

# Multiple Random Variables

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi

# Outline

## 1 Sum of Random Variables

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# Sum of two random variables

## Definition

Let  $W$  be a random variable equal to the sum of two independent random variables  $X$  and  $Y$

$$W = X + Y$$

the probability distribution function is defined by

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$$

$$x + y \leq w$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y) dx dy$$

# Convolution of density functions

## Definition

Let  $W = X + Y$

the probability distribution function is defined by

$$F_W(w) = \int_{-\infty}^{\infty} \int_{x=-\infty}^{w-y} f_{X,Y}(x,y) dx dy$$

$$f_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x) dx$$

the density function of the sum of  
two statistically independent random variables  
is the convolution of their individual density functions

## Sum of two random variables (1)

## Definition

$$F_W(w) = P\{W \leq w\} = P\{X + Y \leq w\}$$

$$x + y \leq w$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_{X,Y}(x,y) dy dx$$

$$f_W(w) = \frac{\partial}{\partial w} F_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x) dx$$

## Sum of two random variables (2)

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_{X,Y}(x,y) dy dx$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{y=-\infty}^{w-x} f_X(x) f_Y(y) dy dx$$

$$F_W(w) = \int_{-\infty}^{\infty} \int_{-\infty}^w f_X(x) f_Y(t-x) dt dx$$

$$F_W(w) = \int_{-\infty}^w \int_{-\infty}^{\infty} f_X(x) f_Y(t-x) dx dt$$

$$f_W(w) = \frac{\partial}{\partial w} F_W(w) = \int_{-\infty}^{\infty} f_X(x) f_Y(t-x) dx$$

<https://math.stackexchange.com/questions/1919545/sum-of-random-variables-and-convolution>



## Sum of several random variables (1)

## Definition

the sum  $Y$  of 3 independent random variables  $X_1, X_2, X_3$

Let  $Y_1 = X_1 + X_2$ , then

$$f_{Y_1}(y_1) = f_{X_2}(x_2) * f_{X_1}(x_1)$$

Let  $Y_2 = X_3 + Y_1$ , then

$$f_{Y_2}(y_2) = f_{X_3}(x_3) * f_{Y_1}(y_1) = f_{X_3}(x_3) * f_{X_2}(x_2) * f_{X_1}(x_1)$$

## Sum of several random variables (2)

## Definition

the sum  $Y$  of  $N$  independent random variables  $X_1, X_2, \dots, X_N$

Let  $Y = X_1 + X_2 + \dots + X_N$ , then

$$f_Y(y) = f_{X_N}(x_N) * f_{X_{N-1}}(x_{N-1}) * \dots * f_{X_1}(x_1)$$

