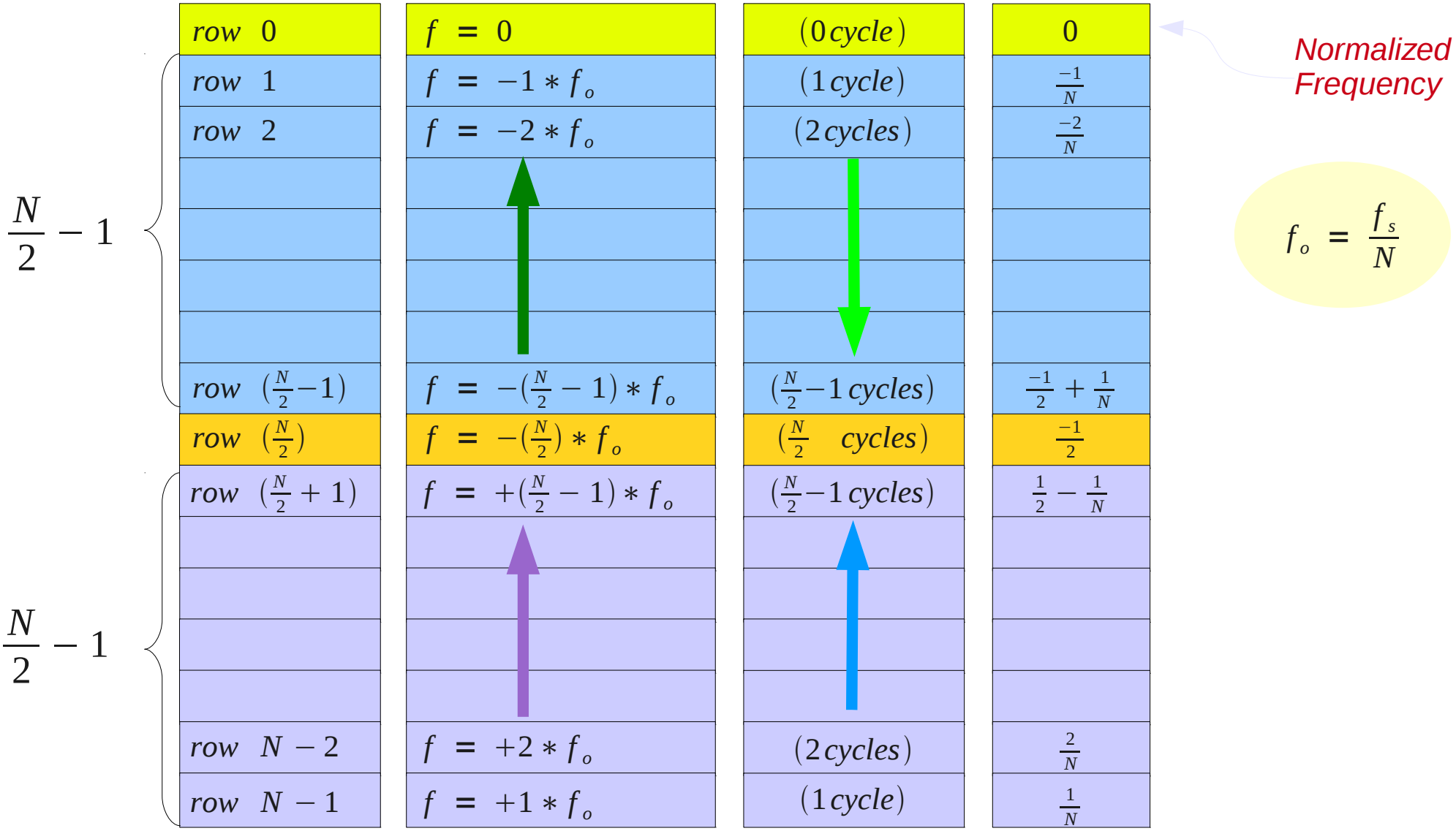
$$\omega_0 = 2\pi \cdot \frac{f_s}{N}$$

0th row:	*samples of*	$\cos(\ \ 0\omega_0)t + j \cdot \sin(\ \ 0\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(-5\omega_0)t + j \cdot \sin(-5\omega_0)t$	(5 cycles)
6th row:	*samples of*	$\cos(-6\omega_0)t + j \cdot \sin(-6\omega_0)t$	(6 cycles)
7th row:	*samples of*	$\cos(-7\omega_0)t + j \cdot \sin(-7\omega_0)t$	(7 cycles)

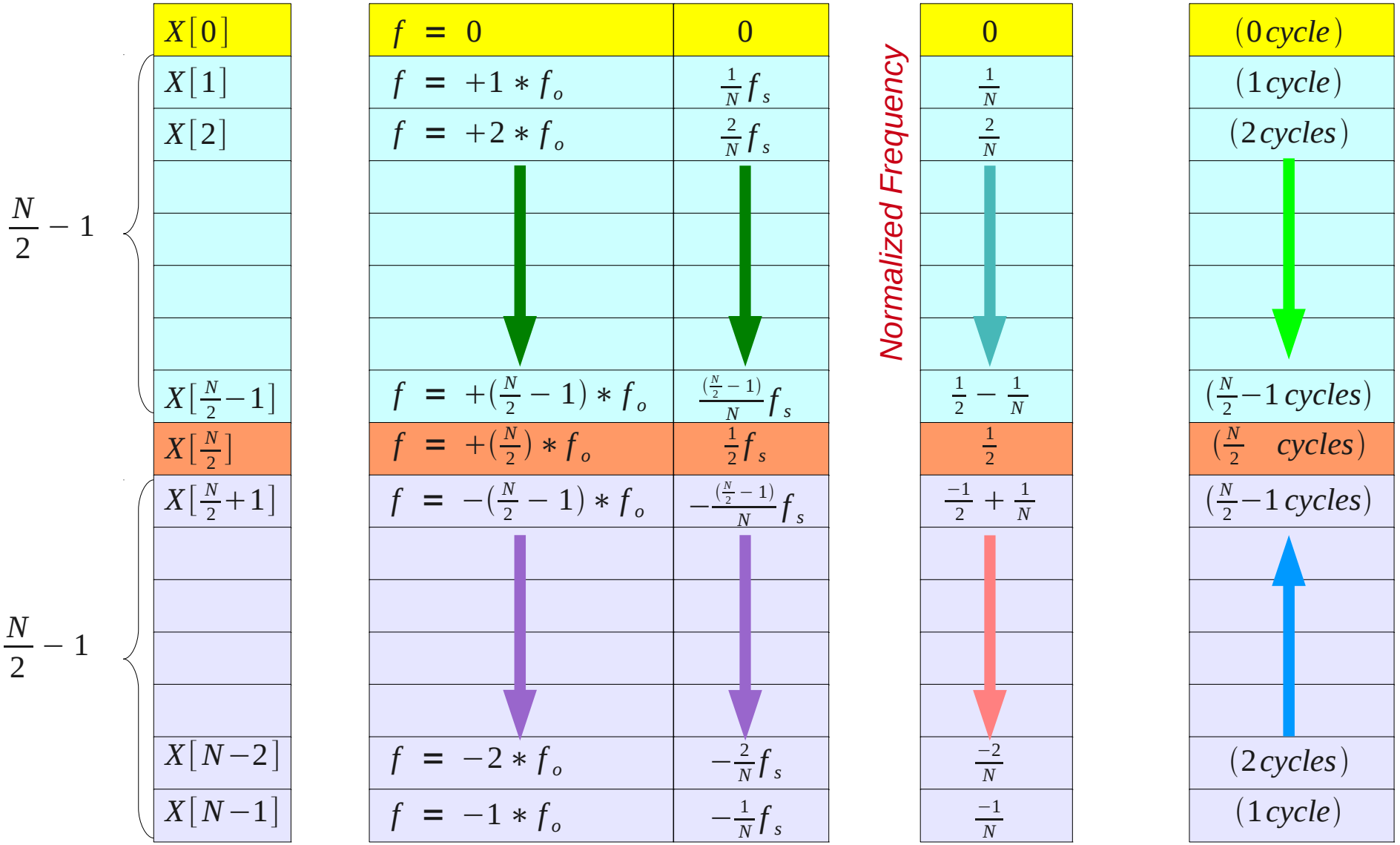
=

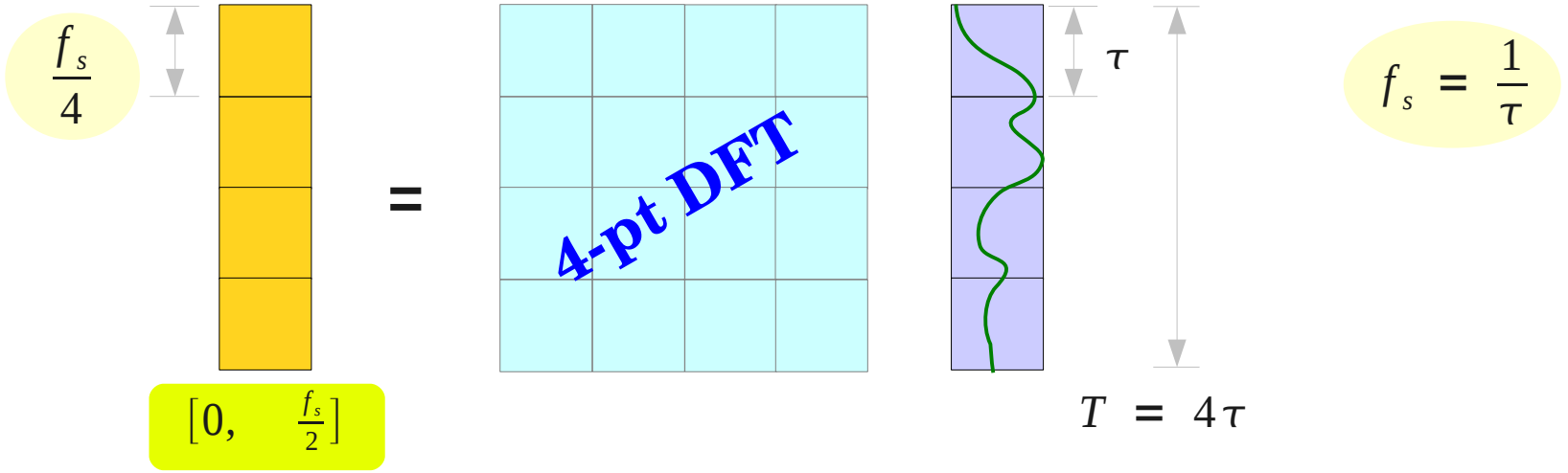
0th row:	*samples of*	$\cos(\ \ 0\omega_0)t + j \cdot \sin(\ \ 0\omega_0)t$	(0 cycle)
1th row:	*samples of*	$\cos(-1\omega_0)t + j \cdot \sin(-1\omega_0)t$	(1 cycle)
2th row:	*samples of*	$\cos(-2\omega_0)t + j \cdot \sin(-2\omega_0)t$	(2 cycles)
3th row:	*samples of*	$\cos(-3\omega_0)t + j \cdot \sin(-3\omega_0)t$	(3 cycles)
4th row:	*samples of*	$\cos(-4\omega_0)t + j \cdot \sin(-4\omega_0)t$	(4 cycles)
5th row:	*samples of*	$\cos(+3\omega_0)t + j \cdot \sin(+3\omega_0)t$	(3 cycles)
6th row:	*samples of*	$\cos(+2\omega_0)t + j \cdot \sin(+2\omega_0)t$	(2 cycles)
7th row:	*samples of*	$\cos(+1\omega_0)t + j \cdot \sin(+1\omega_0)t$	(1 cycles)

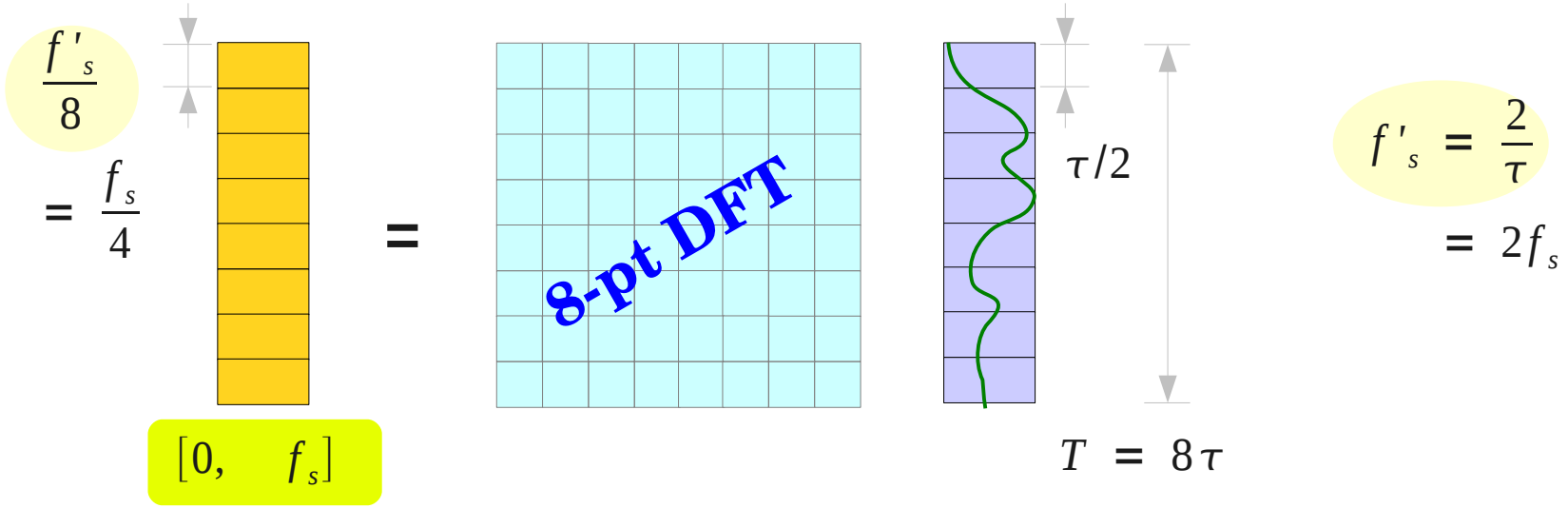
Frequency View of a DFT Matrix

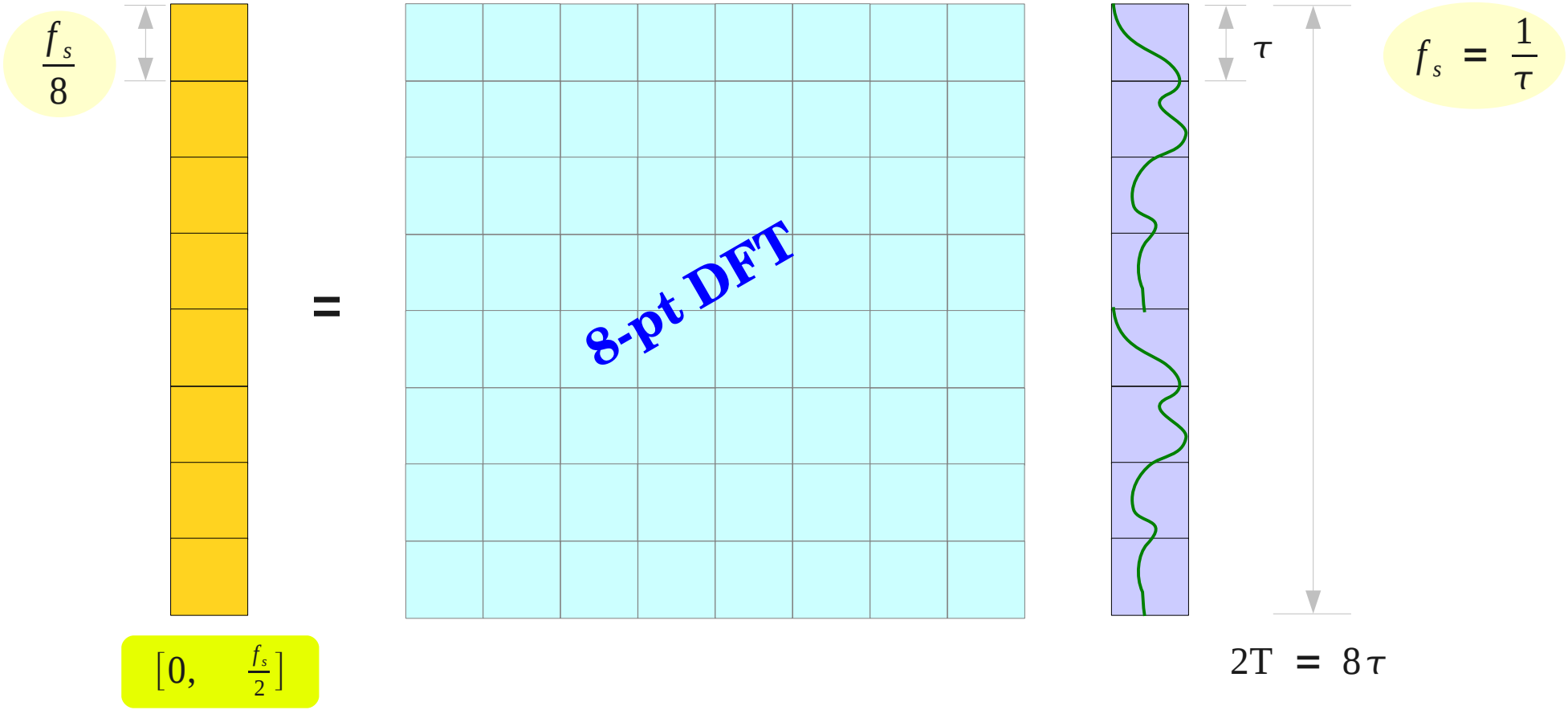


Frequency View of a X[i] Vector




$\frac{f_s}{4}$
=
4-pt DFT
τ
$f_s = \frac{1}{\tau}$
$[0, \quad \frac{f_s}{2}]$
$T = 4\tau$


$\frac{f'_s}{8}$
$= \frac{f_s}{4}$
=
8-pt DFT
$\tau/2$
$f'_s = \frac{2}{\tau}$
$= 2f_s$
$[0, \quad f_s]$
$T = 8\tau$


$\frac{f_s}{8}$
=
8-pt DFT
τ
$f_s = \frac{1}{\tau}$
$[0, \quad \frac{f_s}{2}]$
$2T = 8\tau$

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, by Steve Mann